Nuclear Astrophysics II

Lecture 8,9

Fri. June 22, 29, 2012

Prof. Shawn Bishop, Office 2013,

Ex. 12437

shawn.bishop@ph.tum.de

http://www.nucastro.ph.tum.de/

What are the minimum masses required for nuclear burning?

IGNITION MASSES

Let's take as our equation of state (EOS) the sum of the ion gas pressure (ideal) and the degenerate (non-relativistic) electron pressure:

$$P = P_{ideal} + P_{deg}$$

From Lec. 2 of last Semester: $P_{deg} = \frac{\pi^3}{15 m_e}\hbar^2 \left(\frac{3n_e}{\pi}\right)^{5/3}$ where, $n_e \equiv N_e/V = \frac{N_A}{\mu_e}\rho$

$$\Rightarrow \quad P = \frac{N_A k}{\mu}\rho T + \frac{\pi^3}{15 m_e}\hbar^2 \left(\frac{N_A}{\pi \mu_e}\right)^{5/3} \rho^{5/3}$$

$$\Rightarrow \quad \frac{P}{\rho} = AT + B\rho^{2/3}$$

Where: $A = \frac{N_A k}{\mu}$ and $B = \frac{\pi^3}{15 m_e}\hbar^2 \left(\frac{N_A}{\pi \mu_e}\right)^{5/3}$

From Lec. 2, pg. 35, Sem. 1, it was an exercise for you to show that the central pressure of a star is:

$$P_c = \frac{1}{4\pi(n+1)(d\phi/d\xi)^2_{\xi_*}} \frac{GM_*^2}{R_*^4}$$

We also had the relation between central and average density: $\rho_c \approx 54\bar{\rho} = 54\frac{M_*}{(4\pi/3)R_*^3}$

Use this to obtain R_* and eliminate R_* from central pressure. After the algebra, we have:

$$\frac{P_c}{\rho_c} = \frac{G}{4\pi(n+1)[54/(4\pi/3)]^{4/3}(d\phi/d\xi)^2_{\xi_*}} M^{2/3}\rho_c^{1/3}$$

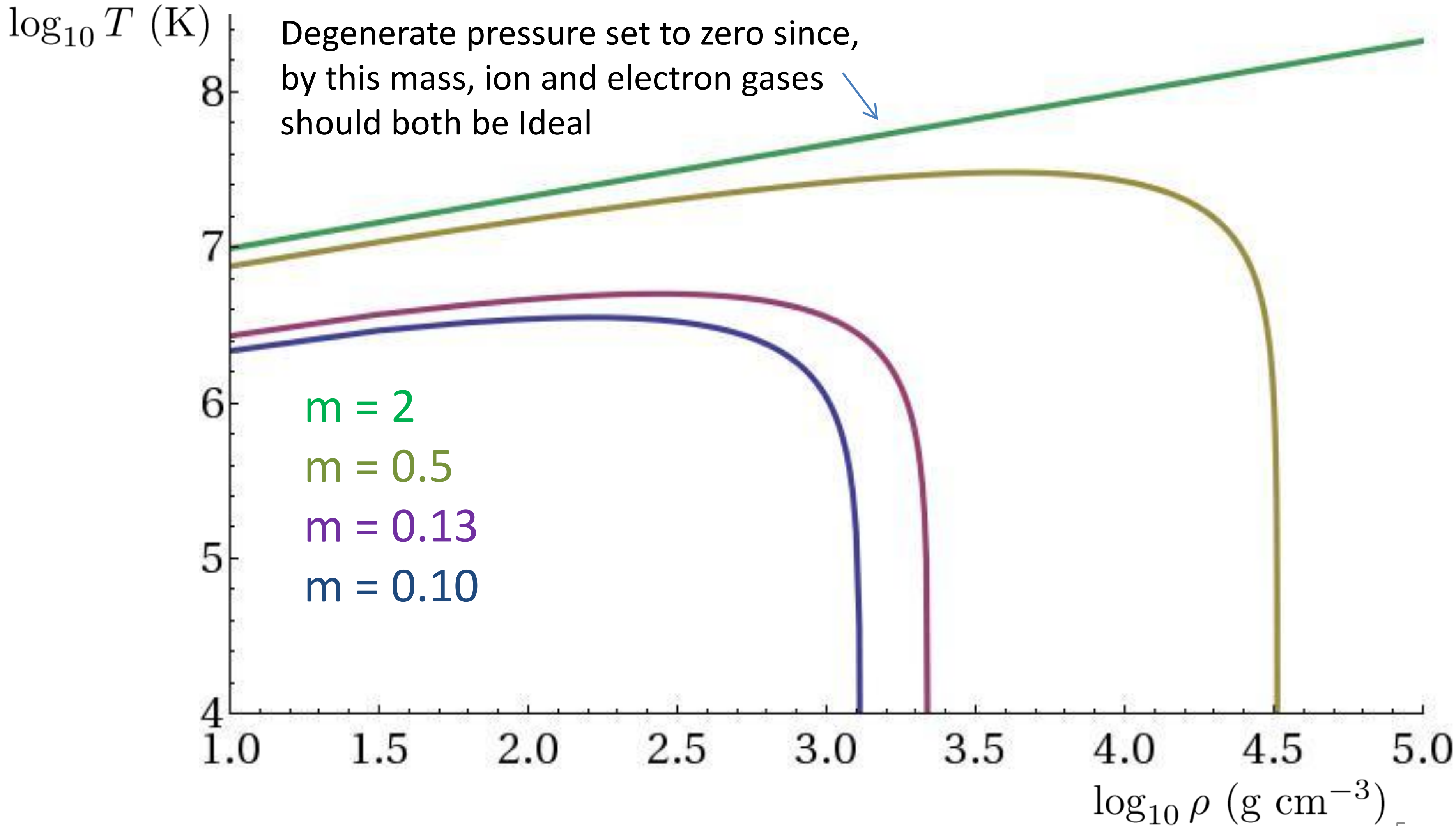
$$= DM^{2/3}\rho_c^{1/3}$$

Let us sub. this into our EOS from previous page:

$$\Rightarrow DM^{2/3}\rho_c^{1/3} = AT + B\rho_c^{2/3}$$

Isolating for temperature, we finally have the temperature as a function of density:

$$T = A^{-1}(DM^{2/3}\rho^{1/3} - B\rho^{2/3})$$



Coming back to the temperature function:

$$T = A^{-1}(DM^{2/3}\rho^{1/3} - B\rho^{2/3})$$

First thing to notice: This function of temperature has a maximum. Taking $dT/d\rho = 0$ yields the following condition:

$$DM^{2/3}\rho_0^{1/3} = 2B\rho_0^{2/3}$$

Here, ρ_0 is the value of ρ at maximum T.

Sub. these conditions back into temperature formula to get max. temperature.

$$\boxed{T_{max} = \frac{D^2}{4AB}M^{4/3}}$$

We will need this to let us determine stellar “Ignition Masses”; the minimum mass required to ignite a particular fuel, such as H, He, C, etc.

Consider a small star, undergoing gradual contraction. As it does so, its internal temperature will rise.

One of two things will happen: Either the mass will reach a temperature where the fuel ignites, and nuclear burning begins, or the mass never reaches this temperature and, instead, ends its contraction supported by electron degeneracy pressure.

On page 6, we saw what the internal temperature function looks like, and we see that, when degeneracy pressure is included, the internal temperature passes through a maximum.

It is at this temperature maximum where our fuel must ignite. If it does not ignite at this maximum temperature, then the star will not burn the fuel and instead becomes an object supported by degeneracy pressure.

We must find a condition connecting mass, this max. temperature, and nuclear burning, to determine what the stellar ignition masses are.

The condition connecting all of these is the luminosity relation:

$$L = M \langle \epsilon \rangle$$

Back in Lec. 7, page 11, we saw that the average nuclear energy generation rate (averaged over mass), is:

$$\langle \epsilon \rangle = \frac{3.26}{(3u+s)^{3/2}} \epsilon_c$$

Remember: u is the exponent on density and s is the exponent on temperature in the nuclear energy generation rate ϵ_c . (u = 2 except for the triple-alpha reaction, u = 3)

$$\epsilon_c \propto (X_i X_j)^{u-1} \rho_c^{u-1} \left(\frac{T_c}{T_0}\right)^s$$

We know the formula for s: it is what we can "n".

$$\epsilon = Q\frac{r_{12}(T)}{\rho} = Q\frac{r_{12}(T_0)}{\rho}\left(\frac{T}{T_0}\right)^{(-2+3\frac{E_{\rm eff}}{\tau})/3}$$

$$3\frac{E_{\rm eff}}{\tau} = 42.487\left(\frac{Z_1^2 Z_2^2 \mu}{T_6}\right)^{1/3}$$

If resonant reaction (3-alpha) then s = n is

$$n = 42.9/T_8 - 3$$

We know that energy conservation requires that the nuclear energy produced in the core must be balanced by the luminosity emitted by the star.

The ignition mass is that minimum mass such that the luminosity is just balanced by the nuclear energy production.

$$L_* = M_* \langle \epsilon \rangle$$

In Lec. 6, Sem. 1, we worked out the Mass-Lum. relation to be:

$$L_* = 1.35 \times 10^{35} \frac{(\mu\beta)^4}{\kappa} \left(\frac{M_*}{M_\odot} \right)^3 \ \mathrm{erg/s}$$

And, when the opacity is manipulated further (Lec. 6, Sem. 1), this result became, for Main Sequence stars:

