Exercises for Nuclear Astrophysics I - WS 2011/12
Sheet 6

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Problems will be discussed Thursday 01/12/2012 at 13:00.

12 General understanding of cross sections and reaction rates

(a) In the classical limit, the reaction cross section can be viewed as the geometrical area in which projectile and target can interact. Use the approximation for the nuclear radius $R = 1.3\ \text{fm} \cdot A^{1/3}$ to calculate the geometrical cross section (in barns) in the following cases:

(i) ^{236}Pu+^{236}Pu

(ii) $p + p$
Compare it to the crutial p-p chain reaction $p(p, e^+\nu)d$ with $\sigma \approx 10^{-20}$ b at $E_p = 2.0$ MeV and interpret the difference.

(b) Consider a reaction of the particles a and X, such that the reaction rate per unit volume is given by

$$r_{aX} = \frac{N_a N_X}{1+\delta_{aX}} \int_0^\infty v\sigma(v)\Phi(v)\,dv = \frac{N_a N_X}{1+\delta_{aX}} \langle\sigma v\rangle \tag{1}$$

Suppose $X = a$ and think of the density N_a as being divided into two components, half of which are stationary and half of which are moving with translational velocity v_0. Then the target density is $\frac{1}{2}N_a$, and the flux is $\frac{1}{2}N_a v_0$, giving the reation rate

$$r = \frac{1}{4}N_a^2 v_0 \sigma(v_0) \tag{2}$$

which, on the face of it, differs from equation (1) by a factor of 2. Resolve this discrepancy. Hint: What is $\Phi(v)$?

13 Nonresonant reaction rate for charged particle reactions

Consider the nuclear reaction rate per particle pair

$$\langle\sigma v\rangle = \left(\frac{8}{\pi\mu}\right)^{1/2} (kT)^{-3/2} \int_0^\infty \sigma(E) E \exp\left(-\frac{E}{kT}\right) dE. \tag{3}$$

Now consider a nonresonant reaction $X_1 + X_2 \rightarrow X_3 + X_4$, where X_1 and X_2 are nuclei.

(a) Introduce the astrophysical S-factor in the form

$$S(E) \equiv \sigma(E) E \exp \frac{2\pi Z_1 Z_2 e^2}{\hbar v}. \tag{4}$$

Show that the cross section can be written as

$$\sigma(E) = \frac{S(E)}{E} \exp\left(-\frac{b}{\sqrt{E}}\right) \tag{5}$$

where $b = 31.28\, Z_1 Z_2 \sqrt{A}\ keV^{1/2}$ and A is the reduced atomic weight: $A = \frac{A_1 A_2}{A_1 + A_2}$.

(b) This way, the cross section $\sigma(E)$ consists of three energy dependent terms. What effects does each of them take into account?

(c) Insert (5) into (3). The resulting integral for the reaction rate per particle pair $\langle\sigma v\rangle$ is non-trivial. Assuming that $S(E)$ is a slowly varying function of energy, use the approximation $S(E) \approx S_0$ =const. to show that the integrand has an extremal value at

$$E_0 = 1.220\,(Z_1^2 Z_2^2 A T_6^2)^{1/3}\ keV \tag{6}$$

where T_6 is the temperature in millions of degrees Kelvin.

(d) Approximate the integrand by a gaussian

$$\exp\left(-\frac{E}{kT} - \frac{b}{\sqrt{E}}\right) \approx C \cdot \exp\left[-\left(\frac{E - E_0}{\Delta/2}\right)^2\right] \tag{7}$$

and show that $C = \exp\left(-\frac{3E_0}{kT}\right)$. Also, match the second derivatives of the functions at E_0 to calculate the $1/e$-width $\Delta/2$ and show that

$$\Delta = 0.75\,(Z_1^2 Z_2^2 A T_6^5)^{1/6} \tag{8}$$

(e) Use the previous results to show that with the gaussian approximation, the reaction rate per particle pair can be written as

$$\langle\sigma v\rangle = \frac{7.20 \cdot 10^{-19}}{A Z_1 Z_2} S_0 \tau^2 e^{-\tau} \quad \frac{\mathrm{cm}^3}{\mathrm{s}} \tag{9}$$

where $\tau = 3E_0/(kT)$ and S_0 is in kev·b.

14 The reaction ^{12}C(p,γ)^{13}N

In the following, use the approximation $S_o = S(E_0)$. This factor can be calculated with the zero-energy intercept $S(0) = 1.20$ keV · b and the slope $dS/dE = 5.81 \cdot 10^{-3}$ b.

(a) Calculate the lifetime of a ^{12}C nucleus against proton capture in a stelar interior containing 80 percent hydrogen by weight and having a density of 15 g/cm^3 at a temperature of $T_6 = 30$. (Do not take screening effects by the electron plasma into account.) Hint: First use equation (9) to calculate the reaction rate per particle pair.

(b) We introduce the power law with some temperature T_0

$$\langle\sigma v\rangle_T = \langle\sigma v\rangle_{T_0} \cdot \left(\frac{T}{T_0}\right)^n . \tag{10}$$

Show that the exponent n is given by $n = (\tau - 2)/3$ and use this result to calculate the temperature dependence of the reaction.