Nuclear Astrophysics II

Lecture 2

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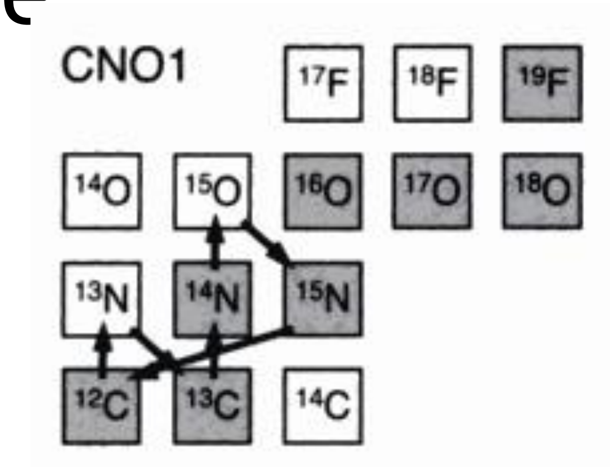
The CNO1 Cycle



1 $$\frac{d\,^{12}C}{dt} = H\,^{15}N\,\langle\sigma v\rangle_{^{15}N(p,\alpha)} - H\,^{12}C\,\langle\sigma v\rangle_{^{12}C(p,\gamma)}$$

2 $$\frac{d\,^{13}N}{dt} = H\,^{12}C\,\langle\sigma v\rangle_{^{12}C(p,\gamma)} - \,^{13}N\,\lambda_{^{13}N(\beta+\nu)}$$

3 $$\frac{d\,^{13}C}{dt} = \,^{13}N\,\lambda_{^{13}N(\beta+\nu)} - H\,^{13}C\,\langle\sigma v\rangle_{^{13}C(p,\gamma)}$$

4 $$\frac{d\,^{14}N}{dt} = H\,^{13}C\,\langle\sigma v\rangle_{^{13}C(p,\gamma)} - H\,^{14}N\,\langle\sigma v\rangle_{^{14}N(p,\gamma)}$$

5 $$\frac{d\,^{15}O}{dt} = H\,^{14}N\,\langle\sigma v\rangle_{^{14}N(p,\gamma)} - \,^{15}O\,\lambda_{^{15}O(\beta+\nu)}$$

6 $$\frac{d\,^{15}N}{dt} = \,^{15}O\,\lambda_{^{15}O(\beta+\nu)} - H^{15}N\,\langle\sigma v\rangle_{^{15}N(p,\gamma)}$$

CNO1

$^{12}C(p,\gamma)^{13}N$

$^{13}N(\beta^{+}\nu)^{13}C$

$^{13}C(p,\gamma)^{14}N$

$^{14}N(p,\gamma)^{15}O$

$^{15}O(\beta^{+}\nu)^{15}N$

$^{15}N(p,\alpha)^{12}C$

$$\lambda_{^{13}N(\beta+\nu)} = 1.1\times 10^{-3}\ \mathrm{s}^{-1}$$

$$\lambda_{^{15}O(\beta+\nu)} = 5.6\times 10^{-3}\ \mathrm{s}^{-1}$$

For temperatures T < 100 MK, the beta decay lifetimes of ^{13}N and ^{15}O are much shorter compared to the lifetimes of ^{12}C and ^{14}N to destruction by protons. In other words, as soon as a ^{12}C + p (or ^{14}N + p) reaction creates ^{13}N (^{15}O) , the newly produced ^{13}N (^{15}O) basically decay before the next new ^{13}N (^{15}O) is made. In this limiting case, the ^{13}N and ^{15}O abundances will quickly reach equilibrium.

$$\lambda_{^{13}N(\beta+\nu)} = 1.1 \times 10^{-3}\ \mathrm{s}^{-1} \ \Rightarrow\ \tau_{^{13}N(\beta+\nu)} = 909\ \mathrm{s}$$

$$\lambda_{^{15}O(\beta+\nu)} = 5.6 \times 10^{-3}\ \mathrm{s}^{-1} \ \Rightarrow\ \tau_{^{15}O(\beta+\nu)} = 179\ \mathrm{s}$$

Therefore, derivatives in equations 2 and 5, page 2, to zero. That system reduces from 6 to 3 differential equations: one for ^{12}C, ^{13}C and ^{14}N.