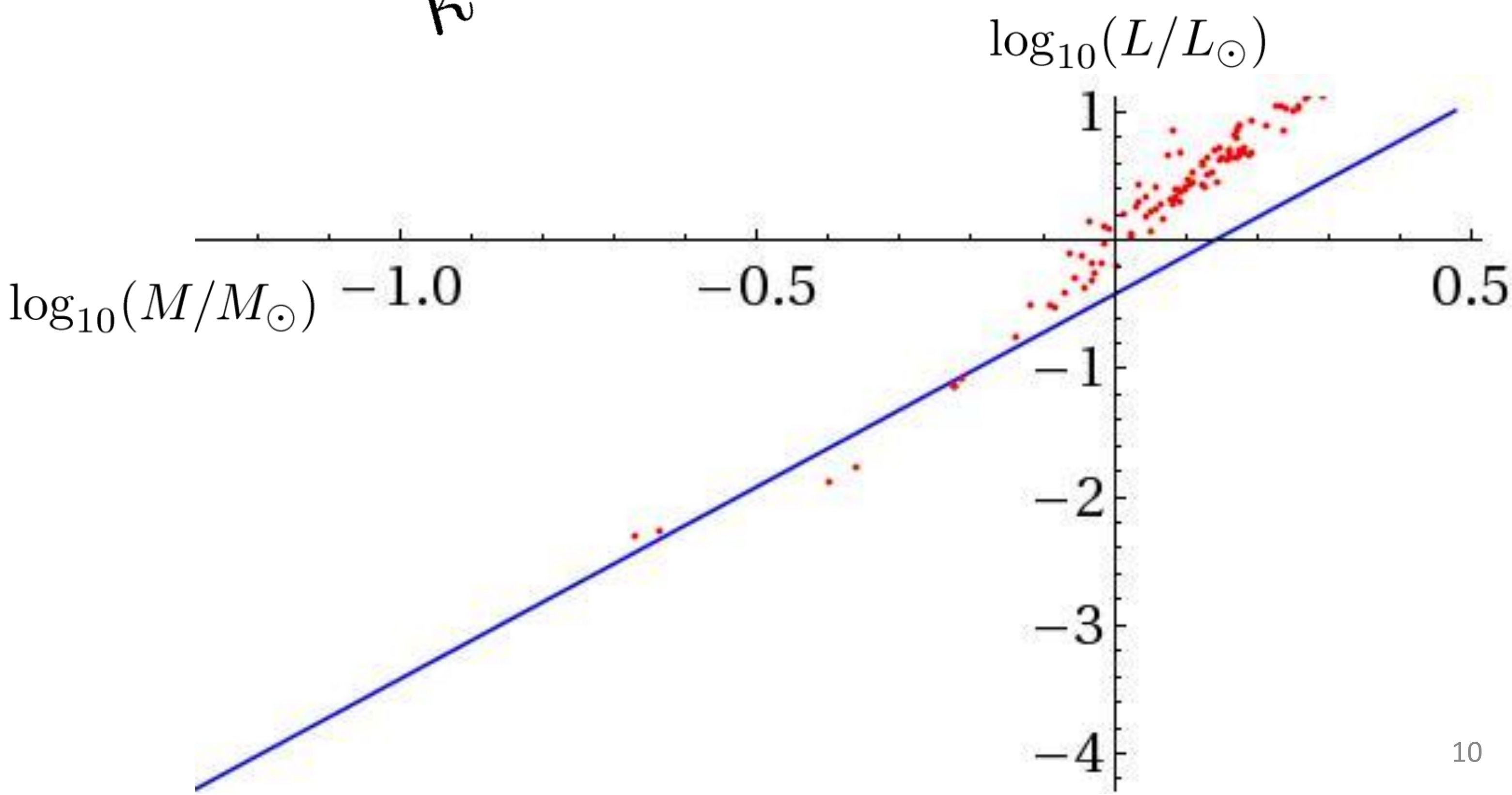
$$\frac{L_*}{L_\odot} = 103 \frac{(\mu\beta)^7 m^5}{m^2(\mu\beta)^3 + 5.94}$$

Ignition Mass: Solar Composition

Reference back to Lect. 6, Sem. 1, we see that the L-M relation does not do a good job describing the Main Sequence data at low mass. Let's "fix" this by taking the derived formula (previous page) and scale the denominator to "force it" through the low mass data.

$$L_* = 1.35 \times 10^{35} \frac{(\mu\beta)^4}{25/\mu_e} \left(\frac{M_*}{M_\odot}\right)^3 \;\; \text{erg/s}$$

$\kappa =$



We use the PP-chain (equilibrium) energy generation rate, with $\epsilon_0 = 0.068$ and $T_0 = 10^7$

$$L_* = 1.35 \times 10^{35} \frac{(\mu\beta)^4}{25/\mu_e} \left(\frac{M_*}{M_\odot}\right)^3 = M_* \, \epsilon_0 \rho X_p^2 \left(\frac{T}{T_0}\right)^n \qquad (**)$$

with the result from Lec. 2 Sem. 1, page 37, $T_c = 4.6 \times 10^6 \mu\beta \left(\frac{M_*}{M_\odot}\right)^{2/3} \rho_c^{1/3}$
to substitute in for ρ. Set $\beta = 1$

With the substitution for ρ, RHS is a function of temperature only and LHS will be a function of stellar mass (after division by M_* on both sides).

For solar composition, X = 0.7, and Y = 0.3, and we assume full ionization, so:

$$\mu = [X_p/m_p + Y/m_\alpha]^{-1} \approx [X_p + Y/4]^{-1} \approx 1.28$$

$$\mu_e = [X_p/m_p + 2Y/m_\alpha]^{-1} \approx [X_p + Y/2]^{-1} \approx 1.18$$

We solve (**) for temperature, by choosing masses and determining (numerically) the corresponding temperature.

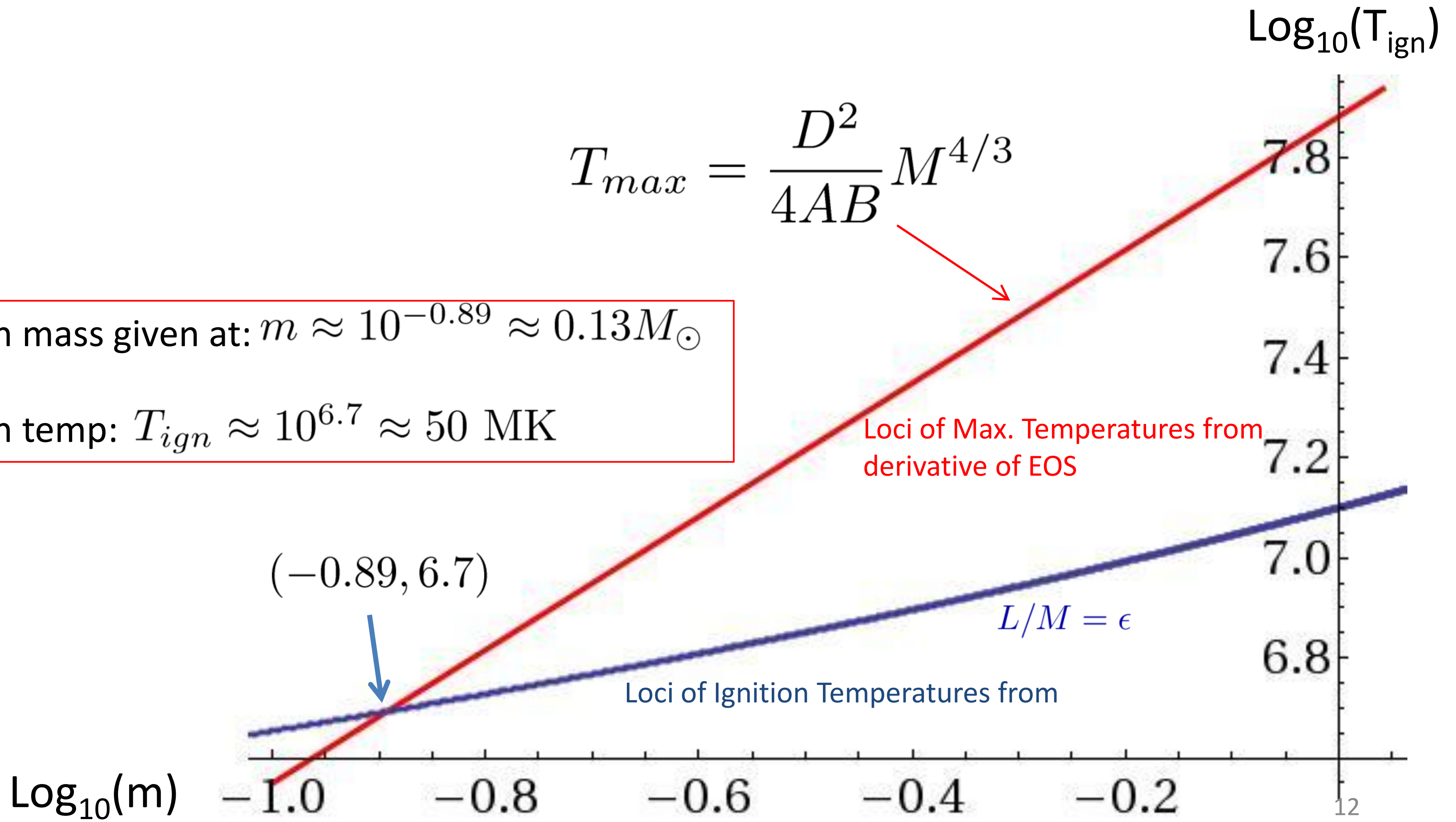
Ignition Mass: Pure Hydrogen

Ignition of fuel will occur when the max. temperature from EOS is at least the same as temperature that satisfies $L/M = \epsilon$, which is: $T_{max} \geq T_{ign}$

$$T_{max} = \frac{D^2}{4AB} M^{4/3}$$

Ignition mass given at: $m \approx 10^{-0.89} \approx 0.13 M_{\odot}$

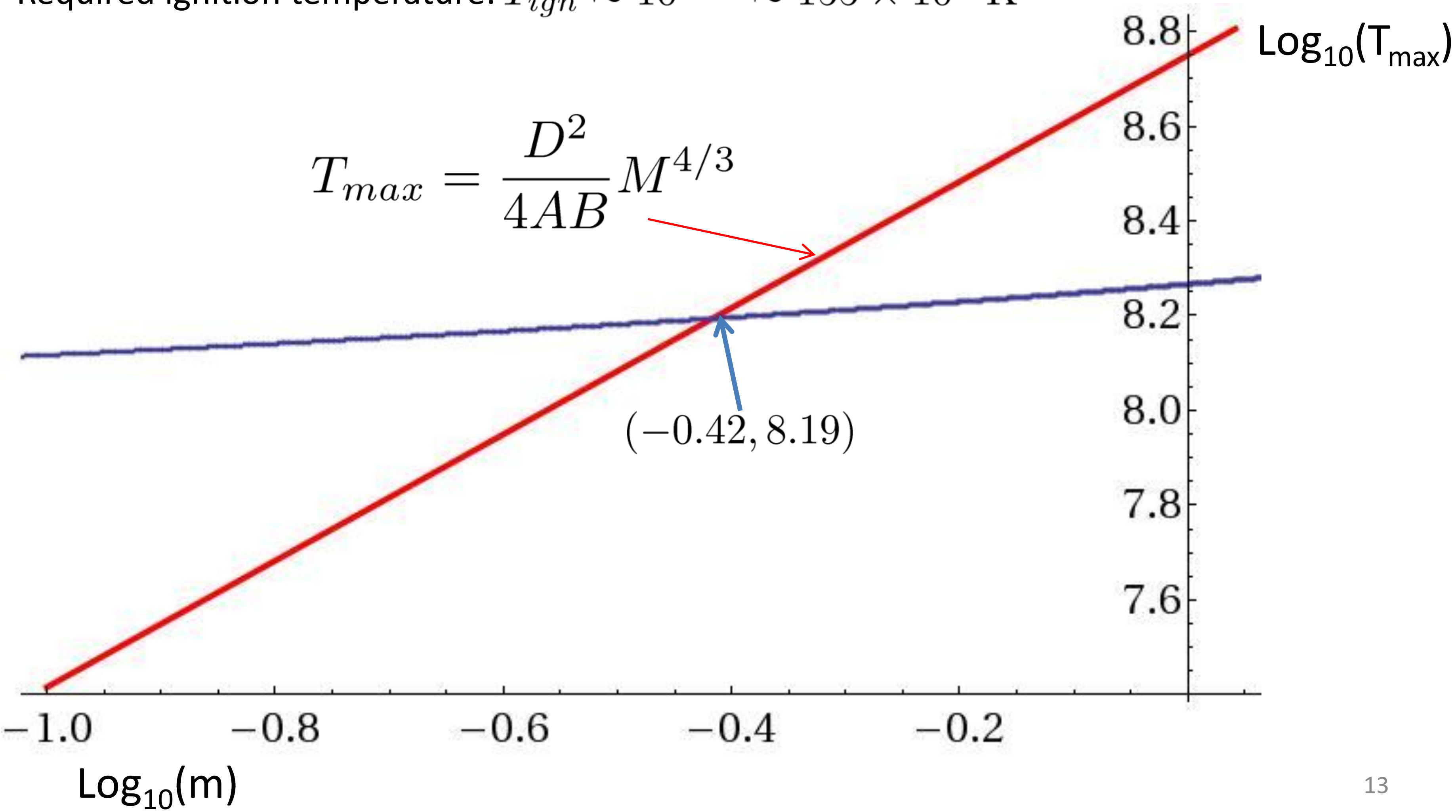
Ignition temp: $T_{ign} \approx 10^{6.7} \approx 50 \text{ MK}$



