Nuclear Astrophysics II

Lecture 11

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Going beyond the Iron Peak

BUILDING THE HEAVY ELEMENTS

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- <u>Big bang (primordial) nucleosynthesis</u>: H, He, D, no elements heavier than Li
- <u>Galactic cosmic ray spallation</u>: Li, Be, B through bombardment of matter by high energy cosmic ray particles

• <u>Stellar nucleosynthesis 1</u>: fusion (burning processes) in stars up to A~56;

<u>Stellar nucleosynthesis 2</u>: *s* process up to Bi-209
<u>Explosive nucleosynthesis</u>: *r* process (heaviest nuclei); *p* process (32 proton-rich, stable isotopes)



BUILDING THE HEAVY ELEMENTS



Neutron Capture

Recall, again, that for a reaction like: $1 + 2 \rightarrow 3 + \gamma$, the reaction rate for 1 and 2 is:

$$r_{12} = N_1 N_2 \left(\frac{8}{\pi\mu}\right)^{1/2} \tau^{-3/2} \int_0^\infty E\sigma(E) \exp\left(-E/\tau\right) dE$$
$$\langle \sigma v \rangle \equiv \left(\frac{8}{\pi\mu}\right)^{1/2} \tau^{-3/2} \int_0^\infty E\sigma(E) \exp\left(-E/\tau\right) dE$$

Neutrons have no Coulomb barrier. The cross section for neutron capture, at low energies is something close to:

$$\sigma(E) = \sigma_0 \frac{\mu}{2E}$$

We therefore have:

$$\langle \sigma v \rangle = \sigma_0 \left(\frac{2\mu}{\pi\tau}\right)^{1/2}$$

The average thermal velocity of a M-B distribution (velocity where M-B has its maximum) is given by:

$$v_T = \left(\frac{2\tau}{\mu}\right)^{1/2}$$

So, we can now write:

$$\langle \sigma v \rangle = \sigma_0 \left(\frac{2\mu}{\pi\tau}\right)^{1/2} = \sigma_0 \frac{2}{\sqrt{\pi}} v_T^{-1}$$

The Maxwellian averaged cross section is defined as (with $E_T = kT$):

$$\langle \sigma \rangle \equiv \frac{\langle \sigma v \rangle}{v_T} = \frac{2}{\sqrt{\pi}} \sigma_0 v_T^{-2} = \frac{2}{\sqrt{\pi}} \sigma(E_T)$$

This is an intriguing result. It tells us that the normally complicated $\langle \sigma v \rangle$ can, for neutron capture, just be taken as the neutron capture cross section, evaluated at some E_T (you choose it), multiplied by the corresponding value for v_T.



Slow Neutron Capture

THE S-PROCESS

S-PROCESS: REACTION PATHS

- s process: along valley of stability, up to Bi-209
- *r* process: very neutron-rich region, up to U-Th
- p process: shielded from both reaction flows!





Consider the path shown in **part (a)**. For nuclide A, the abundance differential equation is: $\frac{dN_A}{dt} = N_n N_{A-1} \langle \sigma v \rangle_{A-1} - N_n N_A \langle \sigma v \rangle_A$ $= N_n \left(N_{A-1} \langle \sigma \rangle_{A-1} v_T - N_A \langle \sigma \rangle_A v_T \right)$

$$= N_n v_T \left(N_{A-1} \langle \sigma \rangle_{A-1} - N_A \langle \sigma \rangle_A \right)$$

Note: v_T is approximately the same for each term because the reduced mass for very heavy nuclei is $\approx m_n$

Now divide both sides by $N_n v_T$ and we finally have:

We finally have: $\frac{dN_A}{d\tau} = N_{A-1} \langle \sigma \rangle_{A-1} - N_A \langle \sigma \rangle_A$

Where: $d\tau = N_n v_T dt$

The integral $\tau = v_T \int_0^\infty N_n dt$ is called the "neutron exposure", and it is basically the time integrated neutron flux in the system.

The above diff. equation is of the "self-regulating" type: given enough time, it seeks to minimize the difference between the two terms on the right. If the magnitudes of the Maxwellian averaged cross sections $\langle \sigma \rangle$ are of the same magnitude, then we should expect that, for reasonable abundances of A and (A-1), that the minimization will be achieved. Then the right hand side will become ≈ 0 . This assumption is called the **Local Approximation**.

So, we have, after sufficient time:

$$N_{A-1}\langle\sigma\rangle_{A-1} = N_A\langle\sigma\rangle_A$$

Neutron Capture Cross Sections at $E_T = 25 \text{ keV}$

Between **closed neutron shells** the cross sections are within a few factors of 3 or 4 of each other. \rightarrow Local Approximation should probably work.



