



Nuclear Astrophysics II

Lecture 3

Thurs. May 3, 2012

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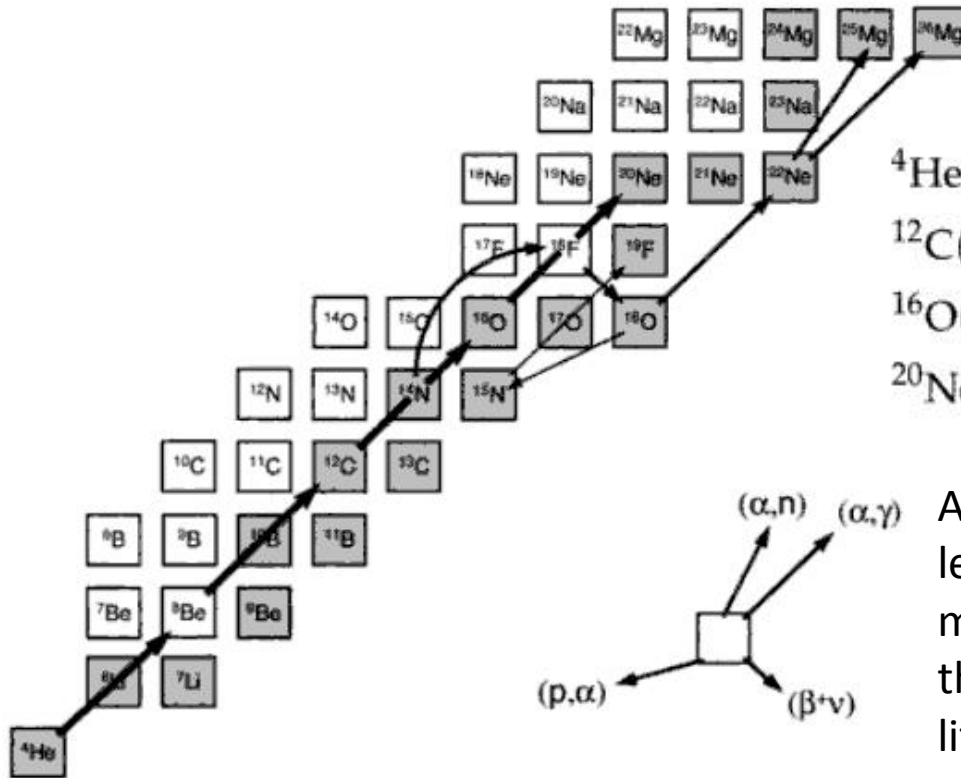
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The next steps in stellar evolution after exhaustion of hydrogen fuel.

HYDROSTATIC HELIUM BURNING & BEYOND

The Reactions



At the end of hydrogen burning, we are only left with alpha particles. This is where we must begin helium burning. One problem, though, is that ${}^8\text{Be}$ is particle unstable, with a lifetime of 2.6×10^{-16} (!) seconds. What does nature do?

Consider, then, we have an equilibrium between $\alpha + \alpha \rightleftharpoons {}^8\text{Be} + \gamma$ and let's see what can happen. Recall from Lecture 1,2 page 10, for a reaction $1 + 2 \rightarrow 3 + \gamma$, the ratio of inverse and forward reaction rates was:

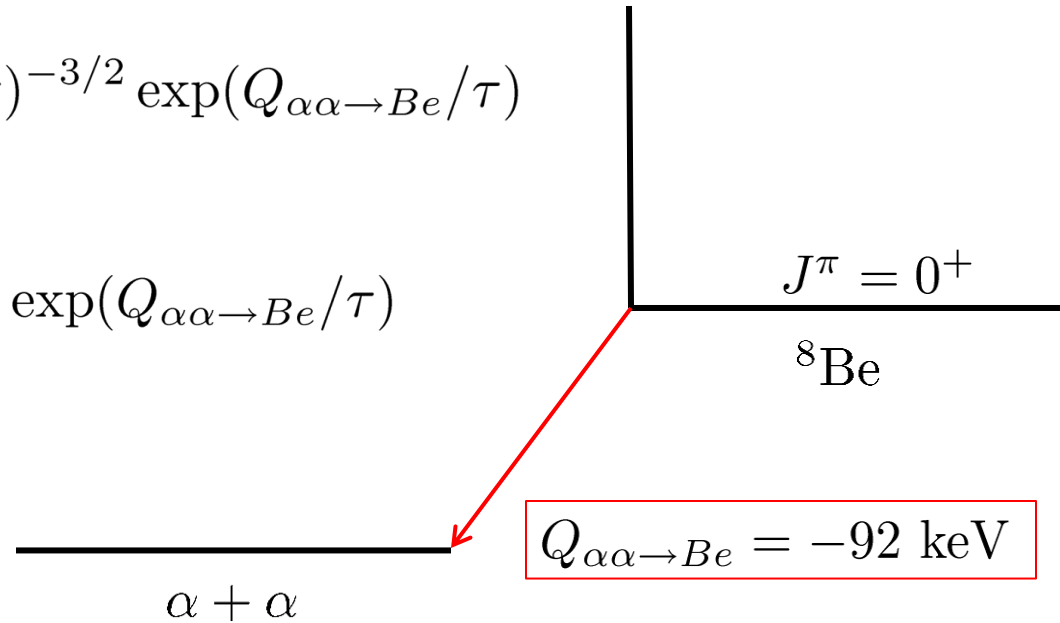
$$\frac{r_{3\gamma}}{r_{12}} = \frac{g_1 g_2}{g_3} \frac{N_3}{N_1 N_2} \left(\frac{2\pi}{h^2} \right)^{3/2} (\mu_{12} \tau)^{3/2} \exp(-Q_{12 \rightarrow 3\gamma} / \tau)$$

Now, set $r_{3\gamma} / r_{12} = 1$ for equilibrium and rearrange for N_3 / N_1 .

Continuing, we have the equilibrium abundance ratio of ${}^8\text{Be}$ to alphas

$$\frac{N_{{}^8\text{Be}}}{N_\alpha} = \frac{g_{{}^8\text{Be}}}{g_\alpha g_\alpha} N_\alpha \left(\frac{h^2}{2\pi} \right)^{3/2} (\mu_{\alpha\alpha}\tau)^{-3/2} \exp(Q_{\alpha\alpha \rightarrow \text{Be}}/\tau)$$

$$= \frac{\rho N_A X_\alpha}{4} \left(\frac{h^2}{2\pi} \right)^{3/2} (\mu_{\alpha\alpha}\tau)^{-3/2} \exp(Q_{\alpha\alpha \rightarrow \text{Be}}/\tau)$$



Note the negative sign, because the exit channel is unbound

Numerically, we have:

$$\frac{N_{{}^8\text{Be}}}{N_\alpha} = 1.5 \times 10^{-20} \frac{\rho N_A X_\alpha}{4} T^{-3/2} \exp(-10.67/T_8)$$

Exercise: take $X_\alpha = 1$, $\rho = 10^5 \text{ g/cm}^3$, $T = 10^8 \text{ K}$ and determine number ratio of alphas to ${}^8\text{Be}$.

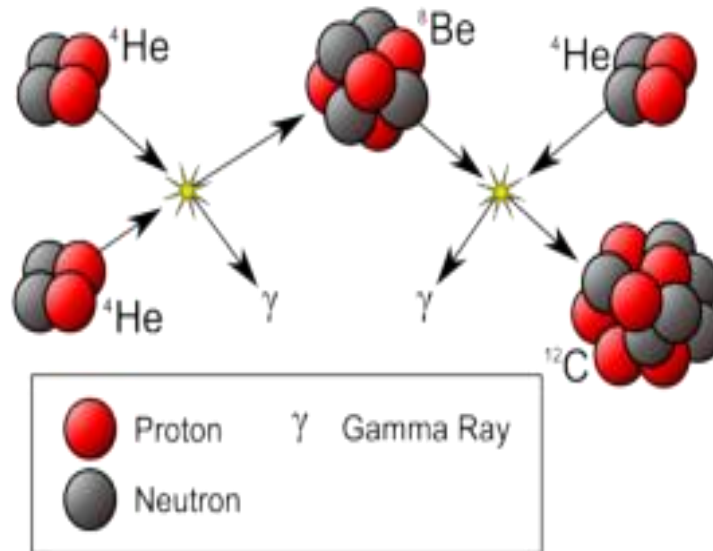
He Burning: Some Details

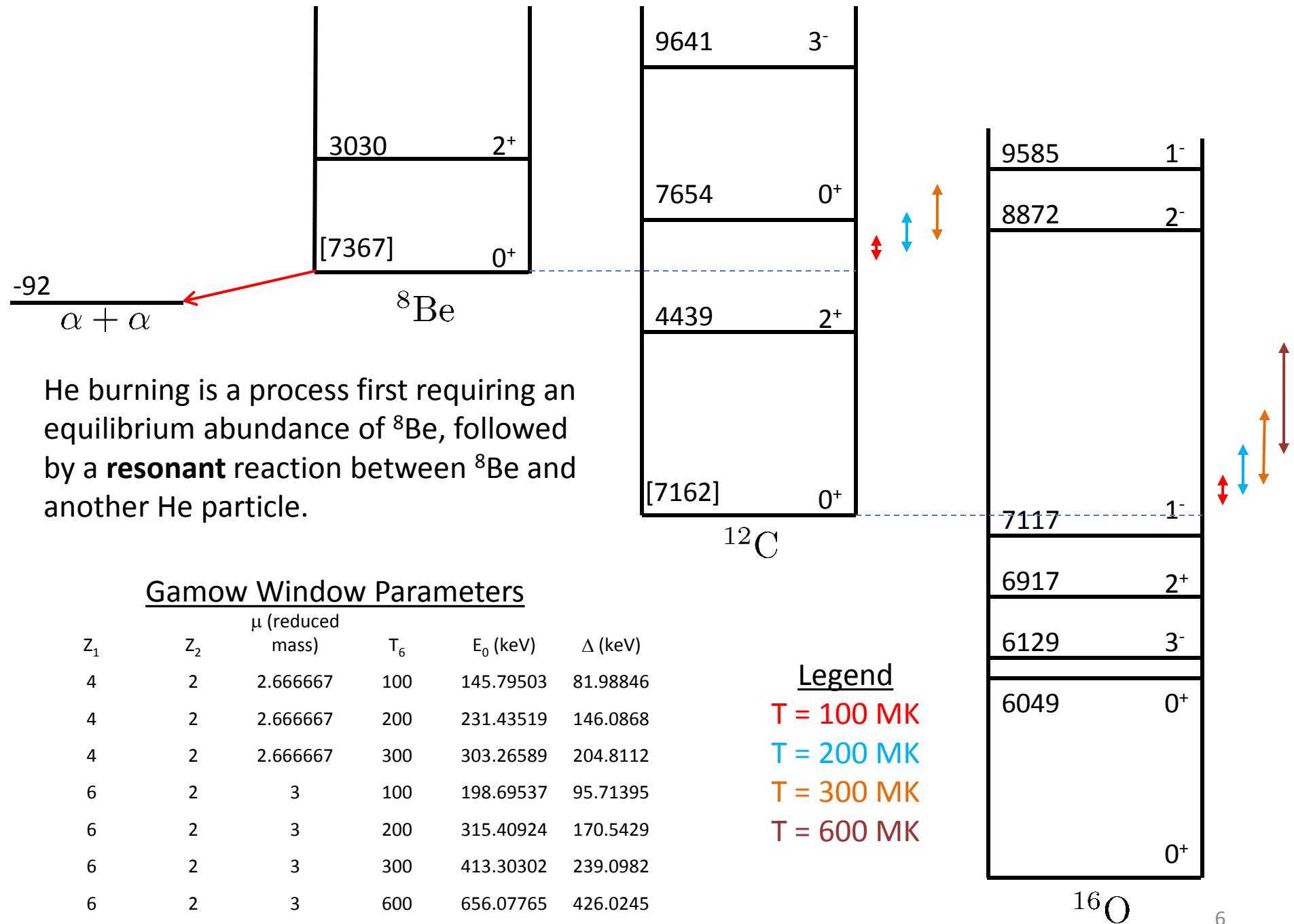
Hydrostatic He-burning is a multistep process, because ${}^8\text{Be}$ is particle-unbound.