He Core: Ignition Mass & Temperature

The minimum mass required for the triple-α reaction is around $m \approx 10^{-0.42} \approx 0.38 M_{\odot}$

Required ignition temperature: $T_{ign} \approx 10^{8.19} \approx 155 \times 10^6$ K



The Fate of Massive Stars

THE PATH TO CORE COLLAPSE

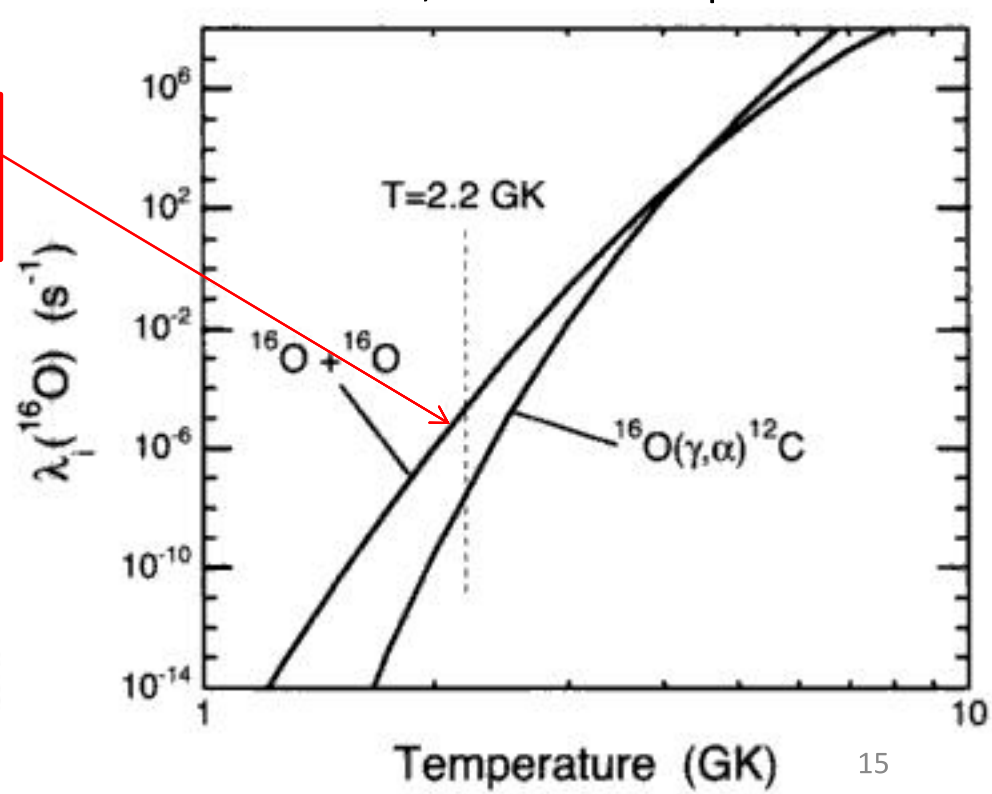
Oxygen Burning

Once the neon fuel has been photodisintegrated down to a negligible mass fraction, the core must contract again. As it does so, it heats up.

Among the products resulting from neon-burning, oxygen has the lowest Coulomb barrier, and a process similar to carbon burning happens with the oxygen.

The p, n and α-binding energies of ^{24}Mg, ^{28}Si are all > 9 MeV. Their photodisintegration rate is small at these temperatures and densities. Same is true for ^{16}O, with the exception of its α-binding energy (7.2 MeV).

^{16}O + ^{16}O dominates over photodisintegration.



$^{16}O(^{16}O,p)^{31}P$ $\quad (Q = 7678\,\mathrm{keV})$

$^{16}O(^{16}O,2p)^{30}Si$ $\quad (Q = 381\,\mathrm{keV})$

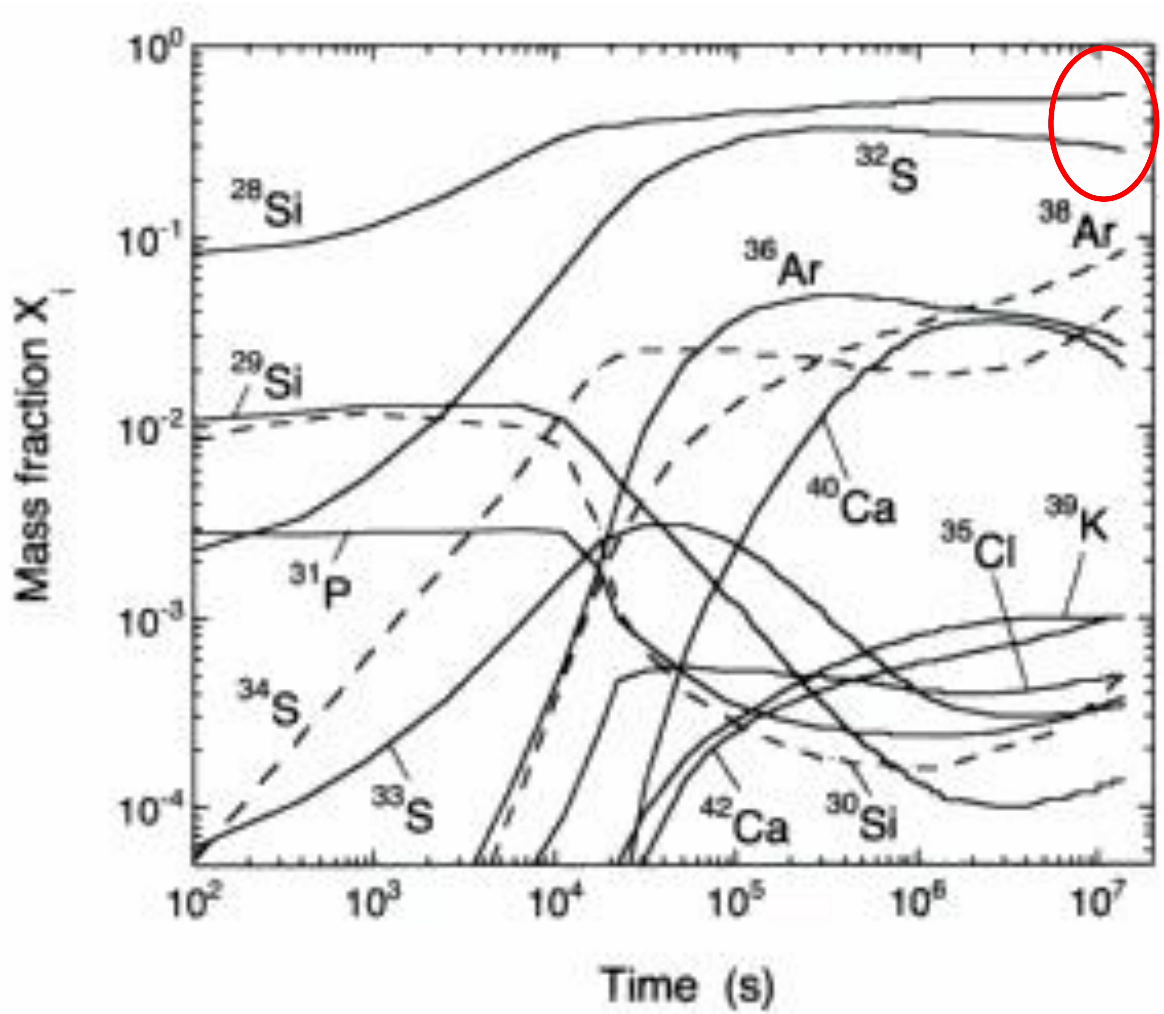
$^{16}O(^{16}O,\alpha)^{28}Si$ $\quad (Q = 9594\,\mathrm{keV})$

