Exercises for Nuclear Astrophysics II - SS 2012
Sheet 6

Peter Ludwig - Lehrstuhl E12 - peter.ludwig@ph.tum.de - http://www.nucastro.ph.tum.de/
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11 Nuclear Statistical Equilibrium

As you already know, massive stars ($M > 11\ M_\odot$) produce predominantly elements in the iron region in their cores when they run out of nuclear fuel. When this happens the star continues to contract and temperature and density increase to a point where the thermal radiation field is able to photodisintegrate the iron peak elements. This can happen on a such short timescale that the abundances of the nuclei can be calculated approximately as if the gas were in chemical equilibrium. This situation is called Nuclear Statistical Equilibrium (NSE). For a simplified model, consider a gas composed only of ^{56}Ni and ^{4}He where the only reaction that is taken into account here is given by

$$14\ ^4\text{He} \leftrightarrow\ ^{56}\text{Ni} + Q \tag{1}$$

(a) Calculate Q.

(b) Write down the Saha statistical equation for the reaction assuming both nuclei are in their ground states.

(c) Convert the Saha equation so that the unknowns are mass fractions X_4 and X_{56} with $X_4 + X_{56} = 1$.

(d) Fix the density to $\rho = 10^7$ g cm^{-3} and solve for X_4 and X_{56} for temperatures in the range $4.5 \leq T_9 \leq 6.5$.

(e) At what temperature T_9 is $X_4 = X_{56}$?

(f) Plot X_4 and X_{56} versus T_9.

12 Reaction yields

Consider a nuclear physics experiment in which you shoot a beam particles with energy E_0 onto a target with thickness d and a concentration of target nuclei $n = N \cdot d$ where N is the number of targets per unit volume. The yield Y of an experiment is defined as the total number of reactions divided by the number of incident beam particles, and the stopping power of the target can be written as:

$$Y = \frac{N_R}{N_b} \quad , \quad \epsilon(E) = -\frac{1}{N}\frac{dE}{dx}. \tag{2}$$

(a) By first considering a small slice of the target with thickness Δx over which the cross section σ is constant, show that the reaction yield can be written as

$$Y(E_0) = \int\limits_{E_0 - \Delta E}^{E_0} \frac{\sigma(E)}{\epsilon(E)} dE \tag{3}$$

(b) Show that in assuming constant ϵ and σ over the whole target thickness, the yield becomes

$$Y(E_0) = n \cdot \sigma(E - \frac{1}{2}\Delta E). \tag{4}$$

Under what conditions can this assumption be valid?

(c) Let the target material be a compound composed of n_X active nuclei per cm^2 and n_Y inactive nuclei per cm^2 with respective stopping powers ϵ_X and ϵ_Y. Show that the effective stopping power of the target can be written as

$$\epsilon_{\text{eff}} = \epsilon_X + \frac{n_Y}{n_X}\epsilon_Y \tag{5}$$

(d) A beam of singly charged protons with a laboratory energy of 200 keV and $1\,\mu$A intensity is incident on a 5 keV thick (in the lab system) natural carbon target for a period of 1h. Calculate the total number of photons originating from the ^{13}C(p,γ)^{14}N reaction, assuming that one photon is emitted per reaction. Assume further that both cross section and stopping power are constant over the target thickness. Use the values $\sigma(E_{\text{lab}} = 200\text{keV}) = 10^{-7}$ b and $\epsilon_{p \to C}((E_{\text{lab}} = 200\text{keV}) = 11.8 \cdot 10^{-15}$ eV cm^2/atom.