- Reaction requires reactants, therefore,
- First, require an abundance of the very short-lived ${}^8\text{Be}$ (previous problem)
- This abundance of ${}^8\text{Be}$ must then react with α particles already present
- Production of ${}^{12}\text{C}$ should result from these steps

Let's look at this in more detail.

$$\tau_{\text{Be}} = 2.6 \times 10^{-16} \text{ s}$$





A closer look at the nuclear physics

Equilibrium abundance of ${}^8\text{Be}$ is given by

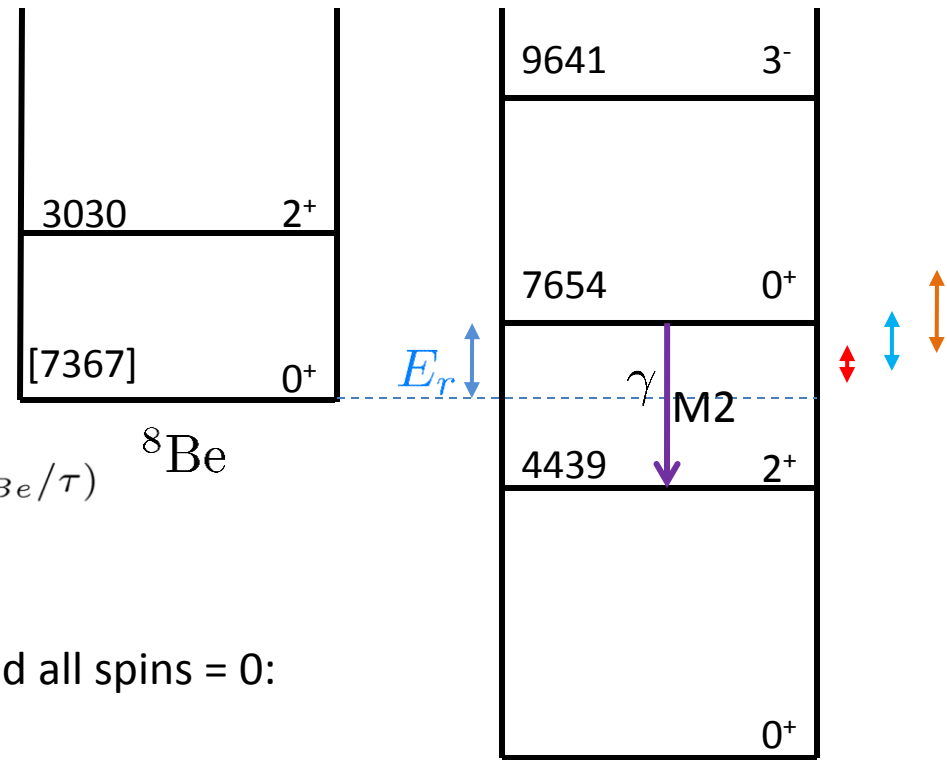
$$N_{Be} = N_{\alpha}^2 \left(\frac{h^2}{2\pi} \right)^{3/2} (\mu_{\alpha\alpha}\tau)^{-3/2} e^{(Q_{\alpha\alpha\rightarrow Be}/\tau)} {}^8\text{Be}$$

Resonant reaction rate, using $m_{Be} \approx 2m_{\alpha}$ and all spins = 0:

$$r_{\alpha Be} = \left(\frac{2\pi}{\mu_{\alpha Be}\tau} \right)^{3/2} \hbar^2 N_{\alpha} N_{Be} \frac{2J_r + 1}{(2J_{\alpha} + 1)(2J_{Be} + 1)} \frac{\Gamma_{\alpha}\Gamma_{\gamma}}{\Gamma} e^{-E_r/\tau} {}^{12}\text{C}$$

$$= \frac{3^{3/2}}{4\pi^2} h^5 \left(\frac{N_{\alpha}}{m_{\alpha}\tau} \right)^3 \frac{\Gamma_{\gamma}\Gamma_{\alpha}}{\Gamma} e^{-(E_r - Q_{\alpha\alpha\rightarrow Be})/\tau}$$

$$= 9.8 \times 10^{-54} \left(\frac{\rho N_A X_{\alpha}}{T_8} \right)^3 e^{-42.94/T_8} \text{cm}^{-3} \text{s}^{-3}$$



$$\Gamma_{\alpha} = 8.3 \pm 1.0 \text{ eV}$$

$$\Gamma_{\gamma} = (3.7 \pm 0.5) \times 10^{-3} \text{ eV}$$

What to notice:

1. Rate depends on the **third** power of alpha-particle abundance (or 3rd power of density).
2. Depends on the $\alpha + \alpha \rightarrow {}^8\text{Be}$ reaction Q-value (from Saha equation)
3. Depends on the resonance energy
4. Depends on the alpha particle and gamma-ray decay widths of the resonance.

All of these are nuclear physics quantities that had to be measured in the laboratory.

Note that the rate depends **exponentially** on the resonance energy and on the **mass difference** (Q-value) between ${}^8\text{Be}$ and 2 alphas.

Something to think about:

How is mass difference of alphas and ${}^8\text{Be}$ measured, given that ${}^8\text{Be}$ has a lifetime of 2.6×10^{-16} seconds?(!)

How are the decay widths for the 7654 keV state measured? It cannot be populated through an electromagnetic transition from its ground state. $0^+ \rightarrow 0^+$ transitions are forbidden from the gamma selection rules.

We will come back to this in another lecture, when we talk about experimental topics. 8

What is the energy generation rate?

$$\begin{aligned}\epsilon_{3\alpha} &= \frac{r_{3\alpha} Q_{3\alpha}}{\rho} & Q_{3\alpha} &= (3m_\alpha - M_{12C})c^2 = 7.274 \text{ MeV} \\ &= 3.9 \times 10^{11} \frac{\rho^2 X_\alpha^3}{T_8^3} e^{-42.94/T_8} \text{ erg g}^{-1} \text{ s}^{-1}\end{aligned}$$

As a temperature power law, this can be written as follows:

$$\epsilon_{3\alpha} = \epsilon(T_0) \left(\frac{T}{T_0} \right)^n \quad \text{where } n = 42.9/T_8 - 3$$

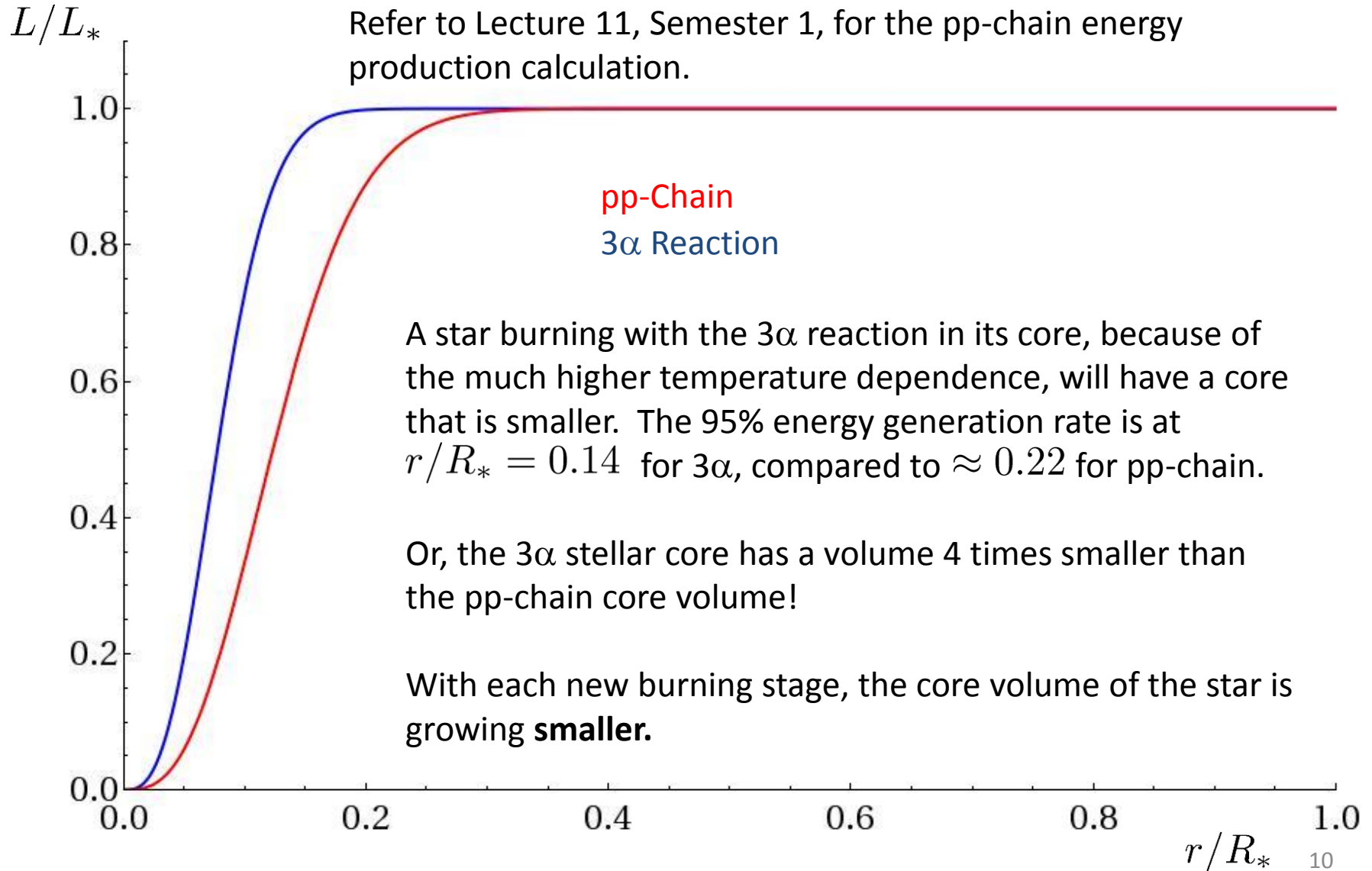
And thus, at $T = 100 \text{ MK}$, the exponent is about 40!

The rate is most sensitive in those regions with highest temperature. This means that the core of a star burning the triple-alpha reaction will be very condensed.

Something else about the star's structure can result from this type of high intensity energy production rate.

Nuclear Energy & Structural Difference

The 3α energy generation rate calculated for $T = 100$ MK.
Refer to Lecture 11, Semester 1, for the pp-chain energy production calculation.