$^{16}O(^{16}O,2\alpha)^{24}Mg$ $\quad (Q = -390\,\mathrm{keV})$

$^{16}O(^{16}O,d)^{30}P$ $\quad (Q = -2409\,\mathrm{keV})$

$^{16}O(^{16}O,n)^{31}S$ $\quad (Q = 1499\,\mathrm{keV})$

Oxygen Burning Abundance Results



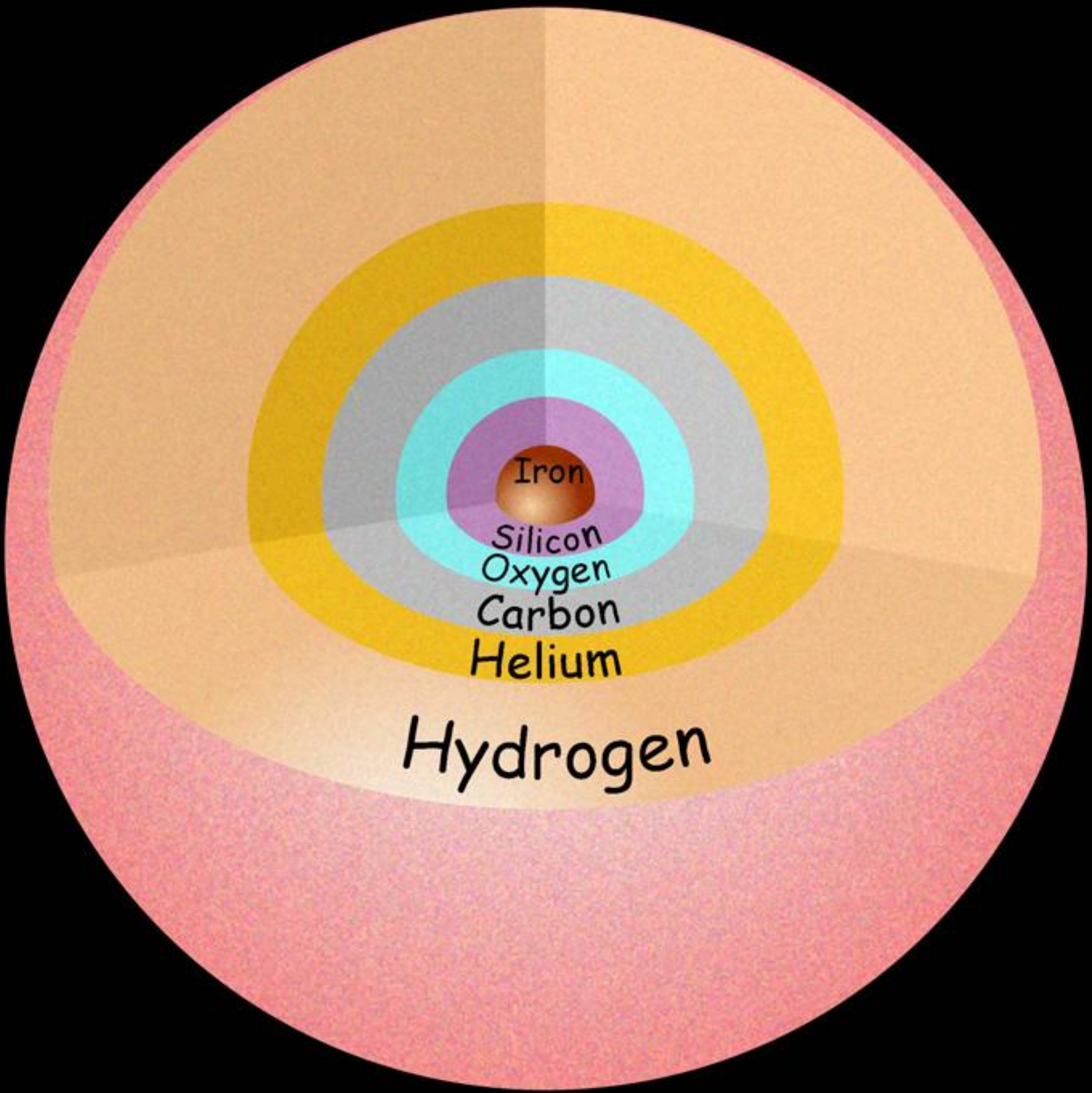
Silicon and sulfur the main products

A few lectures ago we left off at this picture of our 25 solar mass star.

We have covered what happens up to core Oxygen burning, and now it is time to look at what happens next.

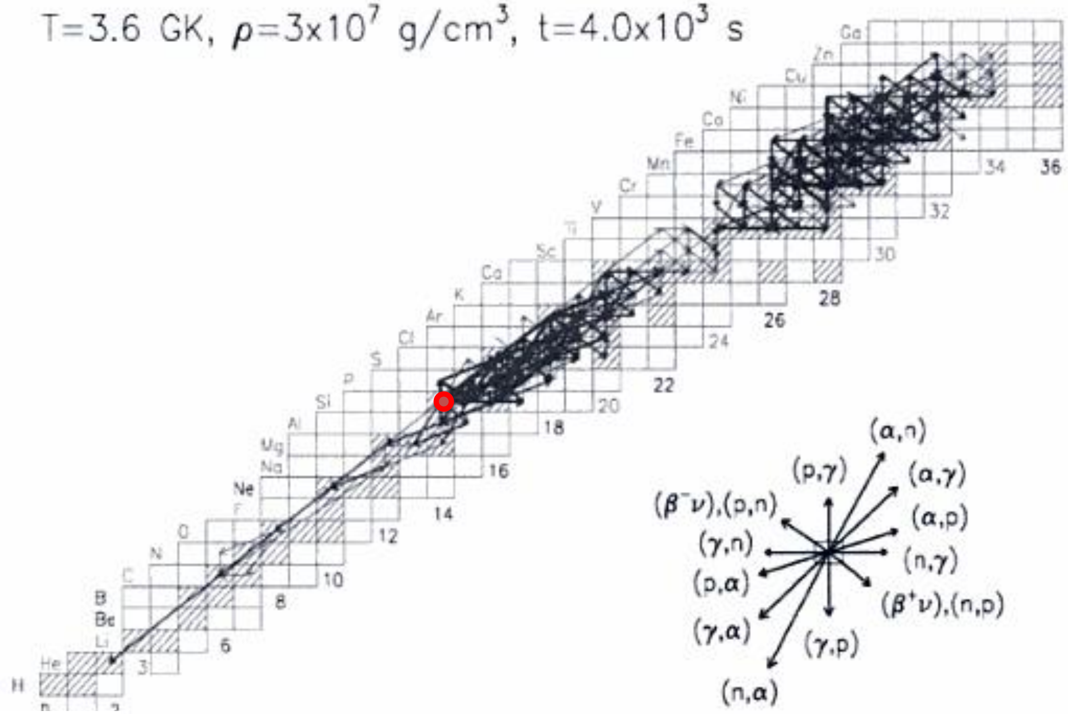
Our star undergoes another core contraction, leaving behind the oxygen burning shell while the central condensate heats up to a temperature of ~ 3GK, whereupon so-called Silicon burning takes place.

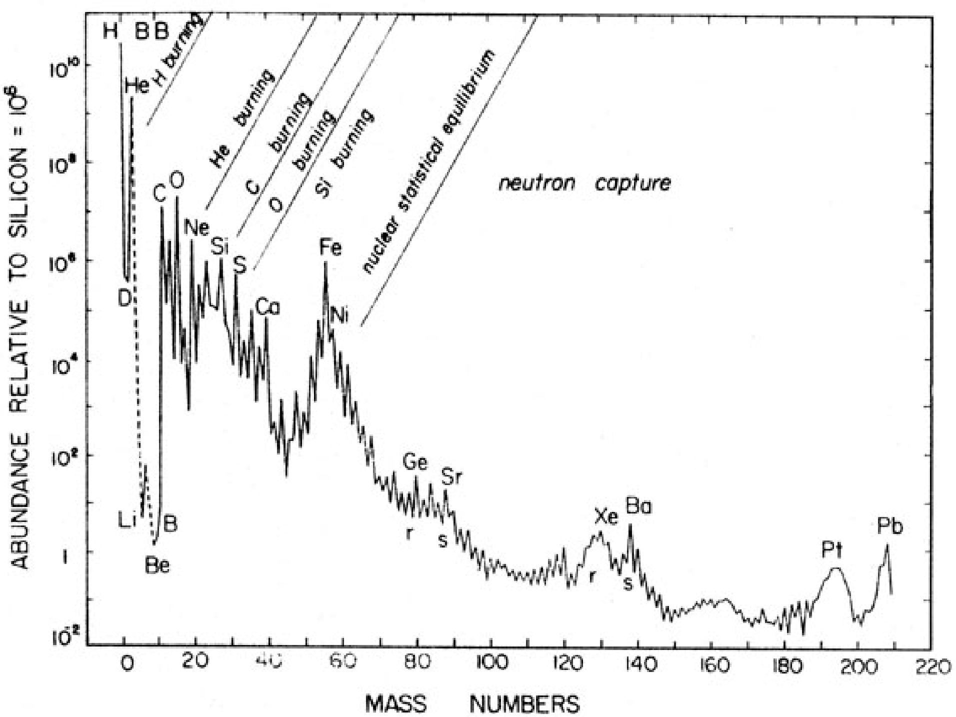
The term "burning" here is not quite correct. It is something more like a "melt". At these extreme temperatures, photo-disintegration becomes intense, and the sulfur and silicon that were produced in Oxygen burning now undergo destruction by photodisintegration reactions. The star's fate is sealed at this point: there is no escape from core collapse.

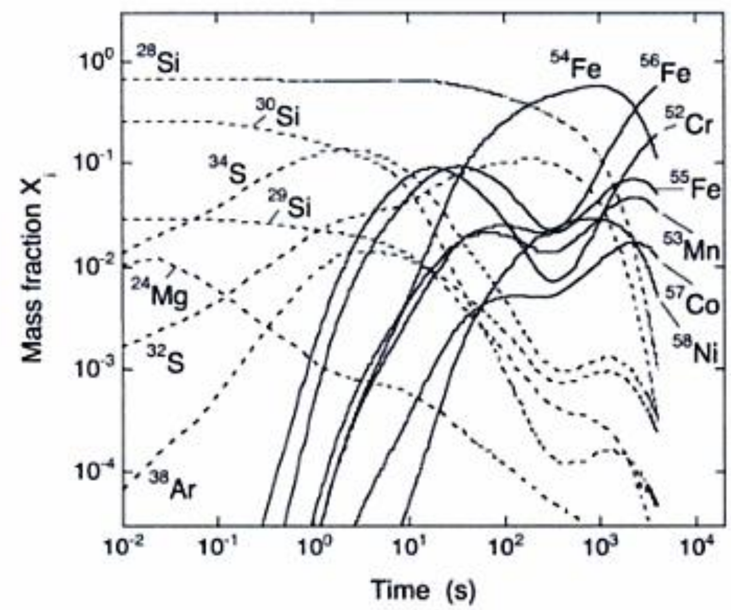


The “Melt”

The result of a network calculation (constant temp and density) is shown here. ^{28}Si is marked by red dot. Note: this is not from a full hydrodynamical calculation: the temperature and density are **both fixed**. The calculation does, however, help us to understand the results of full calculations. So, let’s try to understand the features we see here.







Silicon Burning (Melt)

Most abundant nuclides present after ^{16}O burning are ^{28}Si and ^{32}S.

