

Exercises for Nuclear Astrophysics II - SS 2012

Sheet 4

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7 Carbon Burning

After the fuel for helium burning is exhausted, massive stars ($M > 10M_{\odot}$) experience carbon core burning at temperatures in the range of $0.6 \leq T_9 \leq 1.0$, and later on carbon shell burning at slightly higher temperatures. At the beginning of carbon burning, the star's core is composed primarily of ^{12}C and ^{16}O .

- Why is the reaction $^{12}\text{C} + ^{12}\text{C}$ favored over the other possible heavy ion reactions? Calculate position and width of the Gamow window for a typical temperature of $T_9 = 0.8$.
- Estimate the excitation energy of the compound nucleus ^{24}Mg in the $^{12}\text{C} + ^{12}\text{C}$ reaction using $m(^{24}\text{Mg}) = 23.985042$ u.
- Why does the reaction $^{12}\text{C}(^{12}\text{C}, n)^{23}\text{Mg}$ require a minimum temperature around $T_9 > 1$ to provide a significant contribution to the $^{12}\text{C} + ^{12}\text{C}$ reaction rate? (Hint: $m(^{23}\text{Mg}) = 22.9941237$ u, $m(n) = 1.0086649156$ u)
- Carry out a hypothetical experiment in which you shoot $^{12}\text{C}^+$ ions onto a graphite target (Target density $N_t = 10^{19} \text{ cm}^{-2}$) with the kinetic energy $E_{lab} = 5$ MeV. For simplicity assume the destruction of ^{12}C proceeds only through the reactions $^{12}\text{C}(^{12}\text{C}, p)^{23}\text{Na}$ and $^{12}\text{C}(^{12}\text{C}, \alpha)^{20}\text{Ne}$ contributing 50% each to the total cross section. Do not take energy loss of the beam particles in the target into consideration.
 - Calculate the kinetic energy in the center of mass frame E_{cm} .
 - Can you think of possible detector configurations that would allow the identification of events from these two reactions?
 - (*) Assume you identified 50 events of the $^{12}\text{C}(^{12}\text{C}, \alpha)^{20}\text{Ne}$ reaction with your setup, which has total efficiency of 10^{-4} over a measurement time of 1 hour with a beam current of $I = 1 \mu\text{A}$. Show that the resulting astrophysical S-factor for the $^{12}\text{C} + ^{12}\text{C}$ -reaction is roughly $S = 10^{16}$ MeV b.
 - Assuming the same S-factor, how long would you have to wait for just one count with the same experimental setup at a beam energy of $E_{lab} = 4$ MeV?

8 Helium Core Flash

Low mass stars ($0.5 - 2 M_{\odot}$) undergo a helium core flash upon ignition of helium burning. The exact upper mass limit for this phenomenon is uncertain and also depends on metallicity. The flash occurs because the material is (at least partially) degenerate and is unable to expand in response to a temperature increase. Suppose you have 1 gram of pure helium (as ^4He) in the center of a pre-helium flash red giant star under the conditions $\rho = 2 \cdot 10^5 \text{ g cm}^{-3}$ and $T_9 = 0.15$. All temperatures in this exercise should be kept in the form of T_9 (meaning GK) for simplicity reasons. This is hot enough to burn helium via the triple alpha reaction (the only reaction you will consider here). You already know the energy generation rate of the triple alpha reaction to be

$$\epsilon_{3\alpha} = \frac{5.1 \cdot 10^8 \rho^2 X_{\alpha}^3}{T_9^3} \cdot \exp(-4.4027/T_9) \text{ erg g}^{-1} \text{ s}^{-1} \quad (1)$$

The task is to compute $T_9(t)$ and check if there is indication of a helium flash (which would manifest itself as a sudden, steep temperature increase.) To avoid making things too difficult, make the following assumptions:

- The helium mass fraction X_{α} does not change.
- The density ρ remains constant.

- No heat is allowed to leave the gram of helium. This means that the first law of thermodynamics is reduced to $\frac{dQ}{dt} = \epsilon_{3\alpha} \cdot m$.

The following steps will lead you towards the result $T_9(t)$:

- (a) Find the temperature at which the gram will become non-degenerate. This is the point where you can stop your calculation of $T_9(t)$ later. For this, you can use the non-relativistic demarcation line

$$\frac{\rho}{\mu_e} = 6 \cdot 10^{-9} T^{3/2} \quad (2)$$

which you have already seen in the first semester of the course.

- (b) The helium's reaction to the energy output of the triple alpha reaction will cause a temperature increase. To calculate this increase, calculate the specific heat of the material as a sum of the electron specific heat $c_V(e)$ and the ion specific heat for pure helium $c_V(I)$ using

$$c_V(I) = \frac{3N_A k_B}{2\mu_I} \quad (3)$$

where μ_I is the mean molecular weight in units of u , and

$$c_V(e) = \frac{1.35 \cdot 10^{14}}{\rho} T_9 x_f \sqrt{1 + x_f^2} \text{ erg g}^{-1} \text{ K}^{-1} \quad (4)$$

where x_f is the Fermi momentum, which you can calculate using $\rho/\mu_e = Bx_f^3$ with $B = 9.739 \cdot 10^5 \text{ g cm}^{-3}$.

- (c) Set up a differential equation for $T(t)$ using the results so far. Change the units of this equation so that T is in GK and t is in days.
- (d) Find a way to solve this differential equation numerically (you can of course use a computer program, e.g. Mathematica, Maple, ... for this). Compute the solution until a time where degeneracy is lifted and plot your result.
- (e) Observation shows that during the helium core flash, the absolute magnitude of the star drops by about an order of magnitude. Would you not expect an increase in magnitude? Resolve!