We have a compact core of He burning the triple-alpha reaction, with a power-law dependence that goes like T^{40} at 100 MK.

The core, being compact and acted upon by a large, thick, shell of hydrogen is also highly dense. The triple-alpha rate, as you saw, varies with the **third** power of density.

$$\epsilon_{3\alpha} = \epsilon(T_0) \left(\frac{T}{T_0} \right)^n \quad \Rightarrow \quad \frac{d\epsilon_{3\alpha}}{\epsilon_{3\alpha}} = n \frac{dT}{T}$$

A small temperature perturbation results in a large perturbation in the energy generation. The signs of the perturbations are also the same. This can result in a positive feedback that can then produce a thermonuclear runaway.

We know that the core should expand if such a situation occurs. And by expanding, the runaway is “shut off” because the expanding gas will cool. What would “turn off” this expansion?

If the core He material is electron degenerate, it will not expand as would an ideal gas; this will lead to runaway conditions known as the “Core Helium Flash”.

Hydrogen

${}^4\text{He}$

Back to Thermodynamics

First, let's go back to the **First Law** of Thermodynamics and something already familiar:

$$dQ = dU + PdV$$

Take the internal energy to be functions of T and V: $U = U(T, V)$

Then, by definition: $\Rightarrow dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV$

$$dQ = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV + PdV$$

Let's apply this to the case of the He-core to understand how runaway can happen.

We have (1st Law): $dQ = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV + PdV$

$$\Rightarrow \dot{Q} = \left(\frac{\partial U}{\partial T}\right)_V \dot{T} + \left[\left(\frac{\partial U}{\partial V}\right)_T + P\right] \dot{V}$$

Back in Lectures 2 and 3, last Semester, on page 37, we found that the temperature was related to the density as follows:

$$T \propto \rho^{1/3} \propto V^{-1}$$

$$\Rightarrow 3\frac{\dot{T}}{T} = -\frac{\dot{V}}{V}$$

$$\Rightarrow \dot{Q} = \left(\frac{\partial U}{\partial T}\right)_V \dot{T} - 3\frac{\dot{T}V}{T} \left[\left(\frac{\partial U}{\partial V}\right)_T + P\right]$$

Continuing:
$$\dot{Q} = \left(\frac{\partial U}{\partial T} \right)_V \dot{T} - 3 \frac{\dot{T}V}{T} \left[\left(\frac{\partial U}{\partial V} \right)_T + P \right]$$

If the system is composed of Ideal gas, then: $U = \frac{3}{2}NkT \Rightarrow \frac{\partial U}{\partial T} = \frac{U}{T}$

And, using: $PV = NkT \Rightarrow \frac{U}{V} = \frac{3}{2}P$

And the volume derivative is zero. So, finally: $\epsilon \equiv \dot{Q} = -U \frac{\dot{T}}{T}$

$$\Rightarrow \delta T \approx -\frac{T}{U} \epsilon \delta t$$

We see here something rather strange. We know triple-alpha reaction $\sim T^{40}$ at $T = 100$ MK. So if ϵ increases a little bit, it actually results in a **decrease** in the core temperature!

Why? Because the system can expand, and therefore cool down. Now you see, from 1st Law the proof. What if material is degenerate?

Rearranging the 1st Law relationship, we have:

$$\delta T \approx + \frac{\epsilon}{\left(\frac{\partial U}{\partial T}\right)_V - 3\frac{V}{T} \left[\left(\frac{\partial U}{\partial V}\right)_T + P\right]} \delta t$$

Last Semester, we derived that the energy of zero temperature Fermi gas is:

$$U = \frac{\pi^3}{10m} \left(\frac{\hbar}{L}\right)^2 n_F^5 = \frac{\pi^3}{10m} \left(\frac{\hbar}{L}\right)^2 \left(\frac{3N}{\pi}\right)^{5/3}$$

And the pressure was:
$$P_e = -\frac{\partial U}{\partial V} = \frac{\pi^3}{15m} \hbar^2 \left(\frac{3n_e}{\pi}\right)^{5/3}$$

From these equations, we see that the denominator of δT goes to zero in the limit of a perfectly degenerate system. And the sign is positive. So small changes in the nuclear energy generation rate, under degenerate conditions, results in large and **positive** feedback. We know why: the system cannot expand like an ideal gas. But now you see it explained from the 1st Law. ☺ Thus, stars like our Sun can undergo a rapid thermonuclear runaway at the onset of the triple-alpha reaction after hydrogen burning.

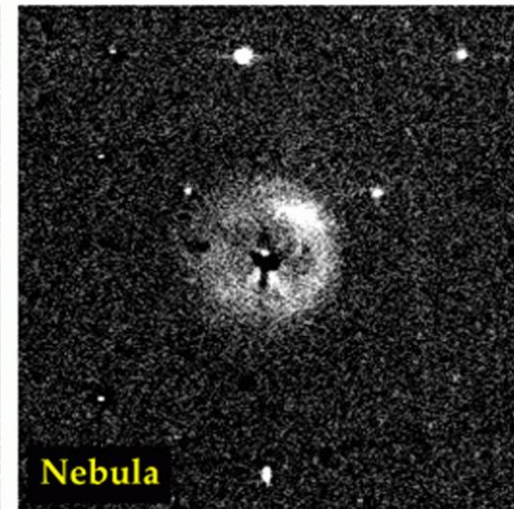
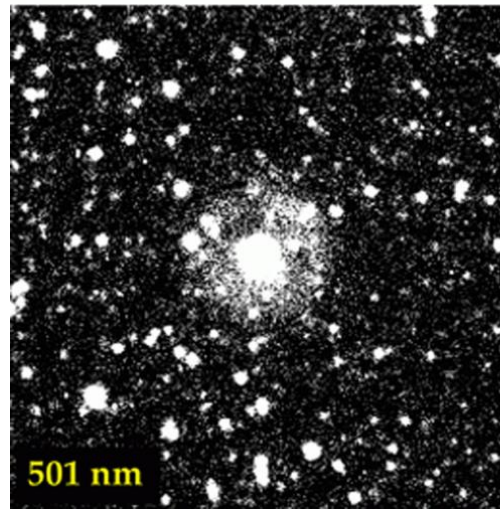
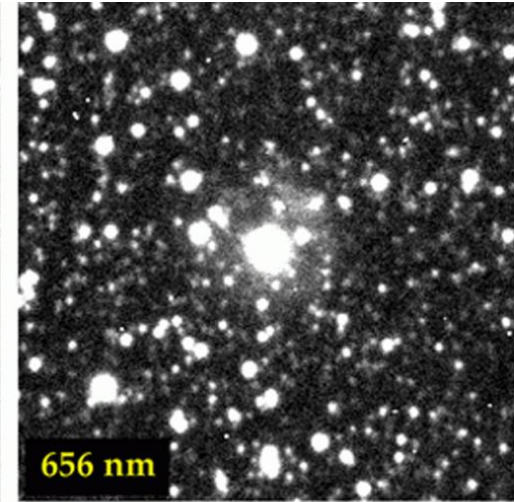
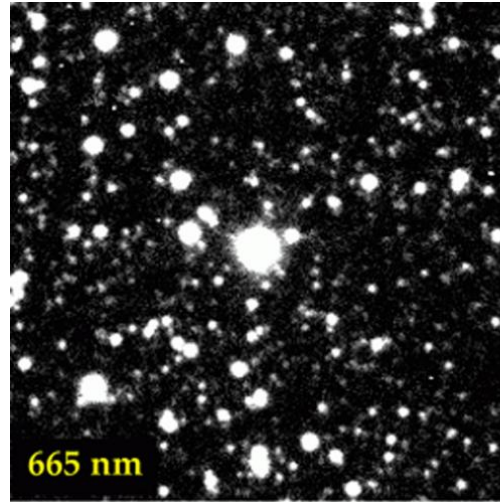
Sakurai's Object

Discovered in 1996 by amateur Japanese astronomer Yukio Sakurai.

Silent star suddenly erupted. It was initially thought to be a “slow nova” explosion, but its subsequent spectra does not have features consistent with nova explosions.

Also, as you can see in the photos of the object, there is already a well established Planetary Nebula (PN) around it.

Giants create PN as they reach the end of their He-burning stage. These data, therefore, demonstrate that this star was already finished (or very nearly finished) core He-burning.



<http://www.eso.org/public/news/eso9619/>

Sakurai's Object: Spectrum

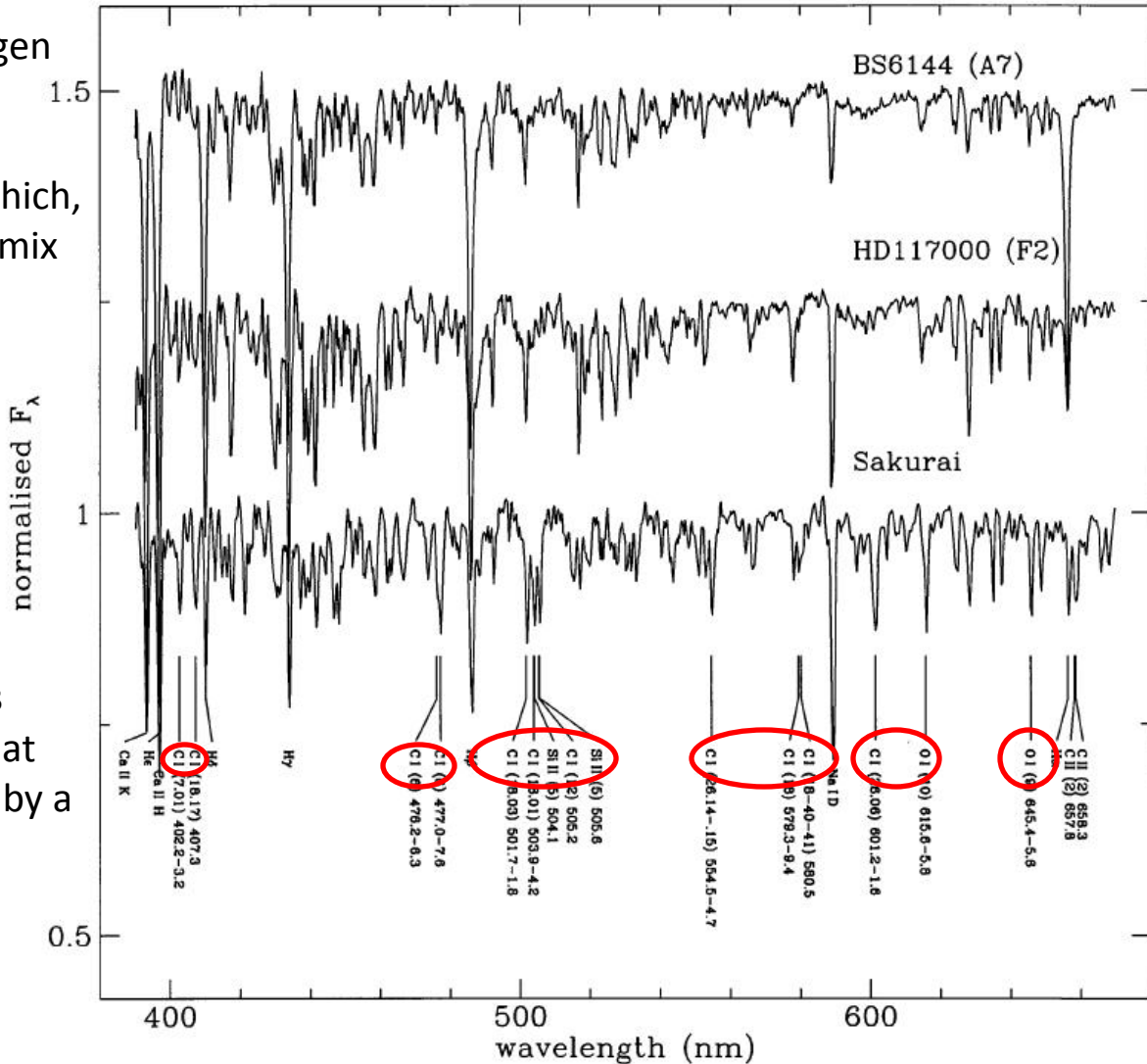
Spectrum dominated by carbon and oxygen lines.