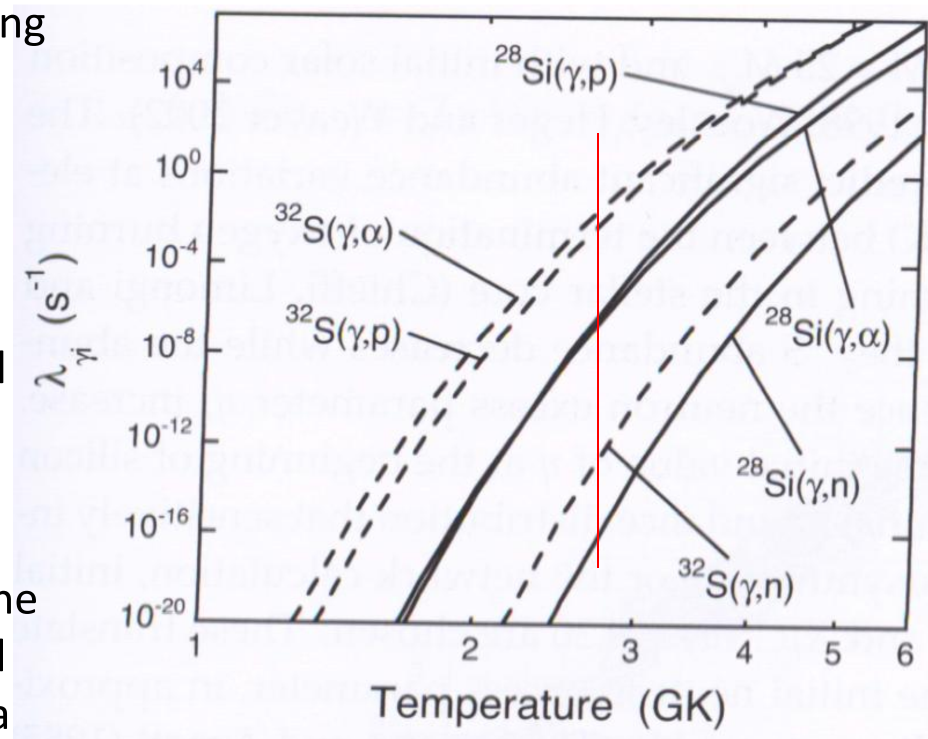
Core contraction occurs again → temperature increases to range around 2.8 – 4.1 GK.

Even with such a high temperature, ^{28}S + ^{28}S will not take place because of prohibitive Coulomb barrier.

Instead, photodisintegration rates on many of the nuclei present result in free alpha-particles (and protons, and neutrons) that can then establish a quasi-equilibrium abundance distribution with the various nuclei present.

Decay constants for photodisintegration shows that ^{32}S is the first to disintegrate.

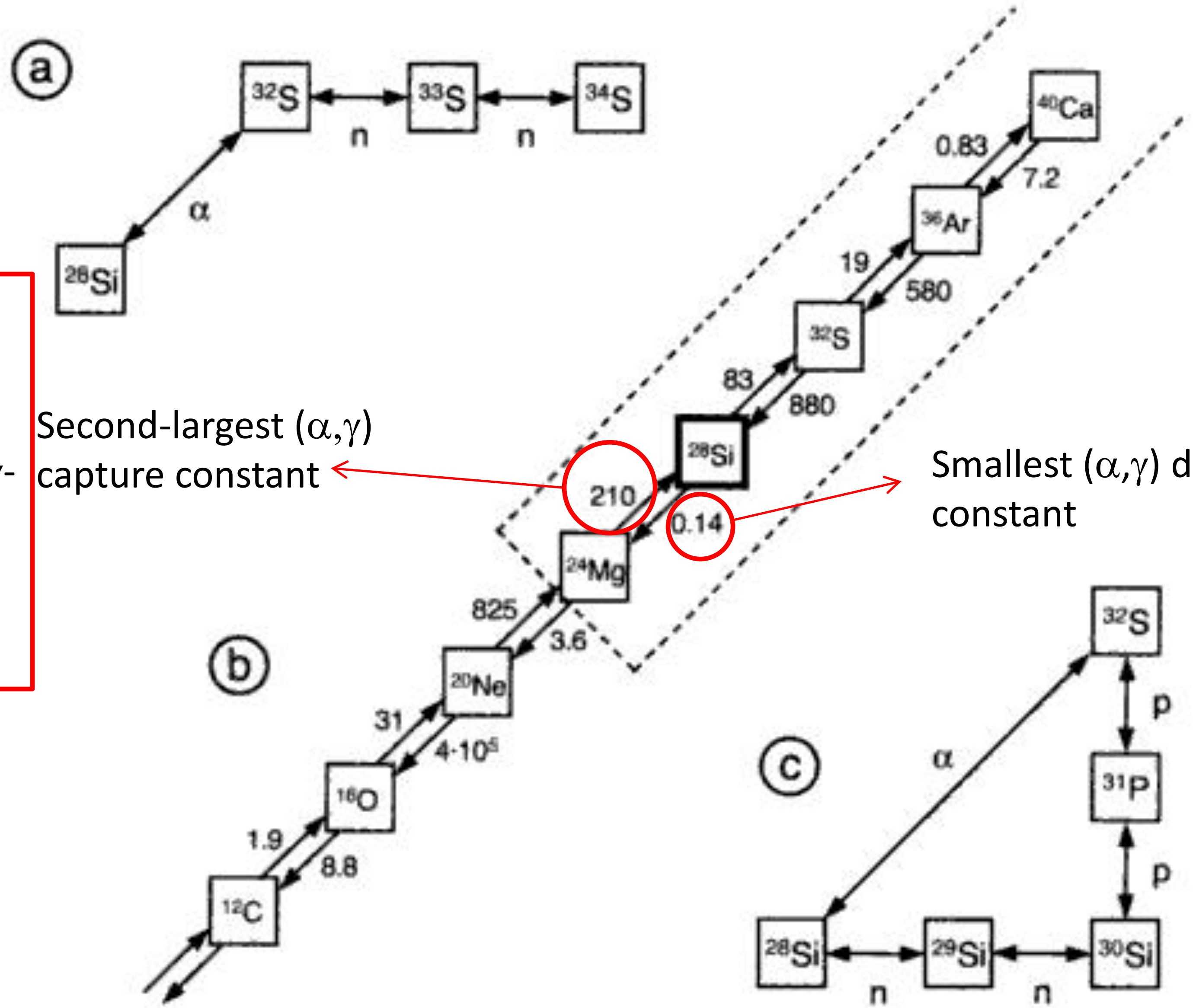
Plot shows the reaction rates and provides hint as to which ones will be in equilibrium.



	B_p (MeV)	B_n (MeV)	B_α (MeV)
^{28}Si	11.60	17.20	9.98
^{32}S	8.90	15.00	6.95

# Photo disintegration and alpha-Capture constants
T = 3.6 GK
rho = 3 x 10^7 g/cm^3

Equilibrium flow will eventually cluster around ^{28}Si, at first, because of its small γ-disintegration rate and the large ^{32}S γ-disintegration rate.



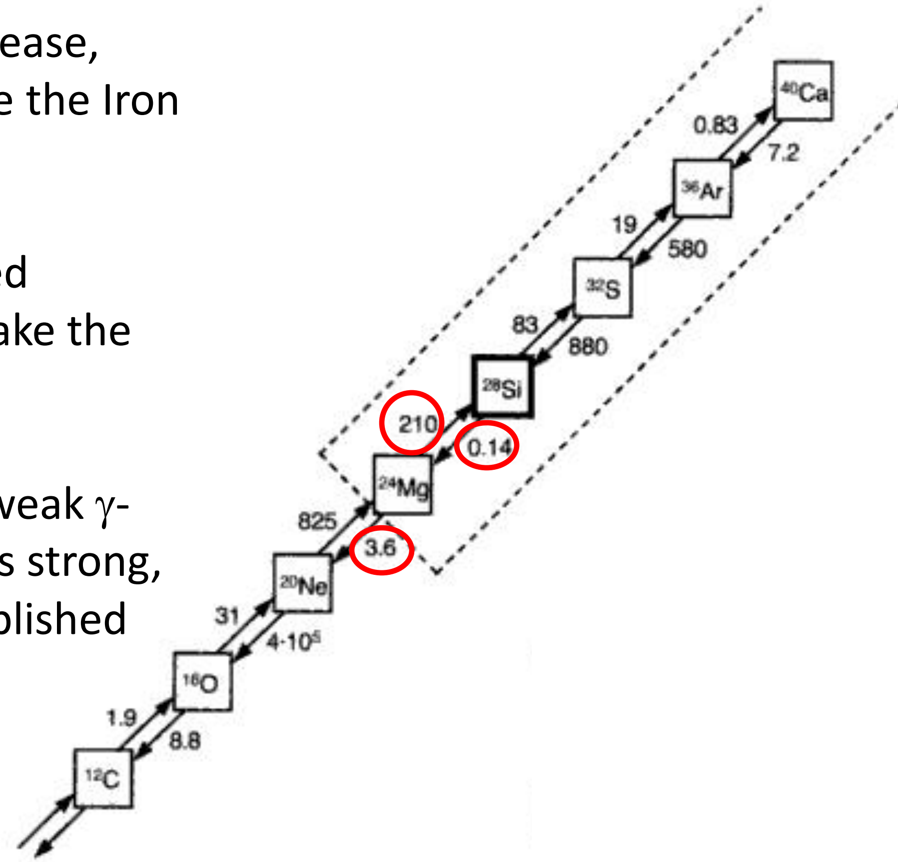
Second-largest (α,γ) capture constant

Smallest (α,γ) decay constant

As the temperature and density continues to increase, material will continue to flow up beyond ^{40}Ca, the the Iron group nuclei, primarily through α-captures.

This takes time, however. Why? Because we need sufficient abundance of free alpha particles to make the forward α-particle rate appreciable.

^{28}Si is where the process starts. And ^{24}Mg has a weak γ-disintegration, too. The forward rate from ^{24}Mg is strong, thus a $(\gamma,\alpha) \leftarrow\rightarrow (\alpha,\gamma)$ equilibrium is quickly established between these two nuclides.



