Exercises for Nuclear Astrophysics II - SS 2012
Sheet 3

Peter Ludwig - Lehrstuhl E12 - peter.ludwig@ph.tum.de - http://www.nucastro.ph.tum.de/
Phone: 089/289/14273 - Discussion of this exercise: 05/10/2012 @ 1 p.m.

5 The triple alpha reaction

Nucleosynthesis needs to bypass the unstable nuclei with $A = 5$ and $A = 8$. Nature solves this problem with the triple alpha reaction. Two alpha particles come together to form the nucleus ^{8}Be, which is unstable against alpha emission by 91.84 keV. In this way, a small equilibrium abundance of ^{8}Be can build up.

(a) Calculate the equilibrium abundance of ^{8}Be at $T_8 = 1$ and $\rho = 10^5$ using the Saha statistical equation.

(b) (*) The alpha capture on ^{8}Be ($Q = 7367keV$) proceeds through the narrow 0+ resonance in ^{12}C at 7654 keV. Use the Saha equation and the formalism for narrow resonances to show that the reaction rate of the triple alpha process can be written as

$$\lambda_{3\alpha} = 8.7590 \cdot 10^{-10} \cdot \frac{(\rho Y)^2}{T_9^3} \exp\left(-4.4040/T_9\right) \ s^{-1}. \quad (1)$$

This problem requires you to start out from the ^{8}B abundance you get from the Saha equation. On your way through the narrow resonance formalism, you will need the resonance strength, which is given as $\omega\gamma = 3.7 \cdot 10^{-3}$ eV.

(c) Use this result to show that the energy generation rate of the triple alpha process can be written as

$$\epsilon_{3\alpha} = 3.1771 \cdot 10^{14} \cdot \frac{\rho^2 Y^3}{T_9^3} \exp\left(-4.4040/T_9\right) \left[\frac{\text{MeV}}{\text{g} \cdot \text{s}}\right]. \quad (2)$$

(d) Show that the temperature dependence of the triple alpha reaction can be written as

$$(\lambda_{3\alpha})_T = (\lambda_{3\alpha})_{T_0} \cdot (T/T_0)^{(4.4040/T_9)-3} \quad (3)$$

and calculate the exponent for typical values for helium burning: $0.1 \leq T_9 \leq 0.4$.

6 The $^{16}\text{O} + \alpha \rightarrow ^{20}\text{Ne} + \gamma$ reaction

Another important reaction in helium buring is the alpha capture on ^{16}O. The level scheme can be seen on page 2.

(a) As you know, light stars like our sun produce mainly ^{16}O and ^{12}C in helium burning. But the figure shows a resonance in ^{20}Ne only 239 keV above the Q-value. Should it then not be light stars (with low temperature) that produce ^{20}Ne by making use of this resonance? Resolve!

(b) Confirm the availability of the three other states above the Q-value to the $^{16}\text{O} + \alpha$ system and determine the required l wave of the alpha-particle capture.

(c) What is the minimum temperature to reach the 5.631 MeV state with the Gamow window?

(d) Calculate the lifetime $\tau(^{16}\text{O})$ due to alpha particle capture into the 5.631 MeV state assuming the partial widths $\Gamma_\alpha = 6 \cdot 10^{-3}$ eV and $\Gamma_\gamma = 4 \cdot 10^{-4}$ eV.

$^{16}O(\alpha,\gamma)^{20}Ne$ Reaction