Exercises for Nuclear Astrophysics II - SS 2012
Sheet 5

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9 Neon burning

At the end of carbon burning, when most of the ^{12}C nuclei have been consumed (see exercise sheet 4 as a reminder), the core of the star consists mainly of ^{16}O, ^{20}Ne, ^{23}Na, and ^{24}Mg. The first reaction of neon burning is the photodisintegration reaction $^{20}\text{Ne}(\gamma,\alpha)^{16}\text{O}$, which occurs due to the unusually small alpha particle separation energy of 4.73 MeV.

(a) To calculate the rate for this reaction, we can make use of the forward alpha capture rate. At $T_9 = 1.5$, let the forward reaction rate be $N_A\langle\sigma v\rangle = 0.125$ s^{-1} cm^3 mol^{-1}. Remember the formalism for inverse reaction rates from last semester:

$$\frac{\lambda_\gamma(3\to 01)}{N_A\langle\sigma v\rangle_{01\to\gamma 3}} = 9.8685\cdot 10^9\ T_9^{3/2}\frac{g_0 g_1}{g_3(1+\delta_{01})}\left(\frac{M_0 M_1}{M_3}\right)^{3/2}\exp\left(-11.605\ Q/T_9\right). \tag{1}$$

Where $g_i = 2j_i + 1$ and the masses are given in u. Use this formula to calculate the life-time of ^{20}Ne against photodisintegration. (Hint: It should be on the order of a few days)

(b) The timescale of neon burning however is roughly a year and the ^{20}Ne is not exhausted in a few days. Can you think of the reaction replenishing the supply of ^{20}Ne? Verify your assumption taking a look at the rate of that reaction.

(c) The two most important reactions concerning energy generation in neon burning are $^{20}\text{Ne}(\gamma,\alpha)^{16}\text{O}$ and $^{20}\text{Ne}(\alpha,\gamma)^{24}\text{Mg}$ ($Q = 9316$ keV). At $T_9 = 1.5$, the $^{20}\text{Ne}(\gamma,\alpha)^{16}\text{O}$ reaction proceeds mainly through ^{20}Ne levels at $E_x = 5621$ keV (3^-) and 5788 keV (1^-), while the most important ^{24}Mg levels for the $^{20}\text{Ne}(\alpha,\gamma)^{24}\text{Mg}$ reaction are located at $E_x = 10680$ keV (0^+), 10917 keV (2^+), and 11016 keV (2^+). Draw an energy level diagram for ^{20}Ne and ^{24}Mg using all the numbers you have seen here so far.

(d) Calculate the energy release per destroyed ^{20}Ne nucleus if the previously mentioned $^{20}\text{Ne}(\gamma,\alpha)^{16}\text{O}$ and $^{20}\text{Ne}(\alpha,\gamma)^{24}\text{Mg}$ reactions contribute 75% of the energy produced in neon burning.

(e) Assume the total energy generation rate is given by

$$\epsilon = 6.24\cdot 10^{33}\frac{(X_{^{20}\text{Ne}})^2}{X_{^{16}\text{O}}}T_9^{3/2}\exp(-54.89/T_9)N_A\langle\sigma v\rangle_{^{20}\text{Ne}(\alpha,\gamma)}\quad[\text{MeV g}^{-1}\ \text{s}^{-1}]. \tag{2}$$

(By the way, you can also derive this assuming a $^{20}\text{Ne} + \alpha \leftrightarrow {}^{24}\text{Mg} + \gamma$ equilibrium but this would be a bit lengthy for this exercise.) The reaction rate can be described by the analytical expression

$$N_A\langle\sigma v\rangle_{^{20}\text{Ne}(\alpha,\gamma)} = 3.74\cdot 10^2\ T_9^{2.229}\exp(-12.681/T_9). \tag{3}$$

Use these equations to derive a power law expression for the energy dependence of the energy generation rate in neon burning. What is the power law exponent n for the typical temperature $T_9 = 1.5$?

10 Hertzsprung Russel Diagram

Discuss the evolution of

(a) a 2 $M_\odot$ star,

(b) a 0.2 $M_\odot$ star,

(c) a 10 $M_\odot$ star.

Consider the different occuring burning stages and sketch the path in the HR diagram for each case.