

Exercises for Nuclear Astrophysics II - SS 2012

Sheet 8

Peter Ludwig - Lehrstuhl E12 - peter.ludwig@ph.tum.de - <http://www.nucastro.ph.tum.de/>
 Phone: 089/289/14273 - Discussion of this exercise: 07/06/2012 @ 09:30 a.m.

14 Explosive hydrogen burning

Explosive hydrogen burning nucleosynthesis proceeds beyond the CNO nuclei by breaking out of the hot CNO cycles from the $A < 20$ region via three possible breakout sequences leading to the nuclei ^{20}Na , ^{21}Na , and ^{20}Ne respectively. This is the starting point for this exercise, which is meant to illustrate the nucleosynthesis path and the concept of waiting point nuclei.

(a) **Choosing the path of nucleosynthesis.**

Imagine the following, for now general, situation in the nucleosynthesis path. A reaction converting nucleus A by particle capture to nucleus B ($A \rightarrow B$) exhibits a small Q -value. A larger Q -value occurs for the subsequent particle capture to C ($B \rightarrow C$). If the stellar plasma can attain sufficiently high temperatures, then the photodisintegration of nucleus B ($B \rightarrow A$) has to be taken into account and may alter the nucleosynthesis. Also, both A and B can beta decay ($A \rightarrow A'$, $B \rightarrow B'$).

- (i) Explain why the following two assumptions have to be valid for the nucleosynthesis to establish an equilibrium between the abundances of nuclei A and B:

$$\lambda_{A \rightarrow B} > \lambda_{A \rightarrow A'} \quad (1)$$

$$\lambda_{B \rightarrow A} > \lambda_{B \rightarrow C} + \lambda_{B \rightarrow B'} \quad (2)$$

- (ii) Assume in addition that the photodisintegration of C is negligible ($\lambda_{C \rightarrow C'} > \lambda_{C \rightarrow B}$). Show that decay constant for the nucleosynthesis path $A \rightarrow B \rightarrow (C \text{ or } B')$ can be written as:

$$\lambda_{A \rightarrow B \rightarrow (C \text{ or } B')} = \frac{\lambda_{A \rightarrow B}}{\lambda_{B \rightarrow A}} (\lambda_{B \rightarrow C} + \lambda_{B \rightarrow B'}) \quad (3)$$

- (iii) The following situation occurs in hydrogen burning environments at high temperatures (thermonuclear explosions). The reaction $^{21}\text{Mg} + p \rightarrow \gamma + ^{22}\text{Al}$ has a small estimated Q -value of $Q_{^{21}\text{Mg} + p} = 163 \text{ keV}$. At $T_9 = 0.6$, $\rho = 10^4 \frac{\text{g}}{\text{cm}^3}$ and $X_H/M_H = 0.7$ the following decay constants are obtained from the tabulated reaction rates and β -decay half-lives:

$$\lambda_{A \rightarrow B} = \lambda_{^{21}\text{Mg} \rightarrow ^{22}\text{Al}} = 1.1 \cdot 10^3 \text{ s}^{-1} \quad (4)$$

$$\lambda_{A \rightarrow A'} = \lambda_{^{21}\text{Mg} \rightarrow ^{21}\text{Na}} = 5.6 \cdot 10^0 \text{ s}^{-1} \quad (5)$$

$$\lambda_{B \rightarrow A} = \lambda_{^{22}\text{Al} \rightarrow ^{21}\text{Mg}} = 3.4 \cdot 10^7 \text{ s}^{-1} \quad (6)$$

$$\lambda_{B \rightarrow C} = \lambda_{^{22}\text{Al} \rightarrow ^{23}\text{Si}} = 3.1 \cdot 10^4 \text{ s}^{-1} \quad (7)$$

$$\lambda_{B \rightarrow B'} = \lambda_{^{22}\text{Al} \rightarrow ^{22}\text{Mg}} = 2.6 \cdot 10^1 \text{ s}^{-1} \quad (8)$$

$$(9)$$

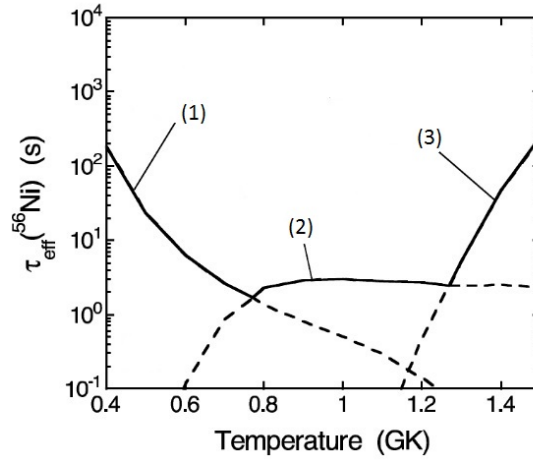
Verify that equations (1) and (2) are fulfilled. Which abundance ratio will come to equilibrium? Sketch the part of the nuclide chart involved in the network and mark the reactions with arrows.

- (iv) Find a way (calculation, not guessing) to decide whether, under these conditions, nucleosynthesis prefers the path $^{21}\text{Mg} \rightarrow ^{21}\text{Na}$, or the path $^{21}\text{Mg} \rightarrow ^{22}\text{Al} \rightarrow (^{22}\text{Mg} \text{ OR } ^{23}\text{Si})$

(b) **The ^{56}Ni bottleneck**

- (i) Consider a situation where the three species A, B, and C achieve equilibrium at elevated temperatures via the reactions $A(p,\gamma)B$ and $B(p,\gamma)C$. In addition to equations (1) and (2), the two conditions $\lambda_{C \rightarrow B} > \lambda_{B \rightarrow C}$ and $\lambda_{B \rightarrow C} > \lambda_{B \rightarrow B'}$ must be fulfilled in order for such an equilibrium to be established. Derive an expression for $\lambda_{A \rightarrow B \rightarrow (C \rightarrow C' \text{ OR } B')}$, that is, the decay constant of species A for consumption via the paths $A \rightarrow B \rightarrow C \rightarrow C' \text{ OR } A \rightarrow B \rightarrow B'$.

- (ii) The nucleus ^{56}Ni is a so-called waiting-point for nucleosynthesis in the rp-process. Waiting-point nuclei have relatively long β -decay half-lives and low Q -values for proton capture. The lifetime of the ^{56}Ni nucleus is shown in the figure.



Use what you have learned so far to explain the following features of the graph:

- (1) What reaction determines the lifetime here? Why is it decreasing with T ? Compare the lifetime to the laboratory lifetime.
 - (2) What causes the plateau? For this, write down equation (3) and inspect the terms.
 - (3) What causes the increase? Which nuclei are in equilibrium here? Use the result of (b)(i) to find an equation describing the situation in a similar way as equation (3).
- (iii) Use the figure to explain why ^{56}Ni is referred to as a bottleneck for nucleosynthesis in the rp-process. At which temperatures do you expect the best chances for nucleosynthesis to significantly proceed past ^{56}Ni ?