Setting equations 2, 5 and 6 to zero lets us eliminate ^{13}N, ^{15}O and ^{15}N from our 3 remaining differential equations.

Example: $0 = H\,{}^{12}C\,\langle\sigma v\rangle_{^{12}C(p,\gamma)} - {}^{13}N\,\lambda_{^{13}N(\beta+\nu)}$, from Eq. 2, let's us eliminate ${}^{13}N\,\lambda_{^{13}N(\beta+\nu)}$ term from diff. equation 3.

Table 5-3 Dependence of $\log\,(\tau\rho X_H/100)$ **on temperature**†

Temperature, T_6	*Reaction*‡						
	$C^{12}(p,\gamma)N^{13}$	$C^{13}(p,\gamma)N^{14}$	$N^{14}(p,\gamma)O^{15}$	$N^{15}(p,\alpha)C^{12}$	$10^4\gamma$	$O^{16}(p,\gamma)F^{17}$	$O^{17}(p,\alpha)N^{14}$
5	16.32	15.73	19.79	15.53	4.649	22.95	21.92
6	14.32	13.73	17.57	13.29	4.598	20.51	20.02
7	12.72	12.13	15.79	11.50	4.551	18.56	18.26
8	11.41	10.81	14.32	10.03	4.508	16.95	16.50
9	10.29	9.69	13.08	8.78	4.468	15.59	15.10
10	9.33	8.73	12.02	7.70	4.431	14.42	14.05
11	8.50	7.90	11.09	6.76	4.396	13.39	13.15
12	7.75	7.15	10.26	5.93	4.363	12.49	12.38
13	7.09	6.49	9.52	5.18	4.332	11.68	11.68
14	6.49	5.89	8.86	4.51	4.303	10.95	11.02
15	5.95	5.35	8.26	3.90	4.275	10.29	10.32
16	5.45	4.85	7.71	3.34	4.248	9.68	9.55
17	5.00	4.39	7.20	2.83	4.223	9.13	8.70
18	4.58	3.97	6.73	2.35	4.198	8.61	7.86
19	4.18	3.58	6.30	1.91	4.175	8.14	7.01
20	3.82	3.21	5.89	1.50	4.152	7.69	6.18
22	3.16	2.55	5.16	0.75	4.110	6.89	4.78
24	2.57	1.97	4.51	0.09	4.071	6.18	3.63
25	2.30	1.70	4.21	−0.21	4.052	5.85	3.10
26	2.05	1.44	3.93	−0.50	4.034	5.54	2.62
28	1.58	0.97	3.41	−1.03	4.000	4.97	1.75
30	1.15	0.54	2.93	−1.51	3.967	4.45	1.05
35	0.23	−0.38	1.91	−2.55	3.893	3.33	−0.42
40	−0.53	−1.14	1.07	−3.42	3.829	2.41	−1.50
45	−1.18	−1.78	0.36	−4.14	3.771	1.64	−2.33
50	−1.73	−2.33	−0.25	−4.77	3.719	0.97	−2.99
55	−2.21	−2.82	−0.78	−5.32	3.673	0.39	−3.53
60	−2.64	−3.24	−1.25	−5.81	3.630	−0.12	−3.97
65	−3.02	−3.63	−1.67	−6.24	3.590	−0.58	−4.33
70	−3.37	−3.97	−2.05	−6.63	3.554	−0.99	−4.65
75	−3.68	−4.28	−2.39	−6.99	3.521	−1.37	−4.91
80	−3.97	−4.57	−2.71	−7.32	3.489	−1.71	−5.14
85	−4.23	−4.83	−2.99	−7.62	3.460	−2.02	−5.35
90	−4.48	−5.08	−3.26	−7.90	3.433	−2.31	−5.52
95	−4.70	−5.30	−3.51	−8.15	3.407	−2.58	−5.68
100	−4.91	−5.51	−3.74	−8.39	3.383	−2.83	−5.82

† Adapted from G. R. Caughlan and W. A. Fowler, *Astrophys. J.*, **136**:453 (1962). By permission of The University of Chicago Press.