Atomic weight A

Time for an estimate of neutron number density in this process:

Previous plot allows us to crudely estimate the Maxwellian average cross section as something around 100 mb across the entire mass range.

The thermal velocity at kT = 25 keV, with the reduced mass being $pprox m_n$ is approximately:

$$v_T \approx 2.2 \times 10^8 \text{ cm/s}$$

Then we have: $\langle \sigma v \rangle \approx 100 \times 10^{-27} \times 2.2 \times 10^8 = 2.2 \times 10^{-17} \text{ cm}^3/\text{s}$

The lifetime of nucleus A against neutron capture is:

$$\tau_A = \frac{1}{N_n \langle \sigma v \rangle}$$

Beta-decay lifetimes along valley of stability are minutes to years. So, let's take 10 years as the **extremely crude order of magnitude** number for τ_A .

Converting 10 years into seconds, and using our numbers above gives a neutron number density $N_n \sim 10^8 \ {\rm cm}^{-3}$

The same procedure using microsecond lifetimes (r-process) gives $~N_n \sim 10^{22}~{
m cm^{-3}}$

Local Approximation Confirmed

Maxwellian Averaged Neutron Cross Sections at $E_T = kT = 30$ keV.



From the neutron capture cross section plot from 3 slides ago, it is obvious that the **Local Approximation** will not hold true across the entire mass range from A = 56 to A = 209. The cross sections at closed neutron shells drop by orders of magnitude, and from A = 56 to A = 209, it can be seen that the cross sections have a systematic increasing trend.

The general solution to the Classical s-Process is then back on page y. For any isotope "A" along the s-Process path, we have:

$$\frac{dN_A}{d\tau} = N_{A-1} \langle \sigma \rangle_{A-1} - N_A \langle \sigma \rangle_A$$

We assume **all** beta-decays are much faster than neutron capture. If the path enters into an unstable nucleus, it beta-decays "instantaneously" to its daughter isobar.

A series of equations like the above are then solved over the mass region A = 56 to A = 209. We are taking ⁵⁶Fe as the seed nuclide that stars the s-process.

Initial condition: only ⁵⁶Fe present as a seed. All others are zero.

The system to solve is something like this:

$$\frac{dN_{56}}{d\tau} = -N_{56} \langle \sigma \rangle_{56}$$

$$\frac{dN_{57}}{d\tau} = N_{56} \langle \sigma \rangle_{56} - N_{57} \langle \sigma \rangle_{57} \qquad 57 \le A \le 209$$
$$A \ne 206$$

$$\frac{dN_{206}}{d\tau} = N_{205} \langle \sigma \rangle_{205} + N_{209} \langle \sigma \rangle_{209} - N_{206} \langle \sigma \rangle_{206}$$

$$\bigvee$$
Production from (n, \alpha) reaction

Our solution to this system starts with the Laplace Transform (LT). Remember from your math courses that the LT of a function $f(\tau)$ is defined as:

$$\tilde{f}(s) = \int_0^\infty f(\tau) e^{-s\tau} d\tau$$

Also, remember that the LT of the derivative function of f(x) is given by:

$$\tilde{f}'(s) = s\tilde{f}(s) - f(0)$$

So, let's start with the first equation from the last page:

$$\frac{dN_{56}}{d\tau} = -N_{56} \langle \sigma \rangle_{56}$$

The Laplace Transform is: $-\sigma_{56} ilde{N}_{56}=s ilde{N}_{56}-N_{56}(0)$

$$\Rightarrow \sigma_{56} \tilde{N}_{56} = \frac{N_{56}(0)}{s/\sigma_{56} + 1}$$

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For the next equation, for ⁵⁷Fe:

$$\frac{dN_{57}}{d\tau} = N_{56}\sigma_{56} - N_{57}\sigma_{57}$$

$$\Rightarrow s\tilde{N}_{57} = \tilde{N}_{56}\sigma_{56} - \tilde{N}_{57}\sigma_{57}$$

And using our previous result:

$$\sigma_{56}\tilde{N}_{56} = \frac{N_{56}(0)}{s/\sigma_{56} + 1}$$

$$\Rightarrow \sigma_{57}\tilde{N}_{57} = \frac{N_{56}(0)}{s/\sigma_{56} + 1} \times \frac{1}{s/\sigma_{57} + 1}$$

This pattern repeats.

