Exercises for Nuclear Astrophysics I
WS 2011/12 - Prof. Shawn Bishop
Sheet 9

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Problems will be discussed in exercises on 2/9/2012 at 13:00.

The topic of this exercise sheet are resonances. For a narrow resonance, we can assume that the Maxwell-Boltzmann factor at the partial widths are approximately constant over the total width of the resonance. Thus, the reaction rate for a single narrow resonance can be written as

$$N_A \langle \sigma v \rangle = N_A \left(\frac{2\pi}{\mu k T} \right)^{3/2} \hbar^2 e^{-E_r/kT} \omega\gamma \tag{1}$$

where μ is the reduced mass of the particles in the entrance channel, E_r is the resonance energy, and $\omega\gamma$ is the resonance strength.

19 Narrow s-wave resonances in the ^{20}Ne(p,γ)^{21}Na reaction

Suppose that four hypothetical narrow s-wave resonances occur at low energies in the ^{20}Ne(p,γ)^{21}Na reaction. The resonance energies are $E_r = 10$ keV, 30 keV, 50 keV, 100 keV. The corresponding resonance strengths are $\omega\gamma = 7.24 \cdot 10^{-33}$ eV, $3.81 \cdot 10^{-15}$ eV, $1.08 \cdot 10^{-9}$ eV, and $3.27 \cdot 10^{-4}$ eV.

(a) Calculate the position E_0 and width Δ of the Gamow-Peak for both temperatures. Which resonance do you expect to dominate the total reaction rates at $T_9 = 0.02$ and $T_9 = 0.08$?

(b) Calculate the reation rates numerically to confirm the result of (a).

20 Temperature dependence of a single resonance

Similar to the derivation of the temperature dependence of non-resonant reaction rates in problem 15, we can obtain a power-law expression for resonant reaction rates.

(a) Start from the power-law

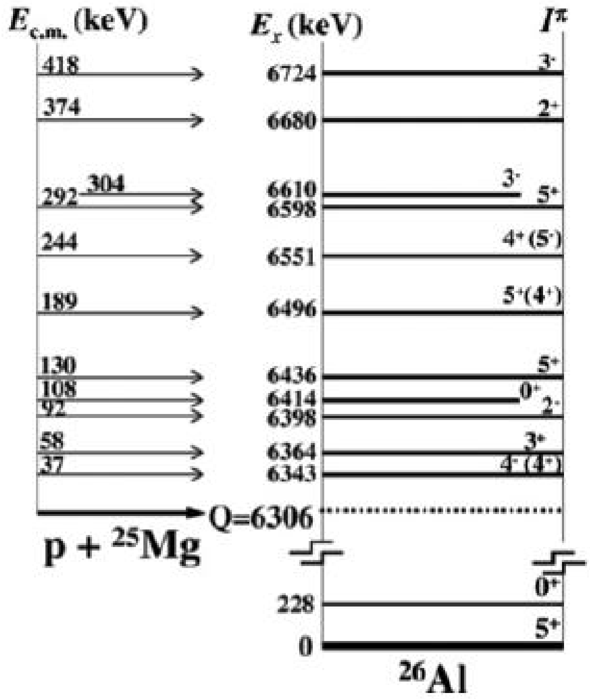
$$\langle \sigma v \rangle_T = \langle \sigma v \rangle_{T_0} \cdot \left(\frac{T}{T_0} \right)^n \tag{2}$$

where T_0 is an arbitrary temperature. Calculate the logarithmic derivative of the reaction rate with respect to temperature to show that the exponent n can be written as $n = 11.605 E_r/T_9 - 3/2$, where E_r is given in MeV.

(b) Calculate the 8 different values n for the 4 resonances and 2 temperatures in problem 19.

21 Resonance strengths in ^{26}Al

Over the last decades, ^{26}Al has become a key isotope in nuclear astrophysics and γ-ray astronomy. ^{26}Al is radioactive and its ground state decays via β^+ and electron capture ($t_{1/2} = 0.716$ s) to the 2^+ first excited state in ^{26}Mg, which deexcites via emission of a 1.809 MeV photon. ^{26}Al can be created by proton capture on ^{25}Mg.



The diagram shows the level scheme of ^{26}Al. Plotted are the ground state, the isomeric state at 228 keV and excited levels above the ^{25}Mg+p threshold (Q = 6306 keV) with corresponding center-of-mass resonance energies ($E_{c.m.}$).

a) Consider the following astrophysical scenarios:

(i) Nova-explosion

(ii) Pre-SN star

(iii) Asymptotic Giant Branch (AGB) star

Estimate the temperature for each of these cases and calculate the center E_0 and width Δ of the Gamow-peak for proton capture on ^{25}Mg. Use this information and the diagram on the left to find out which resonances contribute primarily to the production of ^{26}Al for each of the scenarios.

(b) * Can you think of experimental ways to measure the resonance strengths of the resonances in the diagram? How can a sample of the unstable ^{26}Al be created in the first place? How can extremely small amounts of ^{26}Al be detected in the laboratory? (You will learn more about the experimental techniques in the second half of the course.)