CNO Nuclear Data

Nuclear lifetimes for relevant reactions. Note, the table shows values of:

$$\log_{10}(\rho X_H \tau/100)$$

Within CNO1, it is ^{15}N which has the shortest lifetime, especially after ~ 20 MK.

Next approximation: assume ^{15}N reaches equilibrium fast compared to ^{14}N. Then derivative in Eq. 6 is zero:

$$\left(\frac{^{15}\mathrm{N}}{^{14}\mathrm{N}}\right)_e = \frac{\tau_{15}}{\tau_{14}}$$

The resulting simplified system of equations for CNO1 cycle is then:

$$\frac{d}{dt}\begin{bmatrix} ^{12}C \\ ^{13}C \\ ^{14}N \end{bmatrix} = \begin{bmatrix} -\frac{1}{\tau_{12}} & 0 & \frac{1}{\tau_{14}} \\ \frac{1}{\tau_{12}} & -\frac{1}{\tau_{13}} & 0 \\ 0 & \frac{1}{\tau_{13}} & -\frac{1}{\tau_{14}} \end{bmatrix} \begin{bmatrix} ^{12}C \\ ^{13}C \\ ^{14}N \end{bmatrix}$$

Recall: $\tau_{12}^{-1} \equiv N_p \langle \sigma v \rangle_{12} = \rho \frac{X_p}{M_p} N_A \langle \sigma v \rangle_{12}$

This is a **coupled system** of differential first order differential equations. How to solve it (or any other such system)?

Consider, first, the following simple system of equations, for which the solution is clear:

$$\frac{d}{dt}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

The decoupled nature of all variables (because the coefficient matrix is diagonal) lets us immediately write the solution:

$$\begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix} = \begin{bmatrix} u_1^0 \, e^{\lambda_1 t} \\ u_2^0 \, e^{\lambda_2 t} \\ u_3^0 \, e^{\lambda_3 t} \end{bmatrix}$$

Where u_i^0 denotes the value of the function u at time $t = 0$.

This suggests that, if we diagonalize the CNO1 system of equations, we can get the formal solution.

Consider some linear algebra reminders. Here, A is an $n \times n$ non-diagonal matrix, and y is a column matrix of length "n".

The general system we need to solve is: $$\frac{dy}{dt} = Ay$$

Let's suppose we can write the column vector y, as: $$y = Pu$$
Where P is another n x n matrix.

Then, we can write: $$P\,\frac{du}{dt} = APu$$

$$\Rightarrow \quad \frac{du}{dt} = P^{-1}APu = Du$$ Where: $D = P^{-1}AP$

If matrix D is diagonal, we know the solution from previous page.

To make D, we need the matrix P that diagonalizes matrix A. The matrix P is made, as you know, from column vectors that are the eigenvectors of matrix A.

We also know from Linear Algebra theory that the diagonal matrix D has diagonal entries that are the eigenvalues for matrix A.

So, our solution for column vector y is formally given by, $y = Pu$

Where column vector u is:

$$\begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix} = \begin{bmatrix} u_1^0 \; e^{\lambda_1 t} \\ u_2^0 \; e^{\lambda_2 t} \\ u_3^0 \; e^{\lambda_3 t} \end{bmatrix}$$

And matrix P is given by: $P = [\Omega_1 \mid \Omega_2 \mid \Omega_3]$

Where $\Omega_i = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}_i$ are the eigenvectors of matrix A.

Solution is then:

$$y(t) = A\,\Omega_1\,u_1(t) + B\,\Omega_2\,u_2(t) + C\,\Omega_3\,u_3(t)$$

And $A, \; B, \; C$ are constants to be determined by initial conditions.