T = 3.6 GK, ρ = 3 x 107 g/cm^3, X_α = 10^{-6}

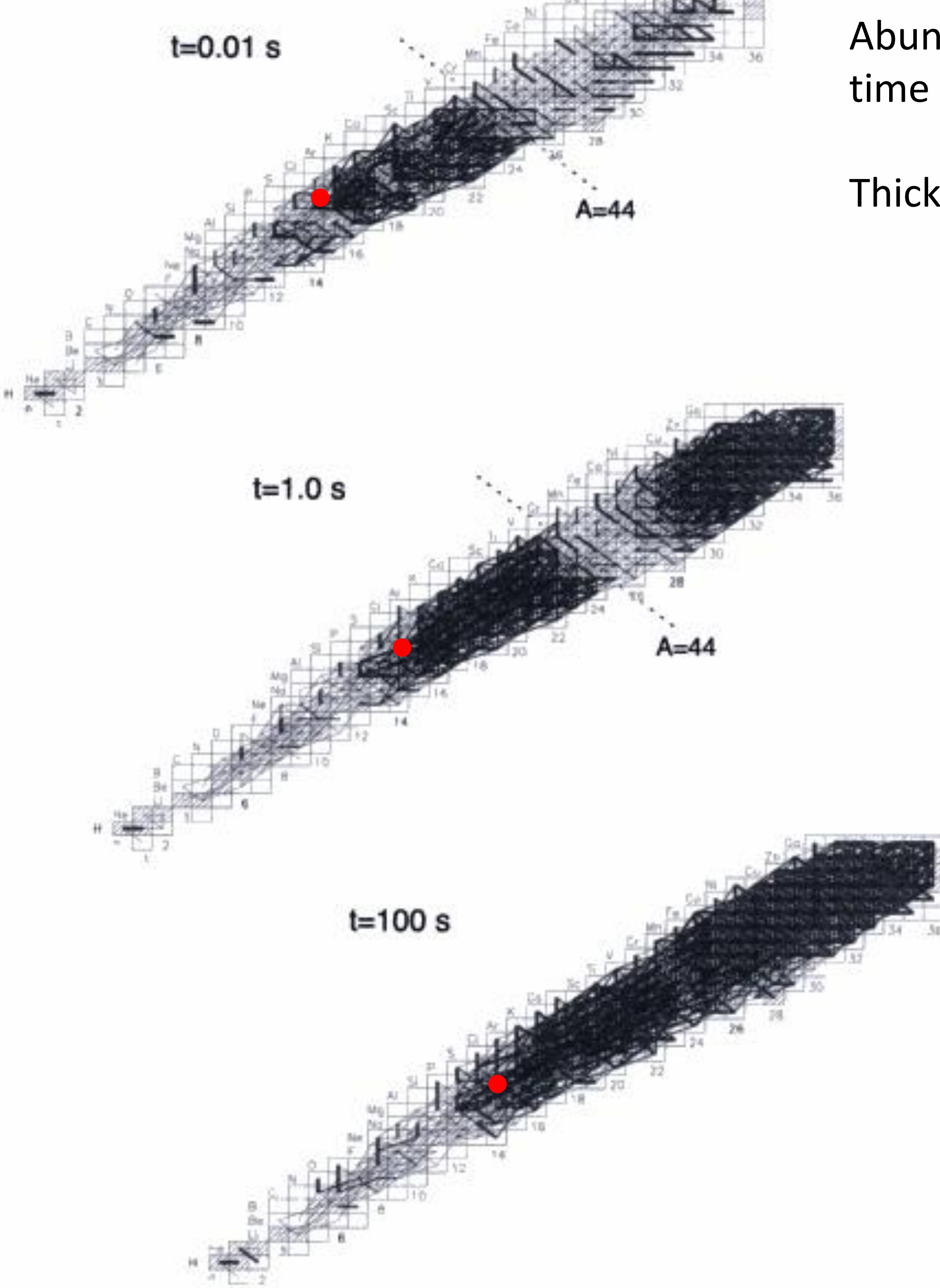
Equilibrium abundances at this first step

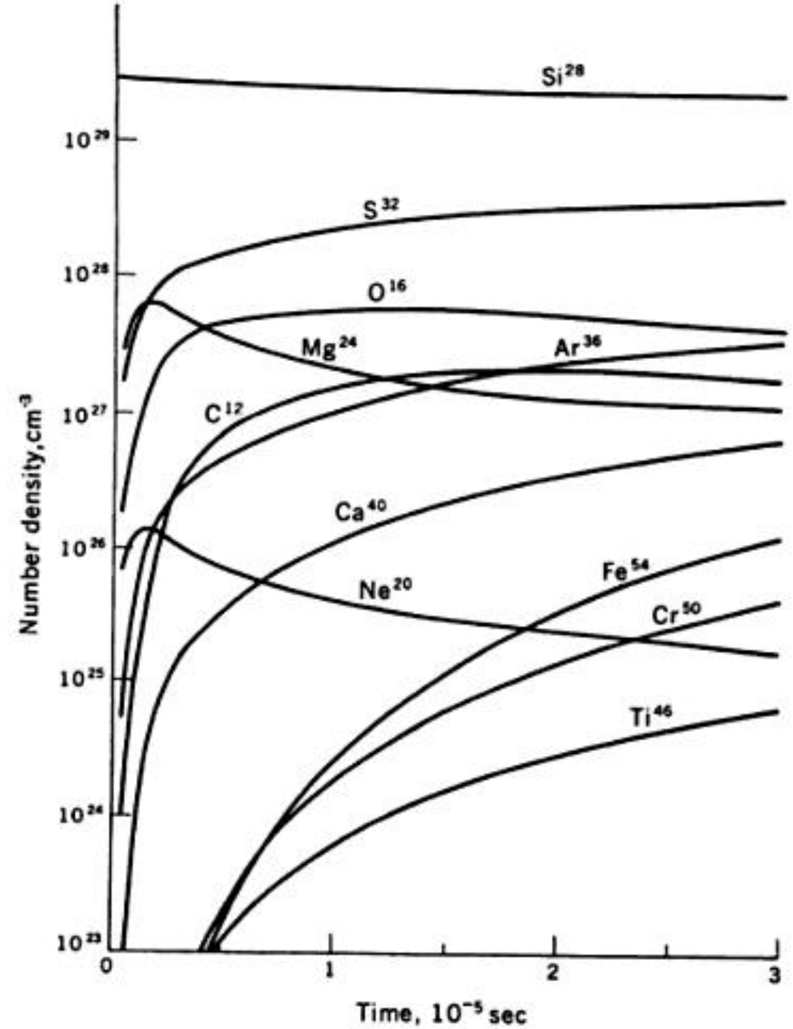
$$\frac{N_{24} N_\alpha}{N_{28}} = \left(\frac{2\pi \mu_{24,\alpha} \tau}{h^2} \right)^{3/2} e^{-Q_{24,\alpha \rightarrow 28}/\tau}$$

Spins are all zero, so g-factors are all unity.

Abundance flows of Si-burning for different time snap-shots.

Thick = strong forward-backward equilibrium





J. W. Truran et al. Can. J. Phys. **44**, 576 (1966)

Eventually, during the contraction and ongoing heat production, the system of all these reactions and their inverses comes into forward-backward equilibrium.

We call this situation Nuclear Statistical Equilibrium (NSE).

Abundances of protons, are then determined by the Saha Equation.

$$\frac{N(A-1,Z)N_n}{N(A,Z)} = \frac{2g(A-1,Z)}{g(A,Z)} \left(\frac{2\pi\mu_{A-1,n}\tau}{h^2}\right)^{3/2} e^{-Q_n/\tau}$$

$$\frac{N(A-2,Z-1)N_p}{N(A-1,Z)} = \frac{2g(A-2,Z-1)}{g(A-1,Z)} \left(\frac{2\pi\mu_{A-2,p}\tau}{h^2}\right)^{3/2} e^{-Q_p/\tau}$$

Repeated application of these will give you:

$$N(A,Z) = \frac{1}{\theta^{A-1}} \frac{N_p^Z N_n^{A-Z}}{2^A} \left(\frac{M(A,Z)}{M_p^Z M_n^{A-Z}}\right)^{3/2} g(A,Z) e^{Q(A,Z)/\tau}$$

$$\theta = \left(\frac{2\pi\mu_{A,p}\tau}{h^2}\right)^{3/2}$$

Binding Energy per Nucleon

Recall from lecture 1 that the a-particle nuclei have proton-binding energies that are, locally, a little bit larger than the nearest neighbours. It is this feature, combined with the alpha-particle photodisintegrations that govern the previous burning steps.