The two other stars are known Giants, which, as you now know, should burn CNO and mix these into their atmospheres.

Sakurai's Object has **much** more C and O than the Giants, but it also has a Planetary Nebula around it, so we also know its core has already completed He-burning.

Note the spectrum is of **absorption lines** rather than emission lines; this means that the gas is being illuminated from behind by a bright "continuum" source.

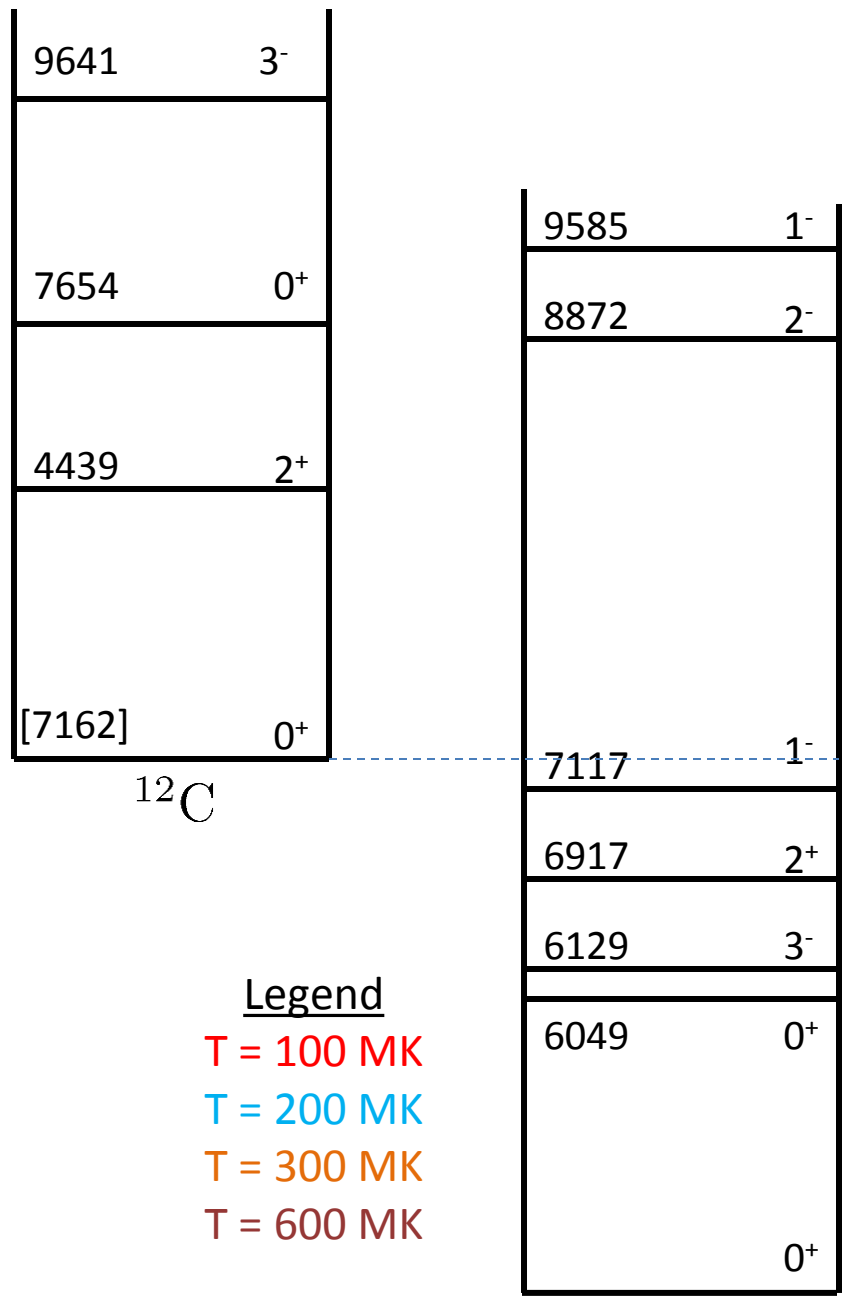
Where did the He-flash come from?



Helium Burning Continued

$^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ Reaction

This reaction rate is somewhat complicated. There are no resonant states in the Gamow Window, but nuclear experiments show that the total width of the 7.117 MeV state, just below the $^{12}\text{C}(\alpha,\gamma)$ threshold is very broad. Also, the 8.872 MeV state is broad. Therefore, the “wings” of these resonances contribute to the reaction rate, and there is no “simple” formula to describe the rate; it must be calculated numerically.



^{12}C

^{16}O

Legend

- T = 100 MK
- T = 200 MK
- T = 300 MK
- T = 600 MK

Gamow Window Parameters

Z1	Z2	A	T6	E0 (keV)	Delta (keV)
6	2	3	100	198.69537	95.71395
6	2	3	200	315.40924	170.5429
6	2	3	300	413.30302	239.0982
6	2	3	600	656.07765	426.0245

$^{16}\text{O}(\alpha,\gamma)^{20}\text{Ne}$ Reaction

Next reaction in the sequence, in principle, is the $^{16}\text{O}(\alpha,\gamma)^{20}\text{Ne}$ reaction.

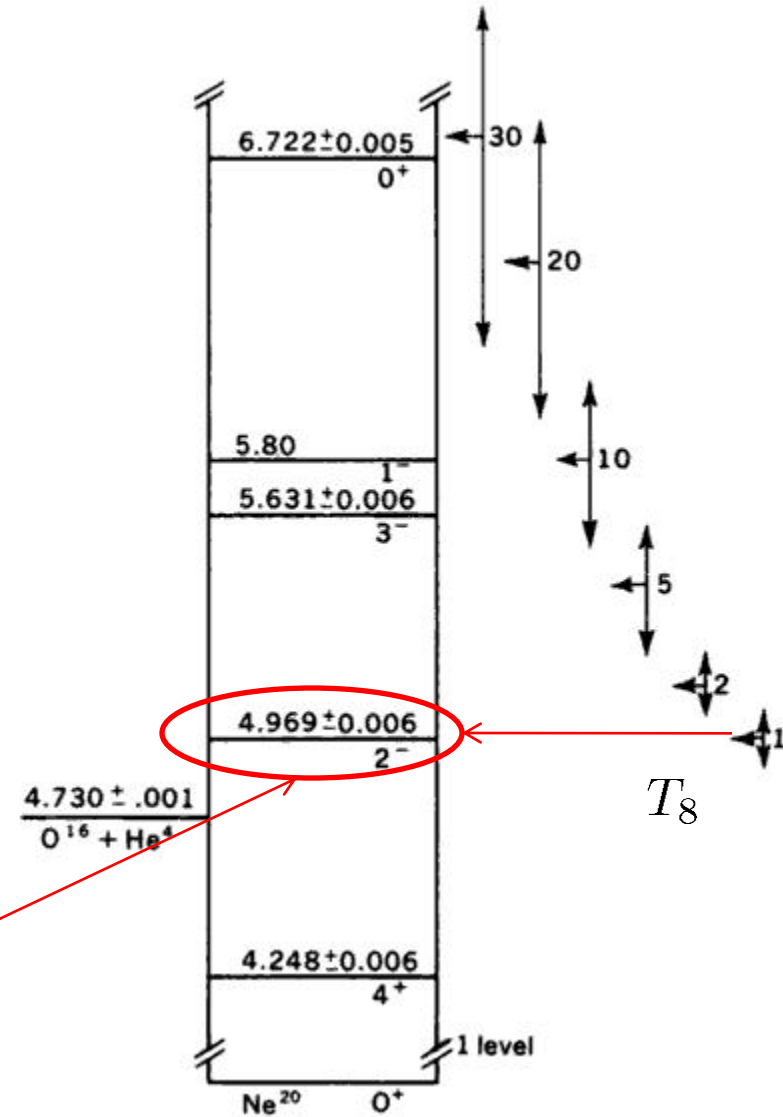
For hydrostatic burning, where $T_8 \sim 300$ MK and less, there is just one resonance level in ^{20}Ne within the Gamow Window, located at $E_r = 4969 - 4730 = 239$ keV.

The presence of this resonance should make the $^{16}\text{O}(\alpha,\gamma)^{20}\text{Ne}$ reaction fast, and little ^{16}O would survive the He-burning phase of stars.

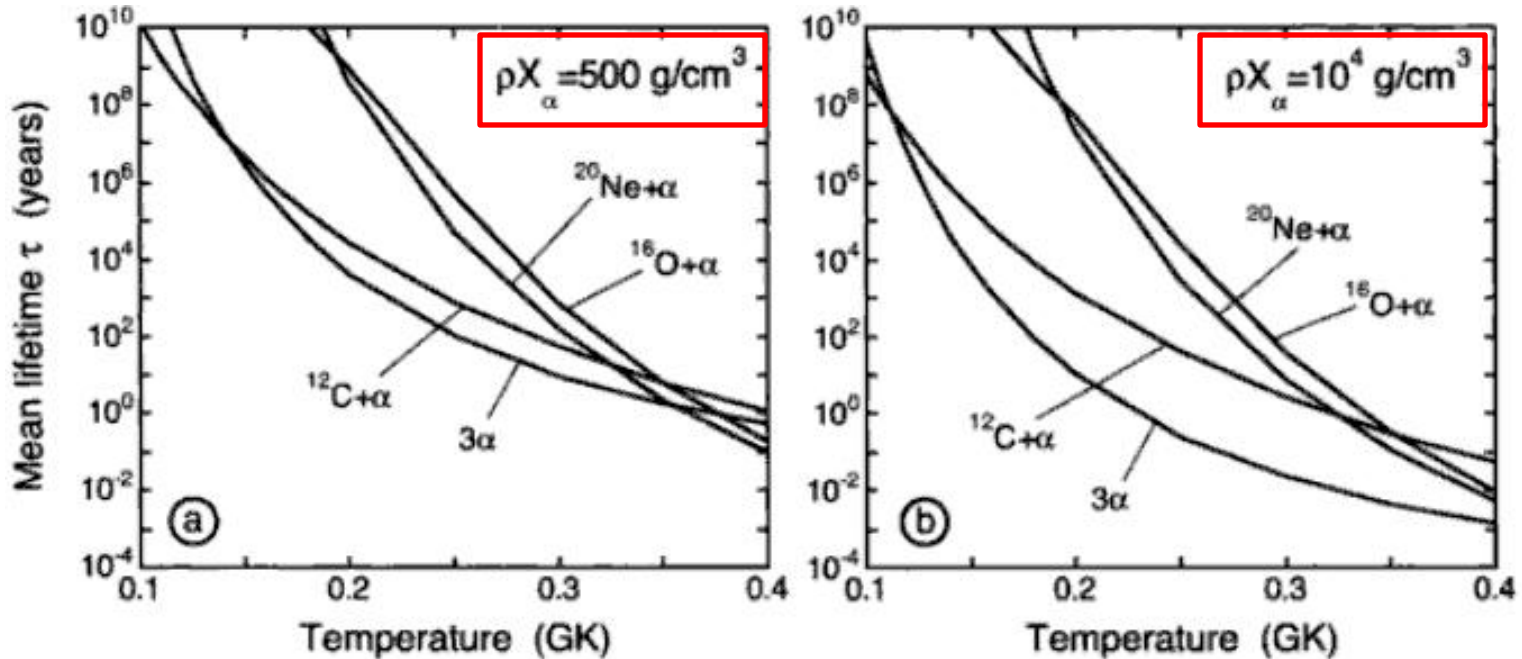
But the abundance ratio of $^{12}\text{C}/^{16}\text{O}$ is about 0.4. This suggests that the destruction of ^{16}O by alpha capture is not much faster than its production rate.

