Exercises for Nuclear Astrophysics I - WS 2011/12
Sheet 7

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Problems will be discussed Thursday 01/19/2012 at 13:00.

15 Reciprocity Theorem

A nuclear reaction rate is related to its inverse reaction by the reciprocity theorem. For a reaction $a + A \to b + B$ it can be written as

$$\frac{p_{aA}^2 (2j_A + 1)(2j_a + 1)\sigma_{aA\to bB}}{1 + \delta_{aA}} = \frac{p_{bB}^2 (2j_B + 1)(2j_b + 1)\sigma_{bB\to aA}}{1 + \delta_{bB}} \tag{1}$$

(a) Show that for massive particle a and A, it follows that the ratio of backward to forward reaction rate is given by

$$\frac{\langle\sigma v\rangle_{bB\to aA}}{\langle\sigma v\rangle_{aA\to bB}} = \frac{(2j_a + 1)(2j_A + 1)(1 + \delta_{bB})}{(2j_b + 1)(2j_B + 1)(1 + \delta_{aA})} \left(\frac{m_{aA}}{m_{bB}}\right)^{3/2} e^{-Q_{aA\to bB}/kT} \tag{2}$$

where $Q_{aA\to bB}$ is the Q-value of the foreward direction of the reaction, which can be written using the kinetic energies as $Q_{aA\to bB} = E_{bB} - EaA$.

(b) * Now consider a photo-disintegration reaction $A + \gamma \to b + B$. The photodisintegration decay constant is given by the expression

$$\lambda_\gamma = \frac{8\pi}{h^3 c^2} \int_{E_t}^{\infty} \frac{E_\gamma^2}{e^{E_\gamma/kT} - 1} \sigma(E_\gamma)\, dE_\gamma \tag{3}$$

where $E_t = Q_{bB\to a\gamma}$ is the threshold energy for the reation (most photodisintegration reactions are endothermic) and E_γ is the photon energy. Show that in this case, the ratio of the reaction rates can be written as

$$\frac{\lambda_\gamma}{\langle\sigma v\rangle_{bB\to A\gamma}} = \left(\frac{2\pi}{h^2}\right)^{3/2} (m_{bB} kT)^{3/2} \frac{(2j_b + 1)(2j_B + 1)}{(2j_A + 1)(1 + \delta_{bB})} e^{-Q_{bB\to A\gamma}/kT} \tag{4}$$

Hint: You can make the assumption that $e^{E_\gamma/kT} - 1 \approx e^{E_\gamma/kT}$ for $E_\gamma >> kT$.

16 Abundance evolution of ^{25}Al

There are several processes that can influence the abundance of an isotope X in a star over time. In general, we can write

$$\frac{d(N_X)}{dt} = \sum_i \pm r_i \tag{5}$$

where we have to sum (+) over all production mechanisms and substract all destruction mechanisms (-) with individual reaction rates r_i.

(a) Find possible production and destruction mechanisms ^{25}Al using the section of the nuclide chart below.

(b) Write down the explicit expression for the time evolution of the ^{25}Al abundance $\frac{dN(^{25}\mathrm{Al})}{dt}$ using your result from (a).

(c) Now we consider two specific reactions that can destroy the nucleus ^{25}Al: the capture reaction ^{25}Al(p,γ)^{26}Si and β^{+}-decay. Neglecting other processes, determine the dominant destruction process at a stellar temperature of $T = 0.3$ GK assuming a proton capture rate of $N_A \langle \sigma v \rangle = 1.8 \cdot 10^{-3}$ cm^3 mol^{-1} s^{-1} and give the total life-time of ^{25}Al with respect to both processes. Assume a stellar density of $\rho = 10^4$ g/cm^3 and a hydrogen mass fraction of $X_H = 0.7$.

(d) Calculate at which temperature T the two processes would contribute equally to the destruction of ^{25}Al. To do this, assume that the beta decay rate is independent of T. Further, assume a non-resonant proton capture reaction rate to calculate the reaction's power law dependence index n first.