CNO1 Cycle: Solution

Our reduced system for CNO1 is given by:

$$\frac{d}{dt}\begin{bmatrix} {}^{12}C \\ {}^{13}C \\ {}^{14}N \end{bmatrix} = \begin{bmatrix} -\frac{1}{\tau_{12}} & 0 & \frac{1}{\tau_{14}} \\ \frac{1}{\tau_{12}} & -\frac{1}{\tau_{13}} & 0 \\ 0 & \frac{1}{\tau_{13}} & -\frac{1}{\tau_{14}} \end{bmatrix}\begin{bmatrix} {}^{12}C \\ {}^{13}C \\ {}^{14}N \end{bmatrix}$$

The 3 eigenvalues are obtained by solving: $\det(A - \lambda I) = 0$

They are given by:

$$\lambda_1 = 0 \qquad \lambda_2 = \frac{-\Sigma + \Delta}{2} \qquad \lambda_3 = -\frac{\Sigma + \Delta}{2}$$

$$\Sigma = \frac{1}{\tau_{12}} + \frac{1}{\tau_{13}} + \frac{1}{\tau_{14}} \qquad \Delta = \left[\Sigma^2 - 4\left(\frac{1}{\tau_{12}\tau_{13}} + \frac{1}{\tau_{12}\tau_{14}} + \frac{1}{\tau_{13}\tau_{14}}\right)\right]^{1/2}$$

After doing the algebra, 3 candidate eigenvectors are found to be:

$$\Omega_1 = \frac{1}{\tau_{14}} \begin{bmatrix} \tau_{12} \\ \tau_{13} \\ \tau_{14} \end{bmatrix} \qquad \lambda_1 = 0$$

$$\Omega_2 = \begin{bmatrix} 1 \\ \frac{1/\tau_{12}}{1/\tau_{13} - (\Sigma - \Delta)/2} \\ -1 - \frac{1/\tau_{12}}{1/\tau_{13} - (\Sigma - \Delta)/2} \end{bmatrix} \qquad \Omega_3 = \begin{bmatrix} 1 \\ -1 - \frac{1/\tau_{12} - (\Sigma + \Delta)/2}{1/\tau_{14}} \\ \frac{1/\tau_{12} - (\Sigma + \Delta)/2}{1/\tau_{14}} \end{bmatrix}$$

$$\lambda_2 = \frac{-\Sigma + \Delta}{2} \qquad \lambda_3 = -\frac{\Sigma + \Delta}{2}$$

$$\Sigma = \frac{1}{\tau_{12}} + \frac{1}{\tau_{13}} + \frac{1}{\tau_{14}} \qquad \Delta = \left[\Sigma^2 - 4\left(\frac{1}{\tau_{12}\tau_{13}} + \frac{1}{\tau_{12}\tau_{14}} + \frac{1}{\tau_{13}\tau_{14}}\right)\right]^{1/2}$$

Looking back at the form of the solution to y(t), we had:

$$y(t) = A\,\Omega_1\,u_1(t) + B\,\Omega_2\,e^{\lambda_2 t} + C\,\Omega_3\,e^{\lambda_3 t}$$

The first eigenvalue is $\lambda_1 = 0$, and so $u_1(t)$ is a constant (exponential with argument = 0) and as time goes to infinity, $u_2,\ u_3 \rightarrow 0$ and the cycle comes into equilibrium. Therefore, the constant A must be such that the first term is just the column vector of equilibrium abundances.