But we saw a moment ago that its production rate is not dominated by resonant alpha-capture. So, why are the abundances comparable, with that resonance sitting squarely in the Gamow Window?

Answer?



Lifetimes Against α -Capture



Recall, by definition, the lifetime for species “z” against destruction by particle capture is:

$$\tau_z = \left[\frac{r_{z\alpha}}{N_z} \right]^{-1} = \left[\frac{\rho X_\alpha}{m_\alpha} N_A \langle \sigma v \rangle_{z(\alpha,\gamma)} \right]^{-1}$$

And for the 3α reaction the lifetime is:

$$\tau_{3\alpha} = \left[9.8 \times 10^{-54} \frac{(\rho X_\alpha)^2}{T_8^3} e^{-42.94/T_8} \right]^{-1}$$

Those nuclides with shorter lifetimes are destroyed faster by α -capture. Data above shows ^4He and ^{12}C are destroyed fastest in He-burning.

Hydrostatic He Burning System

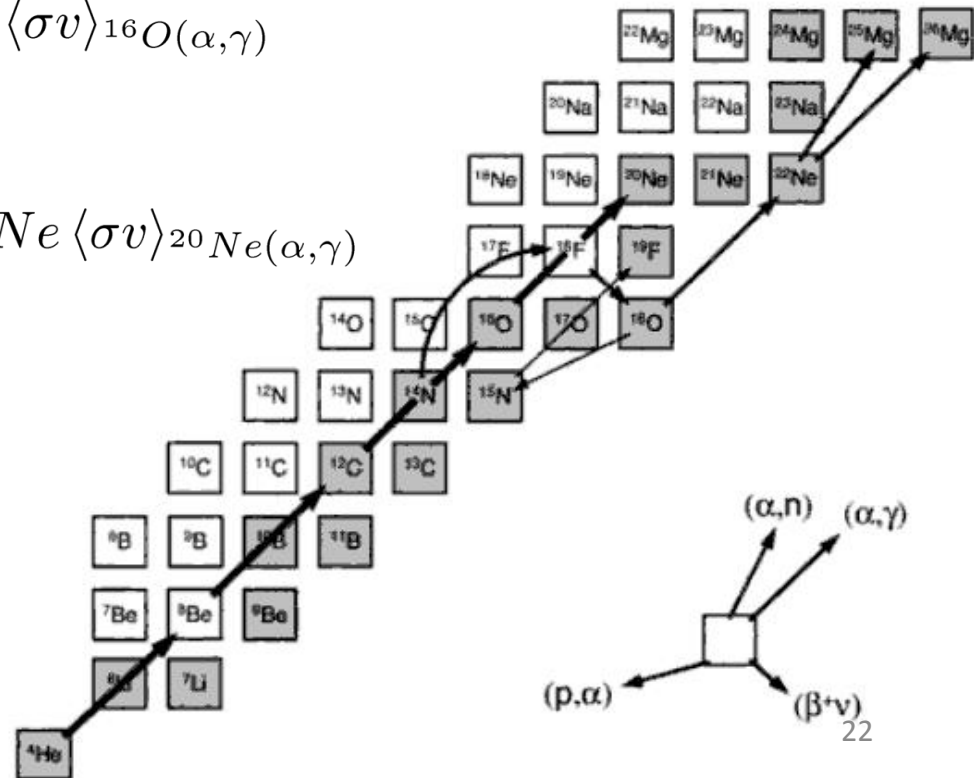
$$\frac{d^4He}{dt} = -3r_{3\alpha} - {}^4He\ ^{12}C \langle\sigma v\rangle_{^{12}C(\alpha,\gamma)} - {}^4He\ ^{16}O \langle\sigma v\rangle_{^{16}O(\alpha,\gamma)}$$

$$\frac{d^{12}C}{dt} = r_{3\alpha} - {}^4He\ ^{12}C \langle\sigma v\rangle_{^{12}C(\alpha,\gamma)}$$

$$\frac{d^{16}O}{dt} = {}^4He\ ^{12}C \langle\sigma v\rangle_{^{12}C(\alpha,\gamma)} - {}^4He\ ^{16}O \langle\sigma v\rangle_{^{16}O(\alpha,\gamma)}$$

$$\frac{d^{20}Ne}{dt} = {}^4He\ ^{12}C \langle\sigma v\rangle_{^{12}C(\alpha,\gamma)} - {}^4He\ ^{20}Ne \langle\sigma v\rangle_{^{20}Ne(\alpha,\gamma)}$$

$$\frac{d^{24}Mg}{dt} = {}^4He\ ^{20}Ne \langle\sigma v\rangle_{^{20}Ne(\alpha,\gamma)}$$



Evolution of ^{12}C & ^{16}O

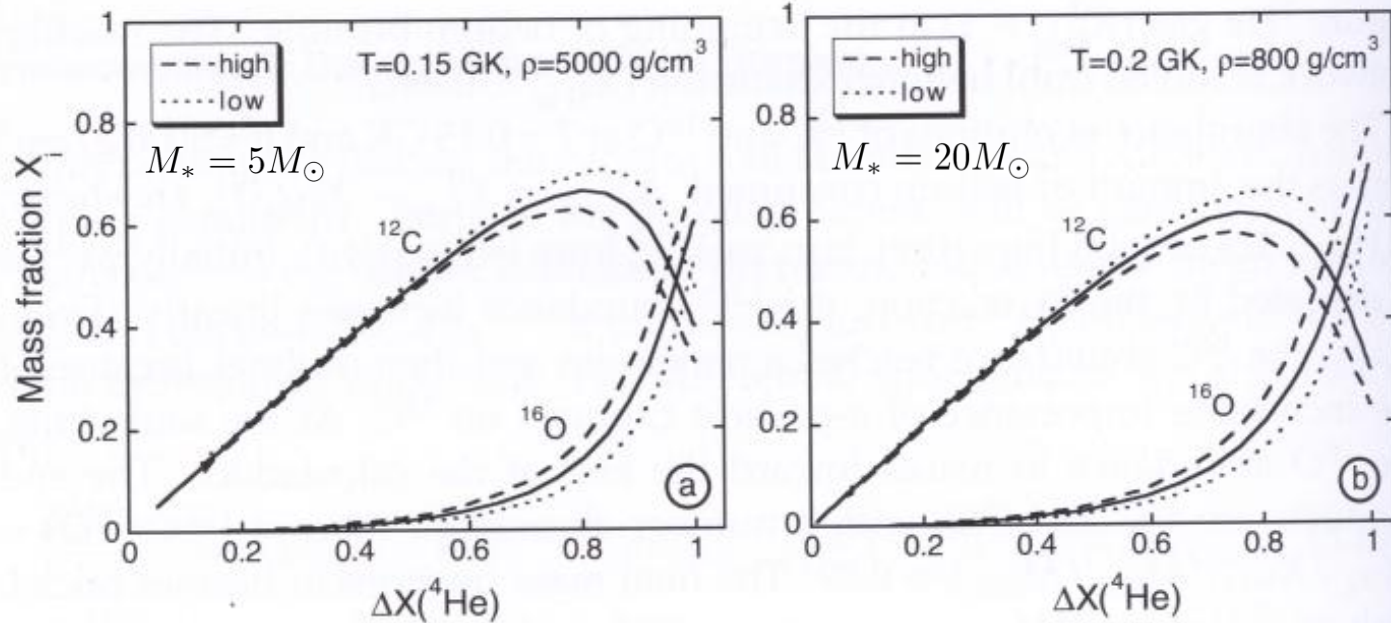


Fig. 5.32 Evolution of ^{12}C and ^{16}O versus the amount of helium consumed during hydrostatic helium burning for constant temperatures and densities of (a) $T = 0.15 \text{ GK}$ and $\rho = 5000 \text{ g/cm}^3$, and (b) $T = 0.2 \text{ GK}$ and $\rho = 800 \text{ g/cm}^3$. The results are obtained by solving the reaction network numerically, assuming a pure ^4He gas at the beginning

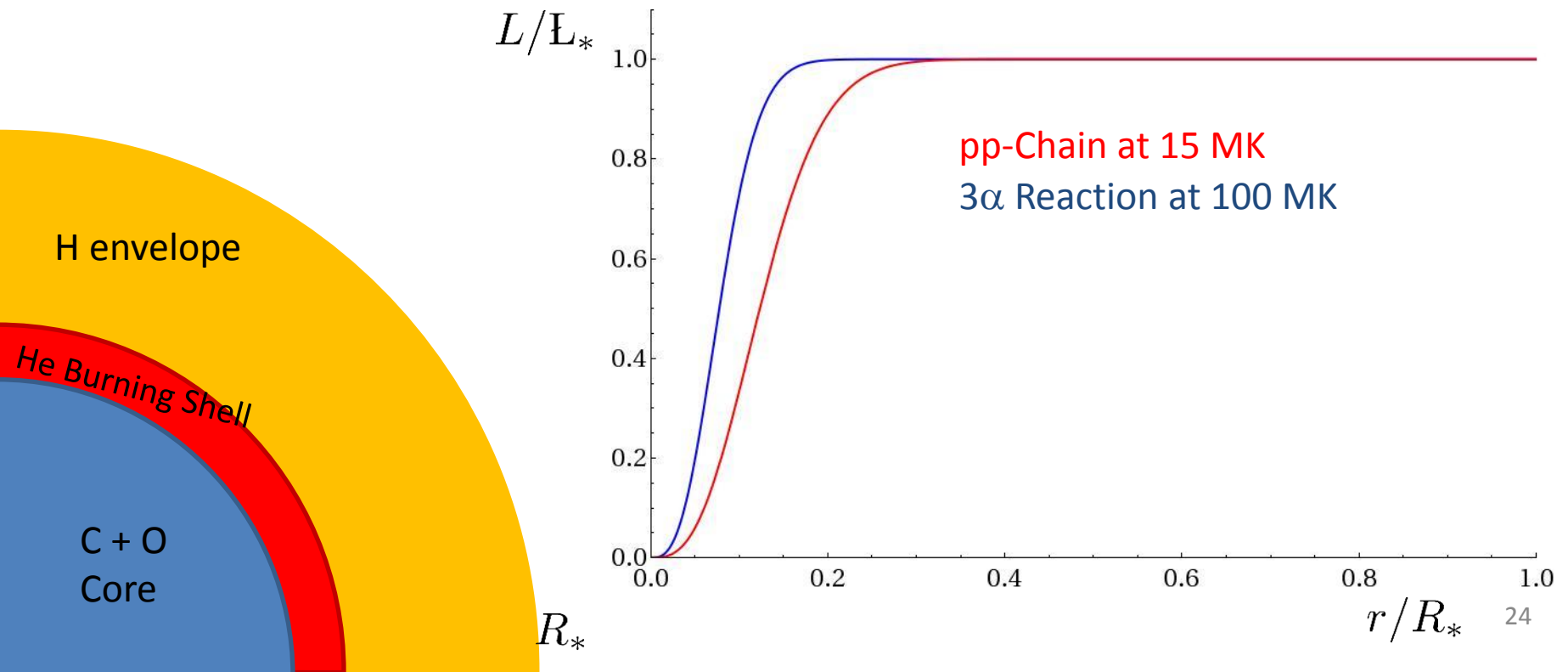
of helium burning. The calculation is terminated when the helium mass fraction falls below $X_{4\text{He}} = 0.001$. The solid lines are obtained by adopting recommended $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rates, while the dotted and dashed lines result from using the lower and upper limit of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rates, respectively.

