

# Exercises for Nuclear Astrophysics II - SS 2012

## Sheet 2

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Phone: 089/289/14273 - Discussion of this exercise: 05/03/2011 @ 1 p.m.

### 2 The pp-I chain

Denote  ${}^1\text{H}$  with the subscript 1,  ${}^2\text{H}$  by 2,  ${}^3\text{He}$  by 3, and  ${}^4\text{He}$  by 4.

- Review the nuclear reactions in the pp-I chain and write down a system of 4 first order differential equations for  $\frac{dn_1}{dt}, \dots, \frac{dn_4}{dt}$  depending on the number densities  $n_1, \dots, n_4$  and the reaction rates per particle pair  $\langle\sigma v\rangle_{ij}$ , where  $ij$  denotes the particles  $i$  and  $j$  in the entrance channel ( $i, j = 1, 2, 3, 4$ ).
- Which of these nuclei can be regarded as being in equilibrium over short time scales compared to the p-p burning itself and why?
- Use the argument from (b) to show that to a good approximation

$$\frac{dn_4}{dt} = \frac{1}{4}\langle\sigma v\rangle_{11}n_1^2 \quad (1)$$

and interpret the result.

### 3 Angular momentum in strong nuclear interactions

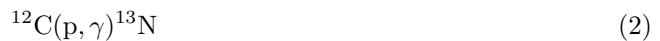
In a nuclear interaction involving only nuclei (including protons and neutrons), several selection rules must be obeyed. Let the incident particles' spins be  $\mathbf{S}_1^{\Pi_1}$  and  $\mathbf{S}_2^{\Pi_2}$  (with parity  $\Pi$ ) respectively and their relative angular momentum vector is  $\mathbf{L}$ .

If necessary, review your quantum mechanics to complete this assignment problem.

- The reaction  ${}^{12}\text{C}(p, \gamma){}^{13}\text{N}$  forms the compound states in  ${}^{13}\text{N}$  with  $J^\Pi$  of  $(1/2)^+$  and  $(3/2)^-$ . What are the permitted values of  $l$  of the entrance channel to form each of these states? (The ground state of  ${}^{12}\text{C}$  is  $0^+$ ).
- For the reaction  ${}^{13}\text{C}(p, \gamma){}^{14}\text{N}$  there are states in  ${}^{14}\text{N}$  with  $2^-$  and  $1^-$ . If the ground state of  ${}^{13}\text{C}$  is  $(1/2)^-$ , what are the permissible values of  $l$ ?

### 4 The CNO-cycle

The main part of the CNO cycle consists of the nuclear reactions



- In reaction (7), a branching into the reaction channel  ${}^{16}\text{O} + \gamma$  occurs with roughly 0.04%. Thus it appears that at this point CNO nuclei are removed from the cycle. What happens to those nuclei? Are they really lost for the CNO cycle?
- Show that assuming  $T_8 < 1$ , the abundance of  ${}^{13}\text{N}$  can be written as

$${}^{13}\text{N}(t) = \frac{\tau_\beta({}^{13}\text{C})}{\tau_p({}^{12}\text{C})} \cdot {}^{12}\text{C} \left( 1 - \exp\left(-\frac{t}{\tau_\beta({}^{13}\text{C})}\right) \right). \quad (8)$$

To do this, write write down the time evolution  $\frac{d({}^{13}\text{N}(t))}{dt}$  and then assume that  ${}^{12}\text{C}$  and  $\tau_p({}^{12}\text{C})$  are constant (which is reasonable for short timescales). What time does it roughly take for  ${}^{13}\text{N}$  to come close to equilibrium?

- (c) In a similar way, the abundance of  $^{15}\text{N}$  will reach an equilibrium value on the order of years, what will be its equilibrium abundance  $\left(\frac{^{15}\text{N}}{^{14}\text{N}}\right)_e$ ?
- (d) Write down the system of first order differential equations describing the time evolution of the nuclei involved in the CNO cycle using all assumptions made so far and ignoring the branching mentioned in (a). How many equations are we left with?