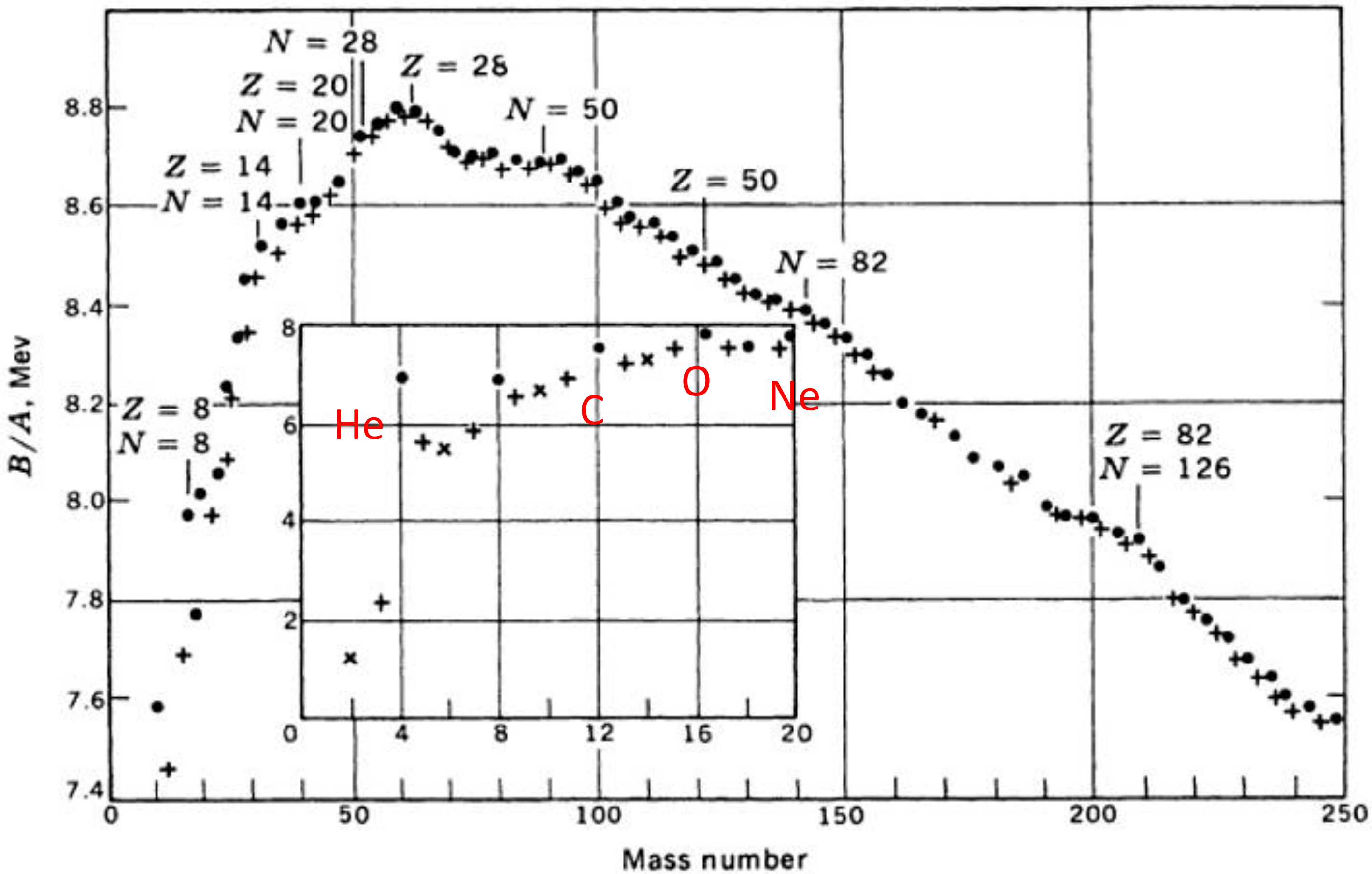
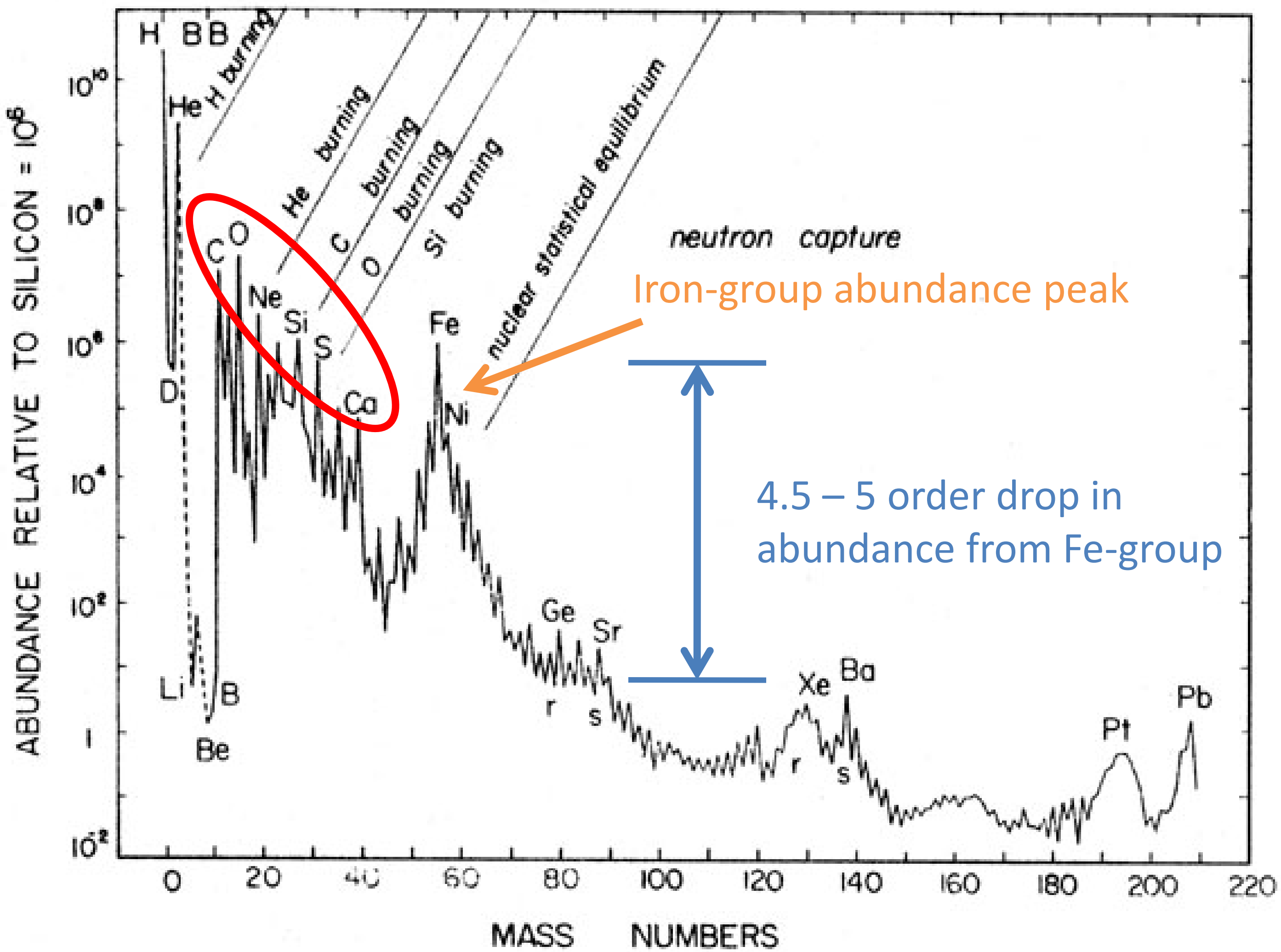


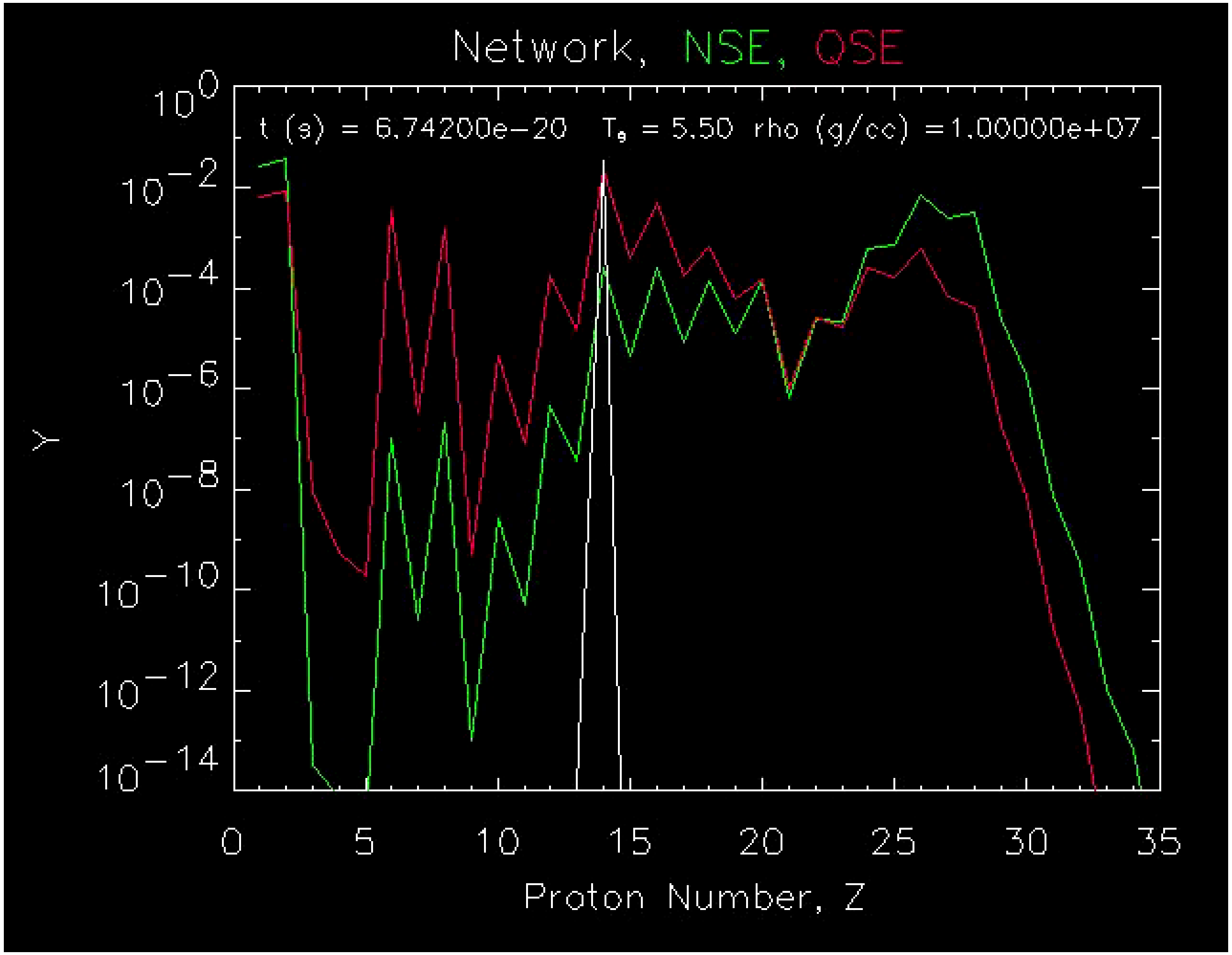
Fig. 7-1 The binding energy per nucleon of the most stable isobar of atomic weight A. The