In general, for any of the nuclides along the s-process path, the LT of their solution is:

$$\sigma_A \tilde{N}_A = N_{56}(0) \prod_{i=56}^A \frac{1}{s/\sigma_i + 1}$$

Now, let $s=1/ au_0$, then:

$$\sigma_A \tilde{N}_A = N_{56}(0) \prod_{i=56}^A \frac{1}{1/\tau_0 \sigma_i + 1}$$

The left hand side is the Laplace Transform of our abundance distributions. In principle, we should now invert the LT to finally obtain $N_A(\tau)$.

It turns out, we do not. If the function on the right hand side, with experimental Mawellian cross sections σ_i , and with τ_0 a fitting parameter (or both τ_0 and N₅₆ as free parameters), is fit to the solar system s-process abundances, we get the following:

What does it mean that the Laplace transform of the actual abundance solutions is what best fits these data?

Consider again the LT:
$$ilde{N}_A(s) = ilde{N}_A(1/ au_0) = \int_0^\infty N_A(au) e^{- au/ au_0} d au$$

The integral on the RHS is the actual abundance distribution folded/convolved over some kind of exponential distribution. This is, of course, just the definition of the LT. But, this is also what beautifully fits our solar system s-process abundances.

So, as it turns out, it seems that the nuclides produced by the s-process must be subjected to a neutron exposure distribution that is a decaying exponential. And the s-process abundances derive from an averaging of the actual abundances over this exposure distribution.

This is analogous to weighting the velocity over the Maxwell-Boltzmann distribution to determine the "average" speed of the atoms in the gas. We do not keep track of the speed of each atom; we instead use the MB distribution to determine the average speed.

In the case of the s-process, the solution to the abundance distribution is, like with the MB situation, determined by averaging the individual abundances over an exponential neutron exposure distribution.

We can use our general result to determine the abundance ratio between any two nuclides:

$$\sigma_A \tilde{N}_A = N_{56}(0) \prod_{i=56}^A \frac{1}{1/\tau_0 \sigma_i + 1}$$

Take this formula and write it for the case of A \rightarrow A – 1. Then take ratios.

$$\sigma_A \tilde{N}_A = \sigma_{A-1} \tilde{N}_{A-1} \frac{1}{1/\tau_0 \sigma_A + 1}$$

Refer back to page 8. Between closed neutron shells, cross section is large, so the term is small. And so $\sigma_A \tilde{N}_A \approx \sigma_{A-1} \tilde{N}_{A-1}$

But near closed neutron shells, the cross section becomes much smaller. And so $1/\tau_0\sigma$ becomes large. This produces a step in the s-process curve at closed neutron locations



S-process conditions

	weak	component	main	component
Mass region	A<90	(Fe - Zr)	A>90	(Zr - Bi)
Stellar site	massive stars	(>8 M _{sun})	TP AGB stars	(1-3 M _{sun})
	core He	shell C	H burning	He shell flashes
т [МК]	300 (kT= 26 keV)	1000 (kT= 91 keV)	90 (kT= 8 ke∀)	250 (kT= 23 ke∀)
Neutron source	Ne-22(α,n)Mg-25	(C-12(C-12,n)Mg-23) Ne-22(α,n)Mg-25	C-13(α,n)O-16	Ne-22(α,n)Mg-25
Neutron density [cm-3]	10 ⁶	10 ¹¹	10 ⁷	10 ¹¹
Duration [y]	10 ⁶	1-20	1 0 ⁴	10



massive star

AGB stars and s-process

Predominantly thought to occur in pulsating, low mass Asymptotic Giant Branch (AGB) stars, between 1.5 and 3 solar masses. These stars have largely exhausted core Heburning, and are instead burning He and H (not at the same time!) in their shells around the core.



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Protons mixed into He-shell by convection. He shell has 12 C from triple- α burning.

$$^{12}C(p,\gamma)^{13}N(\beta^{+}\nu)^{13}C(p,\gamma)^{14}N$$

 $^{13}\mathrm{C}$ builds up in He-shell. As temperature reaches 0.09 GK, then the reaction $^{13}\mathrm{C}(\alpha,n)^{16}\mathrm{O}$ proceeds with a mean lifetime much shorter than the time period between thermal pulses.

The free neutrons produced initiate the s-process.



AGB s-Process Abundances

Solid circles are s-only isotopes. These are scaled with respect to the solar system s-process abundances.

The agreement for masses greater than 100 is striking.



C. Arlandini et al ApJ 525 (1999) 886

Rapid Neutron Capture

THE R-PROCESS

THE R-PROCESS - PATH



Grawe 2007 - Nuclear Structure and astrophysics

We move now from "slow" neutron capture to more extreme conditions.

