Exercises for Nuclear Astrophysics II - SS 2012
Sheet 7

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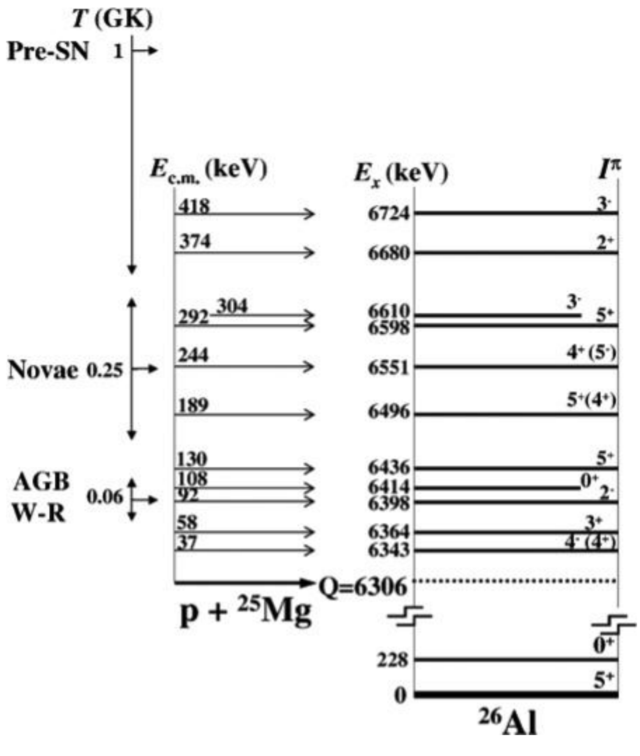
Phone: 089/289/14273 - Discussion of this exercise: 06/29/2012 @ 09:30 a.m.

13 Measuring the ^{25}Mg(p,γ)^{26}Al reaction with AMS

In the last decades, ^{26}Al became a key isotope in understanding and validating stellar nucleosynthesis models. Its ground state ($T_{1/2} = 0.716$ Ma) decays via β^+ and electron capture to the 2+ first excited state in ^{26}Mg, whose deexcitation exhibits a γ-ray of 1809 MeV. When this γ-ray was detected in 1982, it was the first time that extra-solar radioactivity was found and, due to the cosmologically short half-life of ^{26}Al interpreted as proof for ongoing nucleosynthesis within the Milky Way.
For the stellar production of ^{26}Al, there are several scenarios, such as hydrogen burning in massive stars during their Wolf-Rayet-Phase, explosive hydrogen burning in novae, and neon burning in massive stars prior to core collapse supernovae.
In this exercise, you will perform a hypothetical experiment to measure the resonance strengths of a weak resonance contributing to the ^{25}Mg(p,γ)^{26}Al reaction rate at stellar temperature near 1 GK.
We will thus first produce a sample of ^{26}Al by irradiating a thick MgO target with a proton beam.



(a) Review the last exercise sheet about reaction yields. We now want to calculate a resonance strength from a reaction yield Y. The de Broglie wavelength λ_r at the resonance energy in the c.m. system is given by

$$\lambda_r^2 = 4\pi^2 \frac{\hbar^2}{2\mu E_r}. \quad (1)$$

Calculate the reaction yield $Y(E_0)$ for the case of a single narrow resonance and by assuming constant stopping power ϵ, de Broglie wavelength λ, and partial widths Γ_i over the resonance width and show that it can be written as

$$Y(E_o) = \frac{\lambda_r^2}{2\pi}\frac{\omega\gamma}{\epsilon_r}\left[\arctan\left(\frac{E_0 - E_r}{\Gamma/2}\right) - \arctan\left(\frac{E_0 - E_r - \Delta E}{\Gamma/2}\right)\right] \quad (2)$$

(b) Use this result to calculate the projectile energy $E_{0,\text{max}}$ at which the yield is maximum, the maximum yield Y_{max} itself, the energy $E_{0,50\%}$ at which the yield is half of the maximim value, and the FWHM (full width half-maximum) of the resonance yield curve. Also show that assuming an infinitely thick target, meaning $\Delta E >> \Gamma$, the maximum yield can be written as

$$\lim_{\Delta E \to \infty} Y_{\text{max}} = \lim_{\Delta E \to \infty} Y(E_{0,\text{max}}) = \frac{\lambda_r^2}{2} \frac{\omega\gamma}{\epsilon_r} \tag{3}$$

(c) As an example, we will choose to populate the 418 keV state in ^{26}Al. A natural MgO target (200μg/cm^2) is irradiated with a 10μA proton beam. What laboratory energy needs to be chosen for the irradiation to obtain the maximum yield when the stopping power is assumed constant over the target a $\epsilon_{\text{tot}} = 9.23 \cdot 10^{-15}$ cm^2 eV atom^{-1}?

(d) How many protons hit the target during a 1 hour irradiation? Make a crude estimation of how many ^{26}Al nuclei were created using an estimated average cross section of $\sigma = 0.02$ mb.

(e) Such a small number of nuclei of a radioisotope can be identified and counted using accelerator mass spectrometry (AMS). Samples are inserted into a negative ion source (e.g. a Cesium sputter source) and accelerated in a (tandem) accelerator. The desired nuclear species is then isolated using electrostatic and magnetic ion-optical elements and the ions are then individually counted and identified in a particle detection system (e.g. time of flight, ionization chamber, surface barrier detectors). Assume the AMS measurement was able to detect 500 events of ^{26}Al with a total efficiency of $\epsilon_{\text{AMS}} = 5 \cdot 10^{-4}$. What is the yield Y of the experiment?

(f) The proton irradiation will of course also produce the isomeric state ^{26}Al^m (see level scheme). Is this level contributing to the measured yield? If yes, how can it be accounted for? If no, why not?

(g) Calculate the resulting resonance strength $\omega\gamma$ of the 418 keV state in ^{26}Al.