Solar Abundances Continued

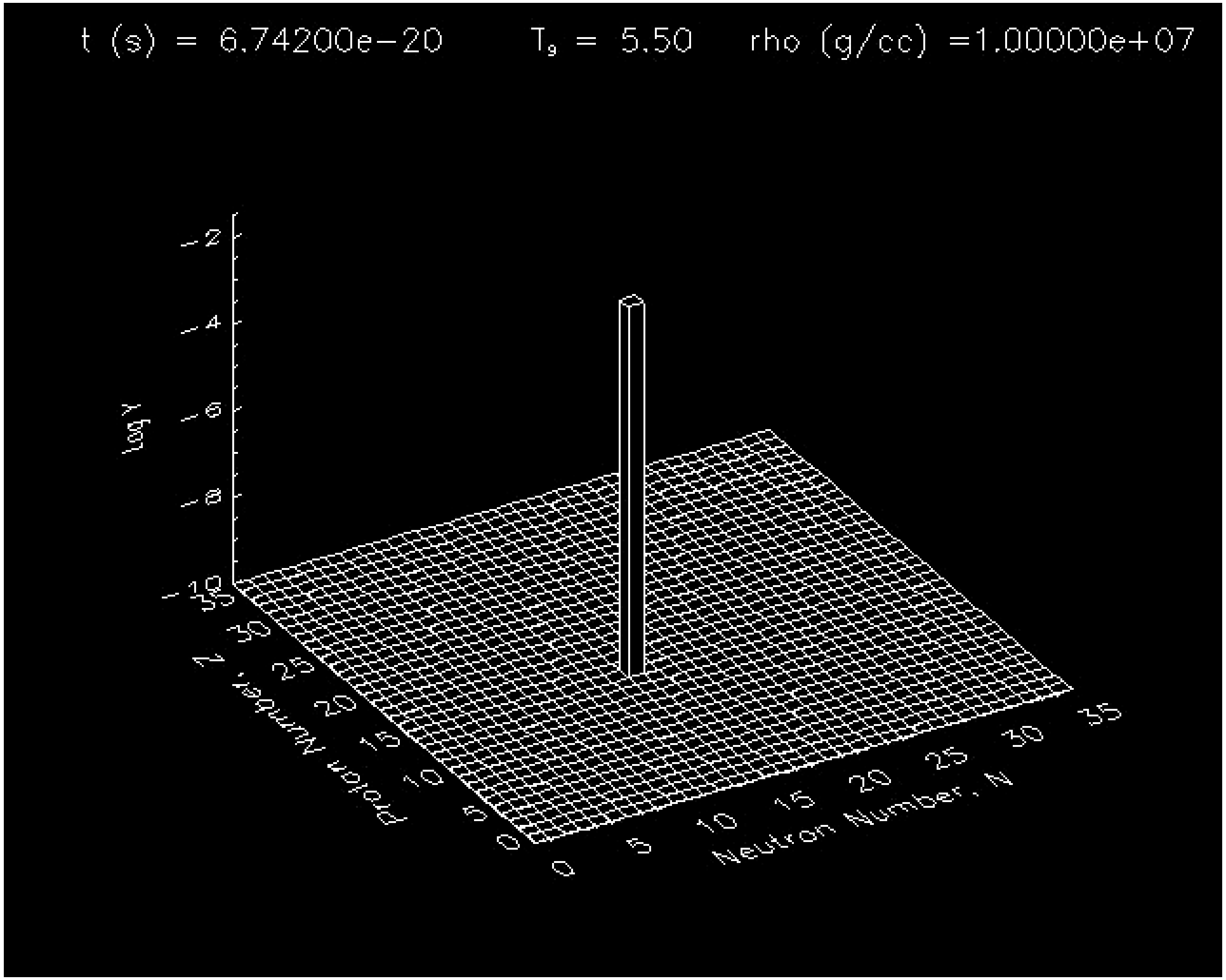


α-particle nuclei have local maxima relative to neighbouring masses

- Will learn more about this further into the course

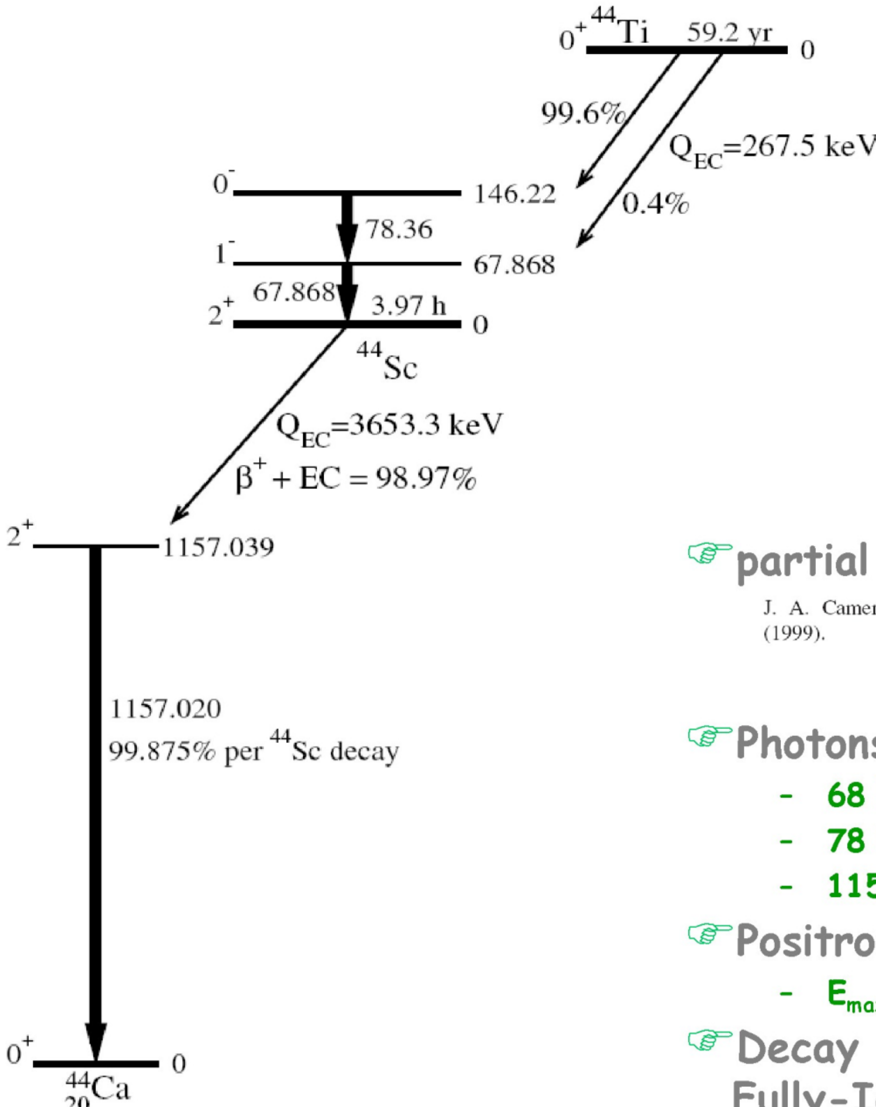


http://nucleo.ces.clemson.edu/home/movies/alpha_rich/mpg/y_network_nse_qse.mpg



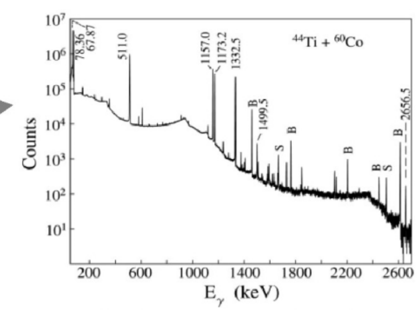
http://nucleo.ces.clemson.edu/home/movies/alpha_rich/mpg/abundance_histogram.mpg

^{44}Ti ($T_{1/2}$ = 60.4 years)

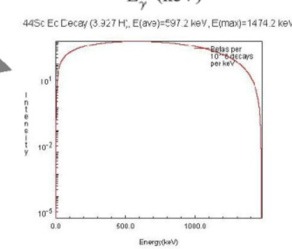


- partial level schemes and ^{44}Ti decay

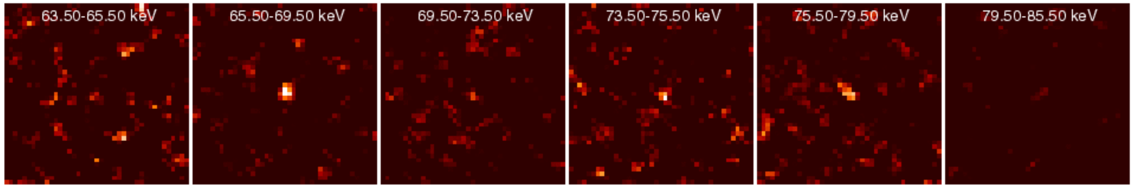
J. A. Cameron and B. Singh, Nucl. Data Sheets **88**, 299 (1999).



- Photons:
  - 68 keV 100%
  - 78 keV 99.6%
  - 1157 keV 99.9%
- Positrons
  - E_{max}=1.47 MeV 98%
- Decay Inhibited for Fully-Ionized ^{44}Ti (EC!)



^{44}Ti is a radio-isotope ($T_{1/2}$ = 60.4 years) that should be produced through Type II SNa and the burning processes we have just seen. Thus far, it has only been (barely) detected in just one Supernova event: SNa Cas A.



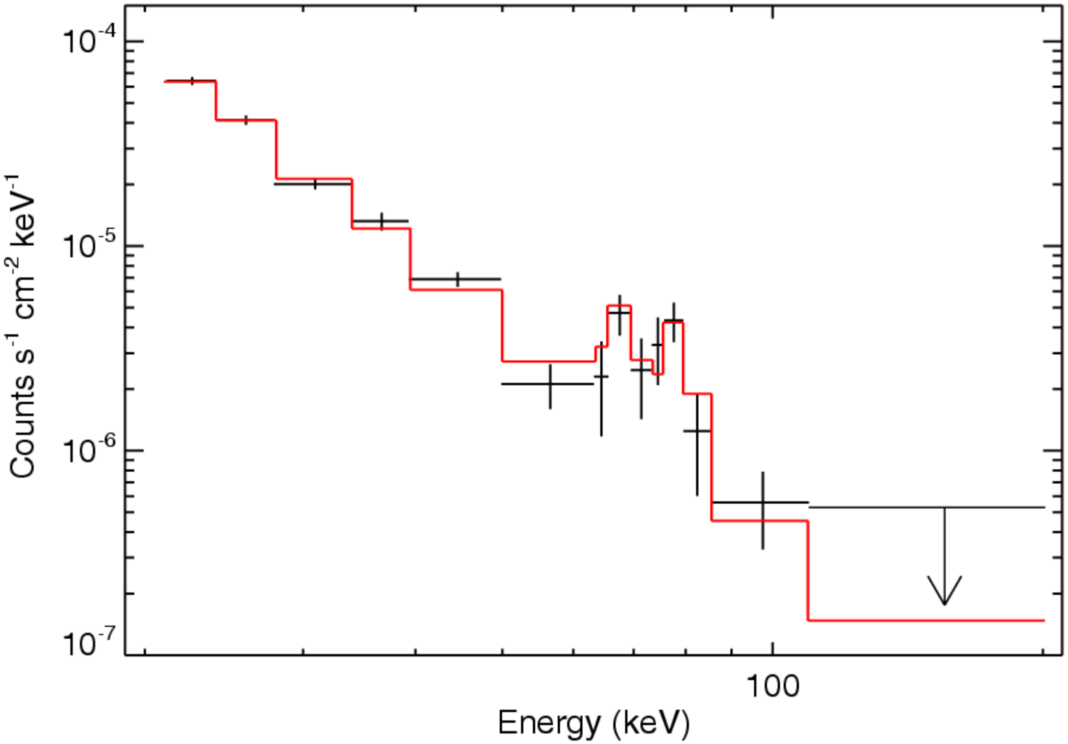


Image graphic: http://sci.esa.int/science-e/www/object/index.cfm?fobjectid=40068
Journal Article: [Astrophysical Journal **647**, L41 (2006)]

Going Beyond the Iron Peak

The binding energy per nucleon beyond ^{56}Fe drops from a maximum, which means that producing these nuclei by charged particle reactions actually steals energy from the star.

Moreover, after Si-burning, the star explodes, preventing any further charged particle captures (Freeze Out).

How do we get beyond the Iron peak to produce the much heavier nuclides?

Suppose the neutrons that are in the NSE mixture can capture onto the Fe peak nuclei.

Suppose the density of these neutrons is very high ~ 10^{22} cm^{-2}

And suppose they can capture on the Fe peak "seeds" faster than most of the competing beta-decays.