Exercises for Nuclear Astrophysics I
WS 2011/12 - Prof. Shawn Bishop
Sheet 5

Peter Ludwig - Lehrstuhl E12 - peter.ludwig@ph.tum.de - Phone: 089/289/14273
No exercise session for this sheet, since I am on vacation, but feel free to email me if there are questions/problems, Peter.

10 Stellar model with linear density distribution

Consider the relatively simple model with the density distribution

$$\rho(r) = \rho_c \cdot (1 - x) \tag{1}$$

with $x = \frac{r}{R}$ where R is the radius of the star.

(a) Show by direct integration of the mass continuity relation that the mass $M(x)$ is given by

$$M(x) = 4\pi R^3 \rho_c \left(\frac{1}{3}x^3 - \frac{1}{4}x^4\right) \tag{2}$$

(b) Integrate the hydrostatic equilibrium equation

$$\frac{dP}{dr} = -G\frac{M(r)\rho(r)}{r^2} \tag{3}$$

to obtain the Pressure function

$$P(x) = -4\pi G R^2 \rho_c^2 \left(\frac{1}{6}x^2 - \frac{7}{36}x^3 + \frac{1}{16}x^4 - \frac{5}{144}\right) \tag{4}$$

(c) Use the ideal gas law to calculate the temperature $T(x)$ and show that the ratio of temperature to mean temperature is given by

$$\frac{T(x)}{T_c} = \frac{144}{5} \cdot \frac{\frac{1}{6}x^2 - \frac{7}{36}x^3 + \frac{1}{16}x^4 - \frac{5}{144}}{1 - x} \tag{5}$$

(d) Examine $\frac{T(x)}{T_c}$ for extreme values and draw conclucions on the stability of this configuration. What is the problem with the density distribution (1)?

11 A realistic model for our sun

Consider the Ansatz,

$$\frac{dP}{dr} = -\frac{4\pi}{3}G\rho_c^2 r \exp\left[-\left(\frac{r}{a}\right)^2\right] \tag{6}$$

where a is a scale length that is chosen here as $a = 0.2 \cdot R_\odot$.

(a) Integrate the Ansatz to derive an expression for the pressure $P(r)$ and show that the central pressure is $P(0) =: P_c = \frac{2\pi}{3}G\rho_c^2 a^2$.

(b) (*) Use the expression derived in problem 9 of exercise sheet 4

$$M^2(r_i) = \frac{8\pi}{G}\left[4\int_0^{r_i} P(r) r^3 dr - P(r_i) r_i^4\right] \tag{7}$$

and the Ansatz (6) to show that the mass $M(x)$ is given by

$$M(x) = \frac{4\pi}{3}\rho_c a^3 \Phi(x) \tag{8}$$

where $x = \frac{r}{a}$ and $\Phi(x) = \sqrt{6 - 3\exp\left(-x^2\right)\left(x^4 + 2x^2 + 2\right)}$.

(c) Determine an expression for the density $\rho(r)$ from the hydrostatic equilibrium equation (3) and confirm that in the limit $r \to 0$, $\frac{d\rho}{dr}$ goes to zero.

(d) Use the results from (a) and (c) to calculate the central values P_c and ρ_c to obtain the central temperature

$$T_c = 4.69 \cdot 10^6 \mu \frac{R_\odot}{a} \tag{9}$$

and estimate the parameter $\frac{R_\odot}{a}$ to match our sun.