Neutron number density $N_n \approx 10^{21}~{
m cm}^{-3}$; temperature is $T \geq 1~{
m GK}$

We see, after subtraction of sprocess abundances, that the remaining r-process nuclides have 3 prominent peaks at atomic mass numbers: 130, 162 and 196.

These are all systematically lower, by approx. 10 mass units, from the peaks in the s-process abundances.

We conclude, then, that these abundance peaks must also be related to the magic neutron numbers: 50, 82 and 126.



C. Arlandini et al ApJ 525 (1999) 886

But how?

Essentials of the r-process path

Let's focus on **path (a)** for now. We take our model to be such that all isotopes within the chain are in reaction equilibrium: forward and inverse reaction rates are the same between pairs of isotopes.

Saha Equation gives us the equilibrium abundance ratios:

$$\frac{N_{A+1}}{N_A} = N_n \left(\frac{h^2}{2\pi\mu_{An}\tau}\right)^{3/2} \frac{g_{A+1}}{g_A g_n} \exp(Q_{n\gamma}/\tau) \xrightarrow{\text{toward valley of stability}} \prod_{\substack{n=1\\ n \neq n}} \prod_{\substack{n=1\\ n \neq n}} \left(\frac{h^2}{2\pi\mu_{An}\tau}\right)^{3/2} \frac{g_{A+1}}{g_A g_n} \exp(Q_{n\gamma}/\tau) \xrightarrow{\text{toward valley of stability}} \prod_{\substack{n=1\\ n \neq n}} \prod_{\substack{n=1\\ n \neq n}}$$



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Let's do a crude approximation to get some feeling for what will happen along the isotopic chain in an equilibrium situation.

First, neglect spin-statistical factors – set them all to unity.

Next, set $N_{A+1} = N_A$. And set reduced mass to be neutron mass.

Then, take $N_n = 10^{22}$ cm⁻³ and the temperature to be T= 1.5 GK.

Finally: Solve for the Q-value that satisfies this crude thought experiment. The number will be around 3 MeV.

$$\frac{N_{A+1}}{N_A} = N_n \left(\frac{h^2}{2\pi\mu_{An}\tau}\right)^{3/2} \frac{g_{A+1}}{g_A g_n} \exp(Q_{n\gamma}/\tau)$$

Back to reality: of course, the Q-values are different, but what this simple numerical game tells us is that, for this temperature and neutron density (typical of r-process), any isotopes in the chain with neutron binding energy close to 3 MeV will tend to be the ones with maximum abundance.

Keep in mind, that larger N_n will shift maxima to larger neutron number, and larger T will shift maximum to smaller neutron number along the chain.

Now let's focus on **path (b)**.

You should know from your nuclear course that there is an even-odd effect in the nuclear binding energy of a nucleus, with the binding energy slightly stronger for nuclei with even Z and even N. Therefore, we expect the abundance to peak at an isotope with the right Q-value that is EVEN-N.

The abundance peak formed in the chain by this isotope will act like a **waiting point**. Most of the abundance distribution sits in that isotope. Its beta-decay will feed matter into the next isotopic chain [**path (b)**]



Now let's focus on **path (c)**.

As matter is fed into the isotopic chains of higher atomic number Z, the equilibrium flow will encounter members in each chain that have a magic neutron number.

These nuclei have large binding energies similar to the alpha-particle nuclei that create waiting points in the rp-process because the protonbinding energy for the next nucleus is very low.

The situation here is similar: once the flow has entered into a magic neutron number nucleus, capturing another neutron is inefficient because the neutron binding energy of the neighbouring (n+1) nucleus is low, and it is therefore easily photo-disintegrated back to the magic nucleus.

The system must, therefore, wait for the magic members of each isotope chain to beta-decay. This is what creates the zig-zag matter flow in **(c)**.



R-process sites: Supernovae Type II M > 15M solar

• Following delayed explosions, neutrinos diffuse out of protoneutron star $p \rightarrow n + e^+ + u$

neutrinos heat up
 surrounding nucleons →
 neutrino driven wind

•High entropy, moderate p-ton-ratio

• favored r-process site



Cassiopeia A Supernova Remnant NASA / JPL-Caltech / O. Krause (Steward Observatory) ssc2005-14c

Spitzer Space Telescope • MIPS Hubble Space Telescope • ACS Chandra X-Ray Observatory

R-process sites: Supernovae Type II M = 8-10M _{solar}



- Form e-degenerate O-Ne-Mg core
- Collapses via e-captures on 24Mg and 20Ne
- Prompt hydrodynamical explorion prior to neutrino heating possible
- heaviest nuclei might be synthesized here

Crab Nebula – M1 Believed to have been created by SNII – progenitor star: M~9-11M_{solar}

R-process sites: Neutron star mergers

- System of 2 neutron stars bound by gravity
- Energy loss by gravitational waves →
 sprialing in → merger
- Extremely high neutron densitites
- Problem: low rate (2-3 orders of mag. lower than SNe !) → high ejection rate necessary → clumping of r-process material



How Does it Look in a Network Calculation?