Previous network solved numerically, for constant temperature and density. Note: in star, both of these change with time. But for our understanding, this simple network suffices to show us the general result.

End of He-Burning

We also see, from the energy generation rates of the pp-chains and triple-alpha reactions that the core size is smaller for each. (Define core size as the radius within which 95% of energy production occurs.)

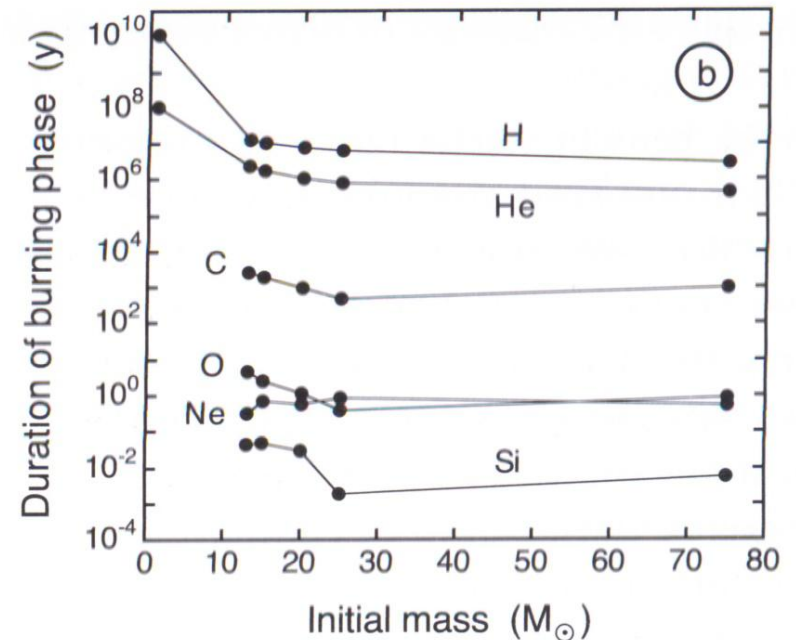
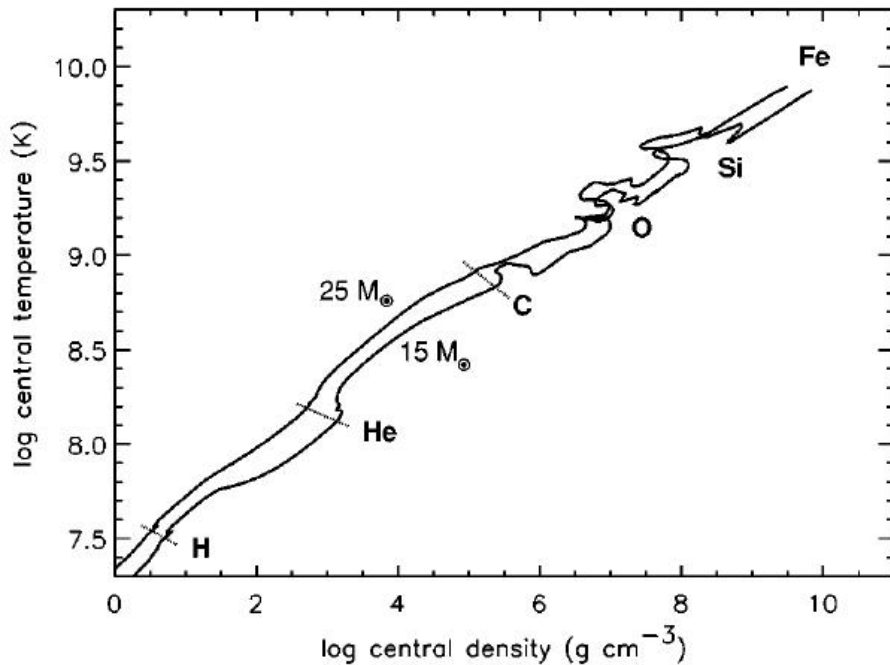
So once the most central parts of core exhaust fuel, and the rate of energy decreases, the outer part of core will still be burning the “old” fuel as the core starts to “shut off”. Central core contracts, becomes more compact, and is surrounded by a burning shell comprised of “old” fuel. At end of He-burning, our star will (schematically) look like we see here.



Burning the Ashes of He-Burning

HEAVY ION BURNING

We will be going through what happens to the star's core after He-burning. The example we take is that of a 25 solar mass star. The plots below, of a fully complete stellar model/evolution calculation will be a guide for what is to come. Note the time scales involved for the various burning stages after H and He burning are complete!

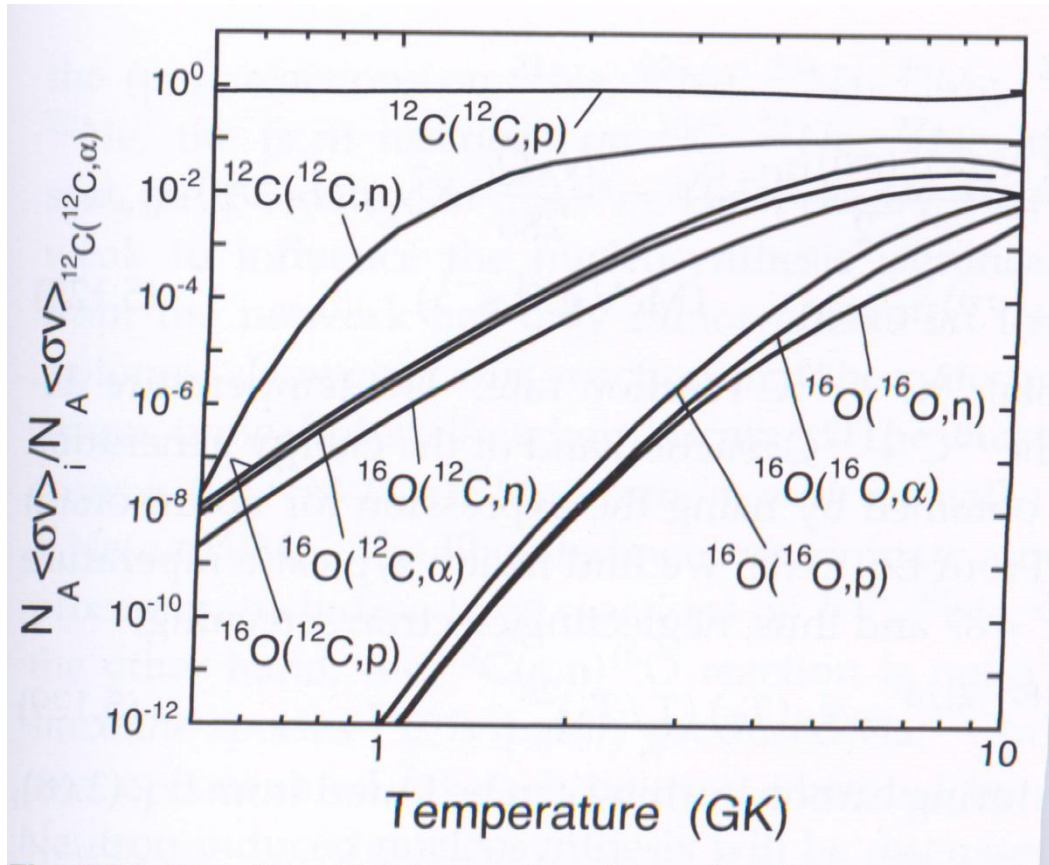
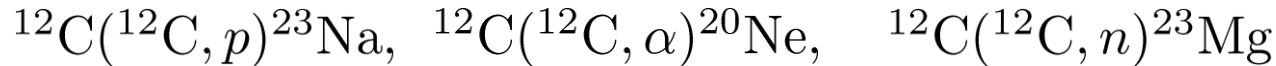


S. E. Woosley et al, Rev. Mod. Phys. **74**, 1015 (2002)

Carbon Burning

Among ^{12}C and ^{16}O , we expect that the next reaction sequence will be $^{12}\text{C} + ^{12}\text{C}$, due to it having the lowest Coulomb barrier.

The predominant reactions that take place are:



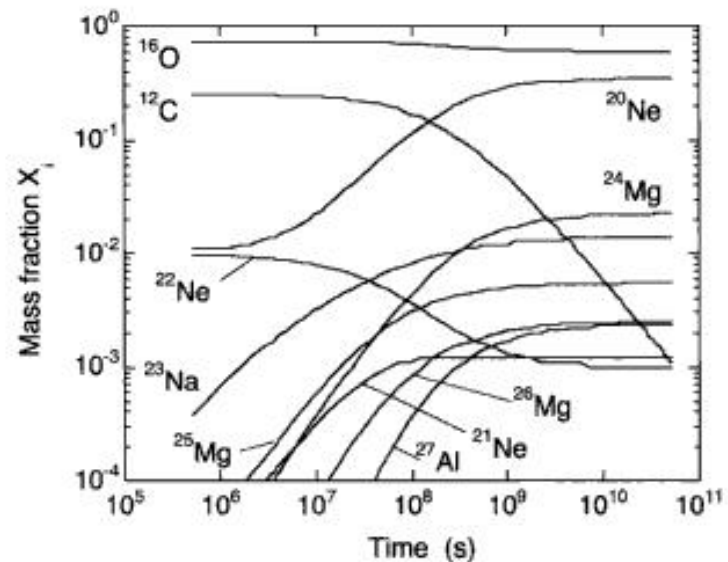
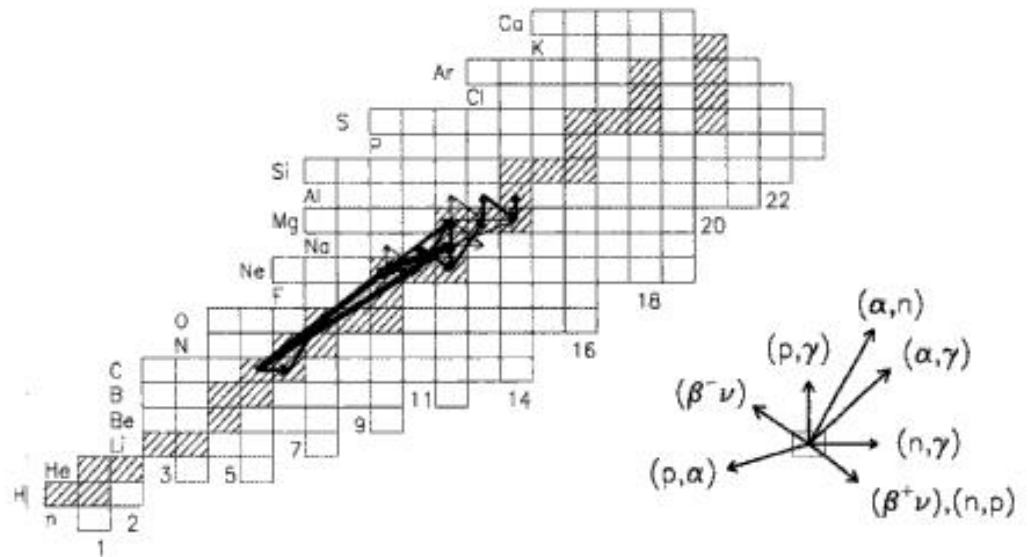
Rates normalized to
 $^{12}\text{C}(^{12}\text{C}, \alpha)^{20}\text{Ne}$ rate