Can we find radioactive s- and r-process isotopes on earth?

• Yes, or course: primordial, radioactive isotopes, there are many, few examples:



But they are just here, because they are so long-lived. They survived since the formation of the solar system

• Isotopes, with shorter half-life that occur on earth can have cosmogenic origin (Supernovae, cosmic ray production, ...) But this origin can also be terrestrial (nuclear bombs, reactors, ...)

• Are there isotopes which are only produced almost exclusively in supernovae? If yes, how and where can we look for them?









Hof, Iow density ISM





The Solar Neighbourhood



a picture based on observations

dense (1cm⁻³) warm (5000K)

thin (<10⁻² cm⁻³) hot (>10⁶ K)

Search for cosmogenic isotopes on earth?



⁶⁰Fe as a messenger from nearby supernovae



Destruction:

- β-decay chain over ⁶⁰Co to ⁶⁰Ni
- (n,γ) reaction at high neutron density
- Photodisintegration (γ ,n) at T > 2 GK

⁶⁰Fe as a messenger from nearby supernovae



M(M_)

Production in stars:

- Shell He burning in massive stars (M > 40 M_{sun})
- Shell C burning in massive stars (M < 40 M_{sun})
- Explosive synthesis in SN when shockwave passes through shells \rightarrow small contribution



Challenge: Isobar separation of $^{60}\text{Ni} \rightarrow$ use of the Gas-filled-Analyzing-Magnet-System (GAMS)





 \rightarrow able to measure down to isotopic ratios of ⁶⁰Fe/Fe~10⁻¹⁶

 \rightarrow worldwide unique setup at the MLL



Deep sea ferromanganese crusts: A terrestrial reservoir for ⁶⁰Fe

- ≈ 15% Fe
- Can be dated with ^{10}Be (T_{1/2} = 1.51 Myr)
- Extremely low growth rate: few mm/Myr
- Cover a time span up to ≈ 20 Myr









AMS results for ⁶⁰Fe/Fe







Supernova signature on the moon?

Another possible reservoir for supernova produced long-lived isotopes is the lunar surface.

Samples were obtained from NASA Apollo missions 12, 15, 16, and 17.



Advantages:

Net sedimentation rates are small

 Low abundance of Ni (target material for spallogenic production of ⁶⁰Fe)

Disadvantages:

Soil gardening

Hard to reach



Picture NASA AS12-49-7286

53Mn and 60Fe measurements at MLL (Garching) 2011



53Mn and 60Fe measurements at MLL (Garching) 2011



Supernova signature in marine sediment?

Current Project at MLL Garching: search for ⁶⁰Fe in ocean drill cores

- Samples from two drill cores from ODP (Ocean Drilling Program) were obtained ~few kg of material total
- Goal: measure depth profile of 60Fe/Fe with resolution ~100.000 years



Microfossils in sediments

 60Fe input from SN event can also be incorporated into organisms

 Magnetotactic bacteria build up chains of magnetite grains (20-80 nm) for orientation in earth's magnetit field (single domain)

Magnetofossils can make up to 60% of the sediments saturation magnetization

 Magnetic measurements of ODP samples show single domain magnetic fraction of

 $\mathscr{C}_{\text{SDFe}}\approx 1.9\times 10^{-5}~g/g$

 This means 100 g of sample material can yield 2 mg of iron from bacteria fossils

 How to get it out? Chemical extraction of fine-grained secondary minerals (mainly iron oxides) with little contamination from primary minerals (e.g. material that came in by wind and does not have a supernova signature)

Measurements currently underway (actually as we speak)

Summary

•r- and s- process responsible for synthesis of 99% of elements heavier than iron
•s-process: slow, moderate n-flux, AGB-stars

•r-process: rapid, high n-flux, Supernovae

•Supernova produced isotope 60Fe has been found using AMS on earth at the GAMS setup at the MLL in Garching in a ferro-manganese crust from the pacific ocean

- One or more supernovae 2-3 Mys ago could be responsible (~40 pc distance)
- Excess of 60Fe has also been found in lunar samples, but analysis still ongoing
- Search for 60Fe in magnetofossils in marine sediment is ongoing