Abundance evolution resulting from Carbon burning in the core of a 25 solar mass star, **for a fixed temperature and density.**

Keep in mind, these are not full stellar model calculations, but instead use our existing nuclear reaction rate information to solve the abundance “network” for a fixed temperature and density.

Our purpose here is to simply see the essential physics output.

$$T=0.9 \text{ GK}, \rho=10^5 \text{ g/cm}^3, t=5.2 \times 10^{10} \text{ s}$$

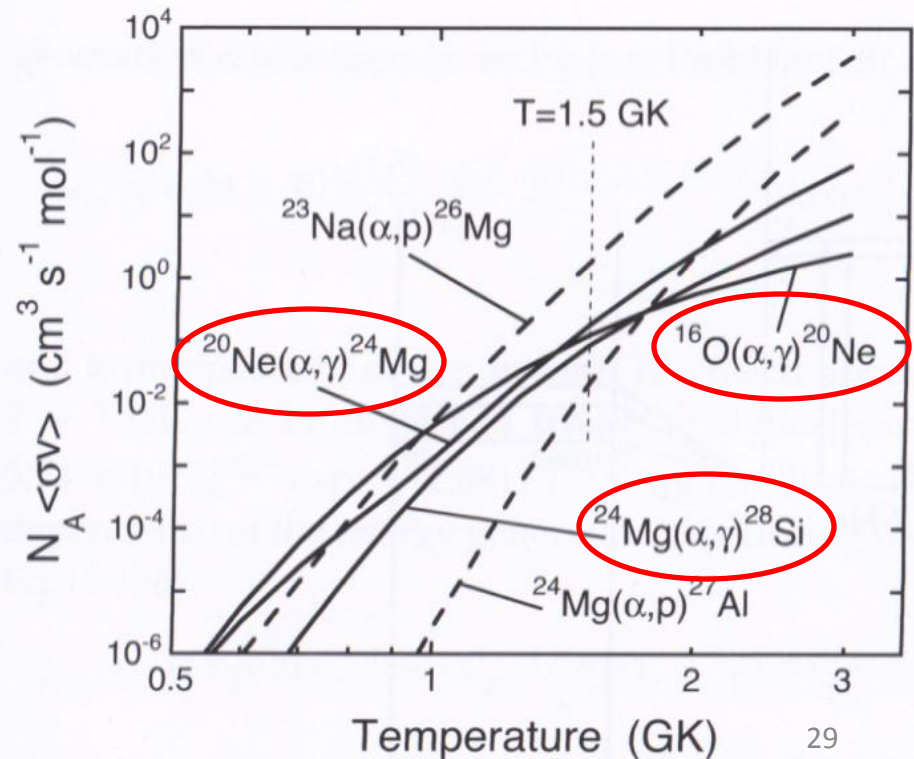
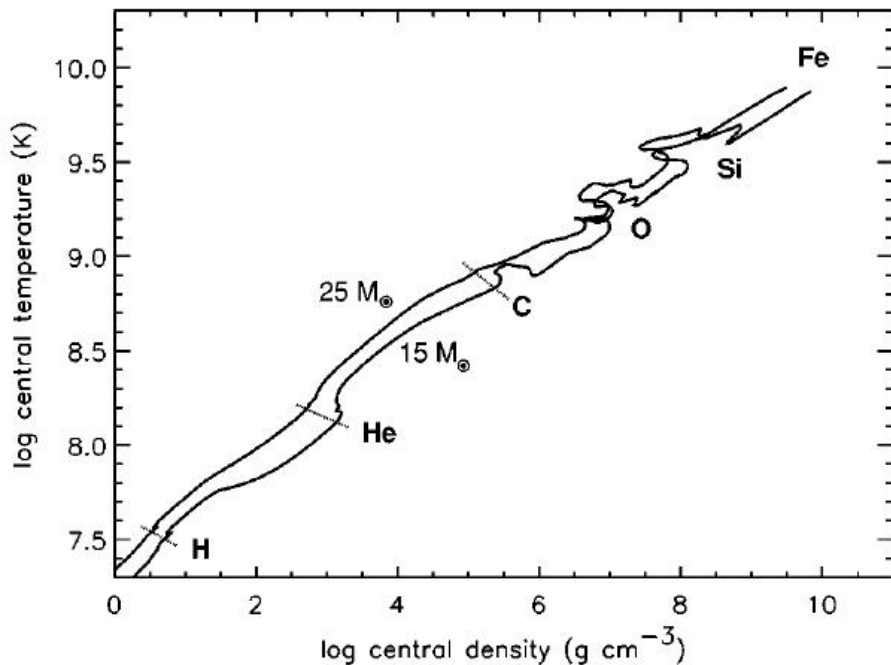


“Neon” Burning

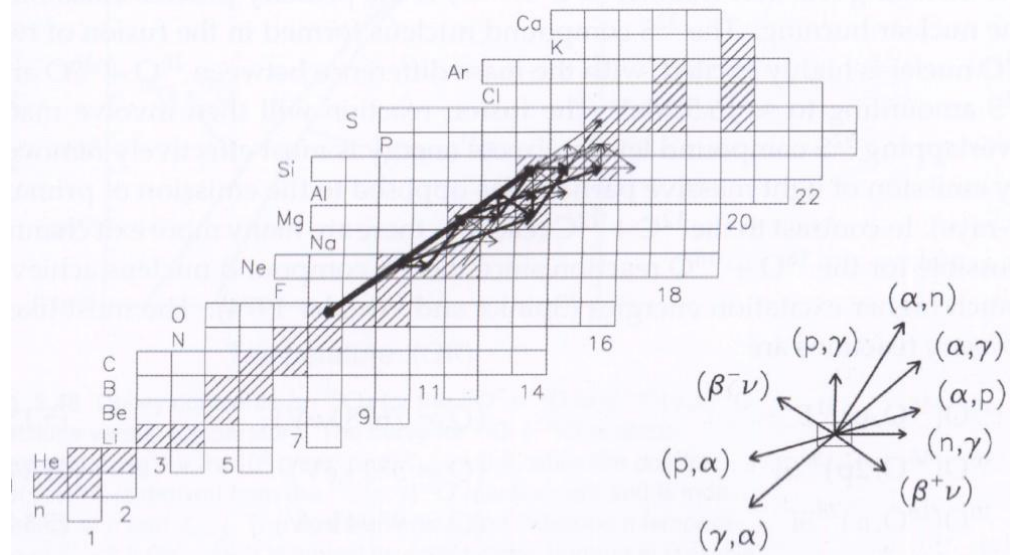
Naively, one might think we next have $^{16}\text{O} + ^{16}\text{O}$ reactions in the next step of the process. **We don't.**

Why? $^{20}\text{Ne}(\gamma, \alpha)^{16}\text{O}$ has a Q-value: $Q = -4.73$ MeV. At ~ 1 GK, this binding energy is small enough that photodisintegration of ^{20}Ne occurs, liberating sufficient alpha-particles which can then fuse with ^{16}O and ^{20}Ne . At 1.5 GK, photo-disintegration rate is:

Of the α -particle species, alpha capture on ^{16}O , ^{20}Ne , and newly made ^{24}Mg are most important. ^{28}Si is, therefore, produced.

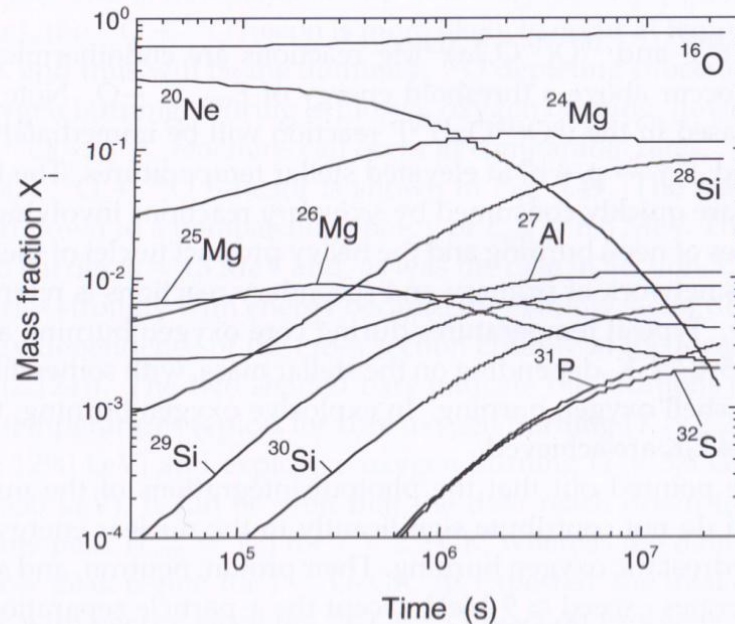


$$T=1.5 \text{ GK}, \rho=5 \times 10^6 \text{ g/cm}^3, t=2.4 \times 10^7 \text{ s}$$



Abundance evolution of the model during the course of “neon” burning in the 25-solar mass star.

End products are mostly: ${}^{16}\text{O}$, ${}^{24}\text{Mg}$, ${}^{28}\text{Si}$.



Oxygen Burning

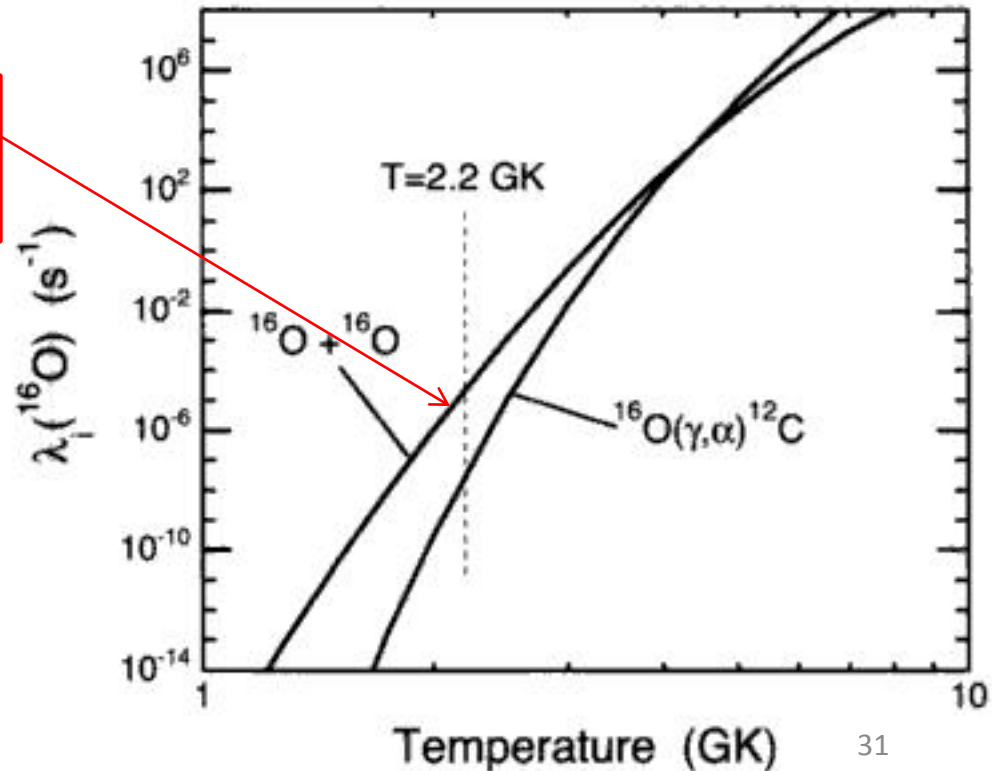
Once the neon fuel has been photodisintegrated down to a negligible mass fraction, the core must contract again. As it does so, it heats up.

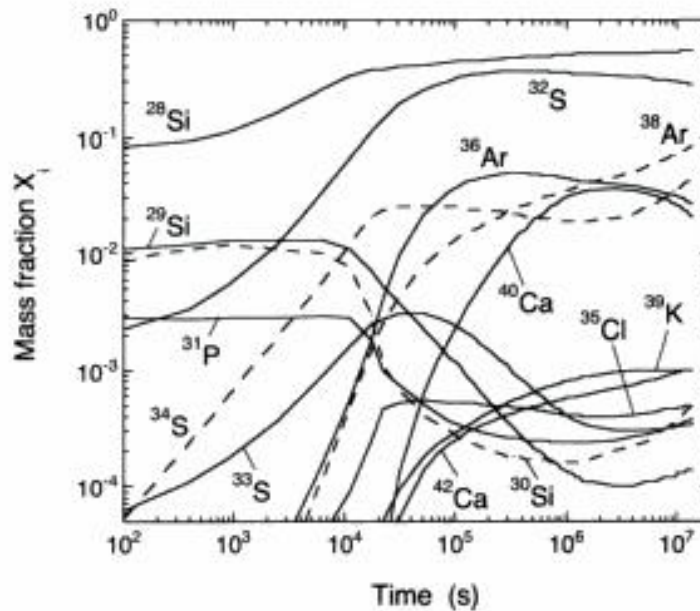
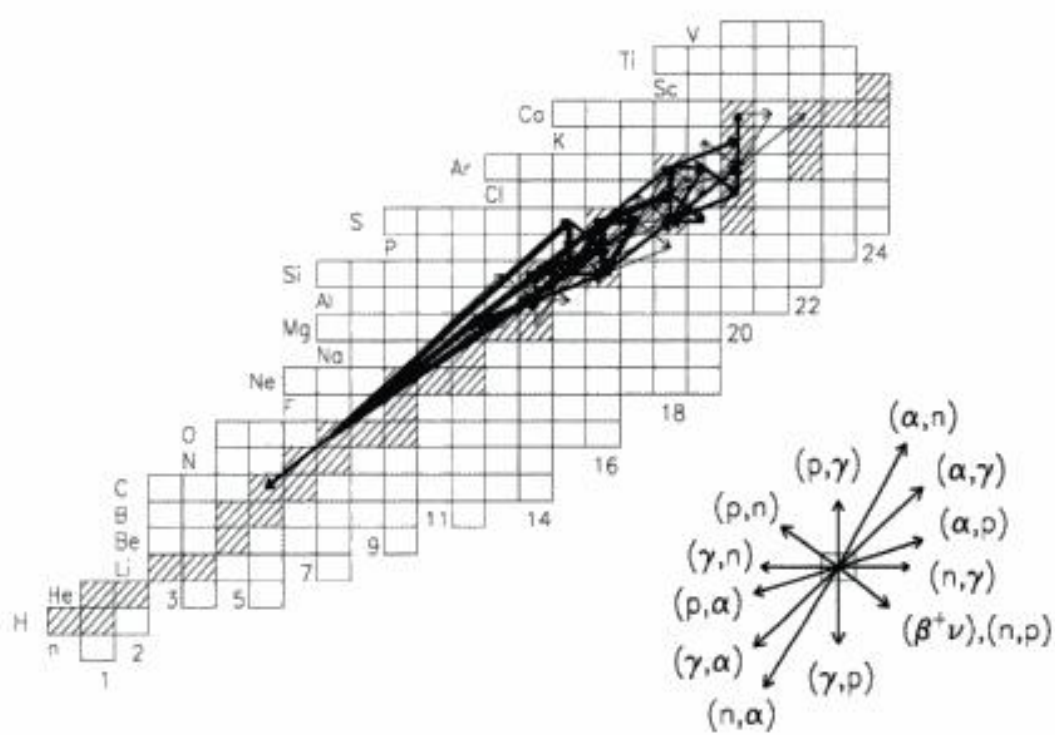
Among the products resulting from neon-burning, oxygen has the lowest Coulomb barrier, and a process similar to carbon burning happens with the oxygen.

The p, n and α -binding energies of ^{24}Mg , ^{28}Si are all > 9 MeV. Their photodisintegration rate is small at these temperatures and densities. Same is true for ^{16}O , with the exception of its α -binding energy (7.2 MeV).

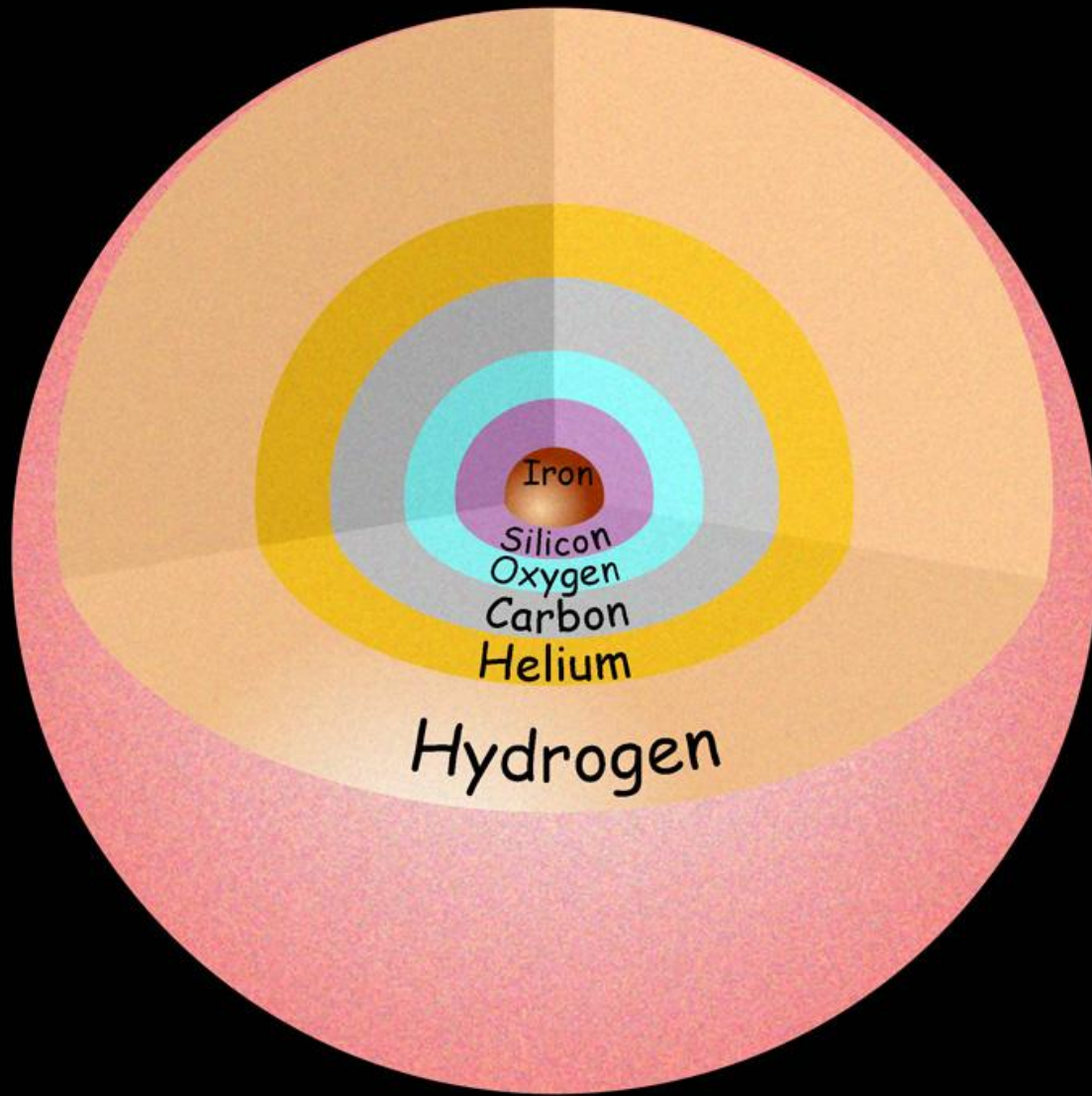
$^{16}\text{O} + ^{16}\text{O}$ dominates over photodisintegration.

$^{16}\text{O}(^{16}\text{O}, p)^{31}\text{P}$	($Q = 7678$ keV)
$^{16}\text{O}(^{16}\text{O}, 2p)^{30}\text{Si}$	($Q = 381$ keV)
$^{16}\text{O}(^{16}\text{O}, \alpha)^{28}\text{Si}$	($Q = 9594$ keV)
$^{16}\text{O}(^{16}\text{O}, 2\alpha)^{24}\text{Mg}$	($Q = -390$ keV)
$^{16}\text{O}(^{16}\text{O}, d)^{30}\text{P}$	($Q = -2409$ keV)
$^{16}\text{O}(^{16}\text{O}, n)^{31}\text{S}$	($Q = 1499$ keV)





Most abundant nuclides after ^{16}O burning stage.



Where to from here?

- We are now at a crossroads for next set of topics:
- Stars with initial mass $M_* > 10$ solar masses end up with the structure on previous page
 - From that point on, the star's fate is sealed: it will become a core collapse supernova
- Stars with initial mass $M_* < 8$ solar masses will not explode
 - They terminate their lives after helium burning is complete and become White Dwarfs
- The remaining topics we have now deal with explosive nuclear burning
- We will begin by looking at the “simplest” explosive objects: Novae