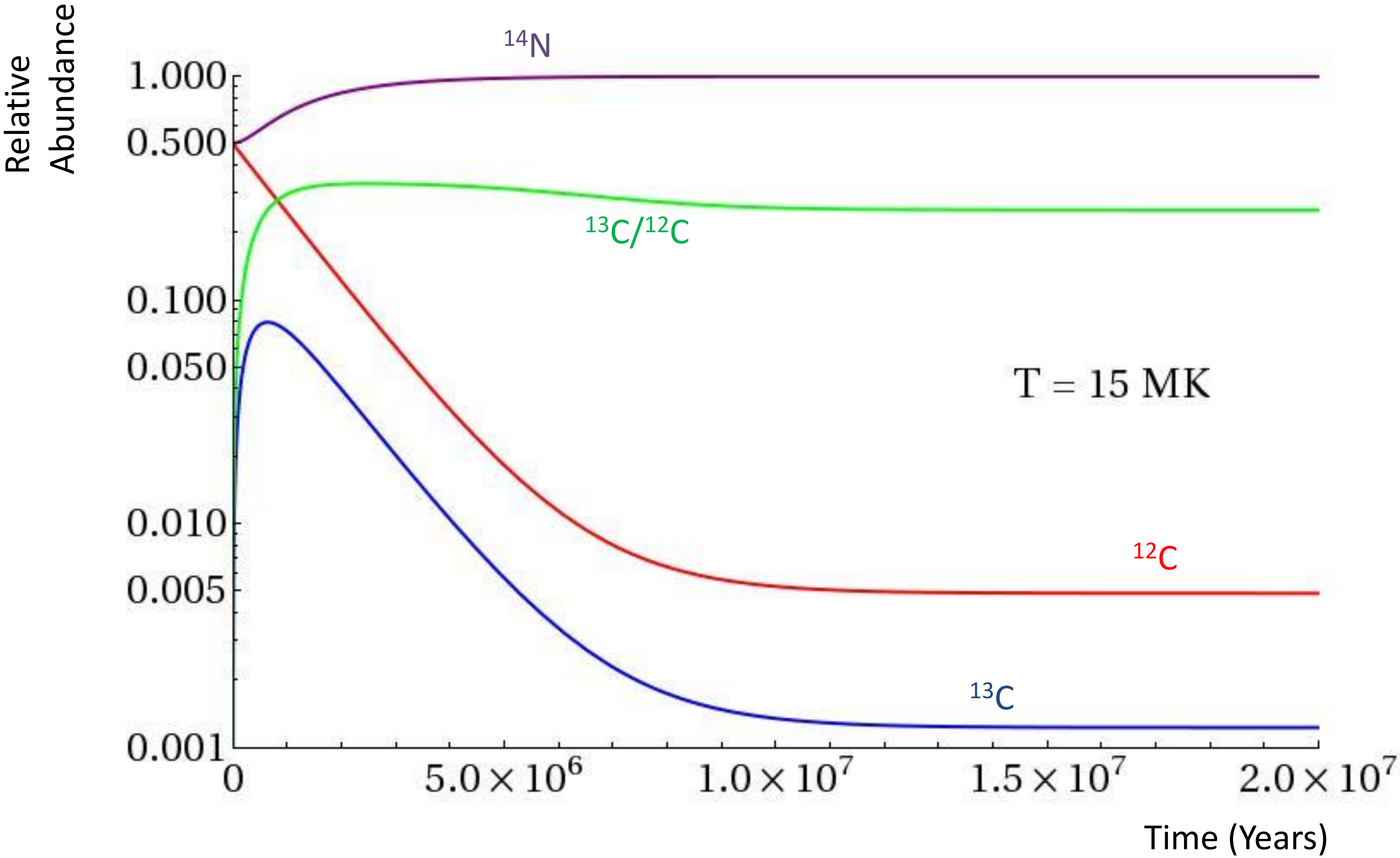
$$A\,\Omega_1 u_1 = \frac{1}{\tau_{12} + \tau_{13} + \tau_{14}} \begin{bmatrix} \tau_{12} \\ \tau_{13} \\ \tau_{14} \end{bmatrix} = \begin{bmatrix} ^{12}\mathrm{C}_e \\ ^{13}\mathrm{C}_e \\ ^{14}\mathrm{N}_e \end{bmatrix}$$

For coefficients B and C, we have from inspection:

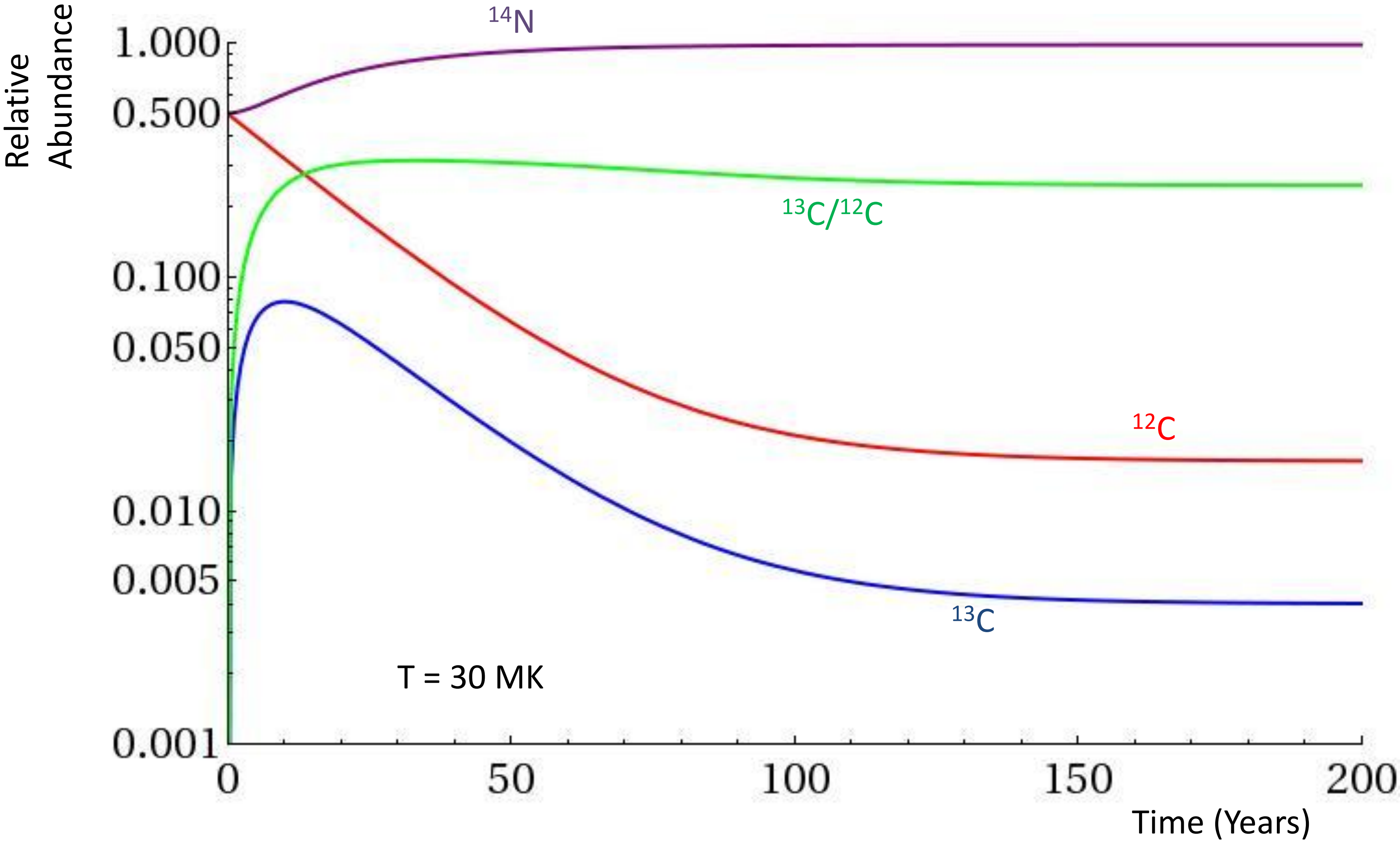
$$^{12}\mathrm{C}(0) - ^{12}\mathrm{C}_e = B + C$$

$$^{13}\mathrm{C}(0) - ^{13}\mathrm{C}_e = B\left(\frac{1/\tau_{12}}{1/\tau_{13} - (\Sigma - \Delta)/2}\right) - C\left(1 + \frac{1/\tau_{12} - (\Sigma + \Delta)/2}{1/\tau_{14}}\right)$$

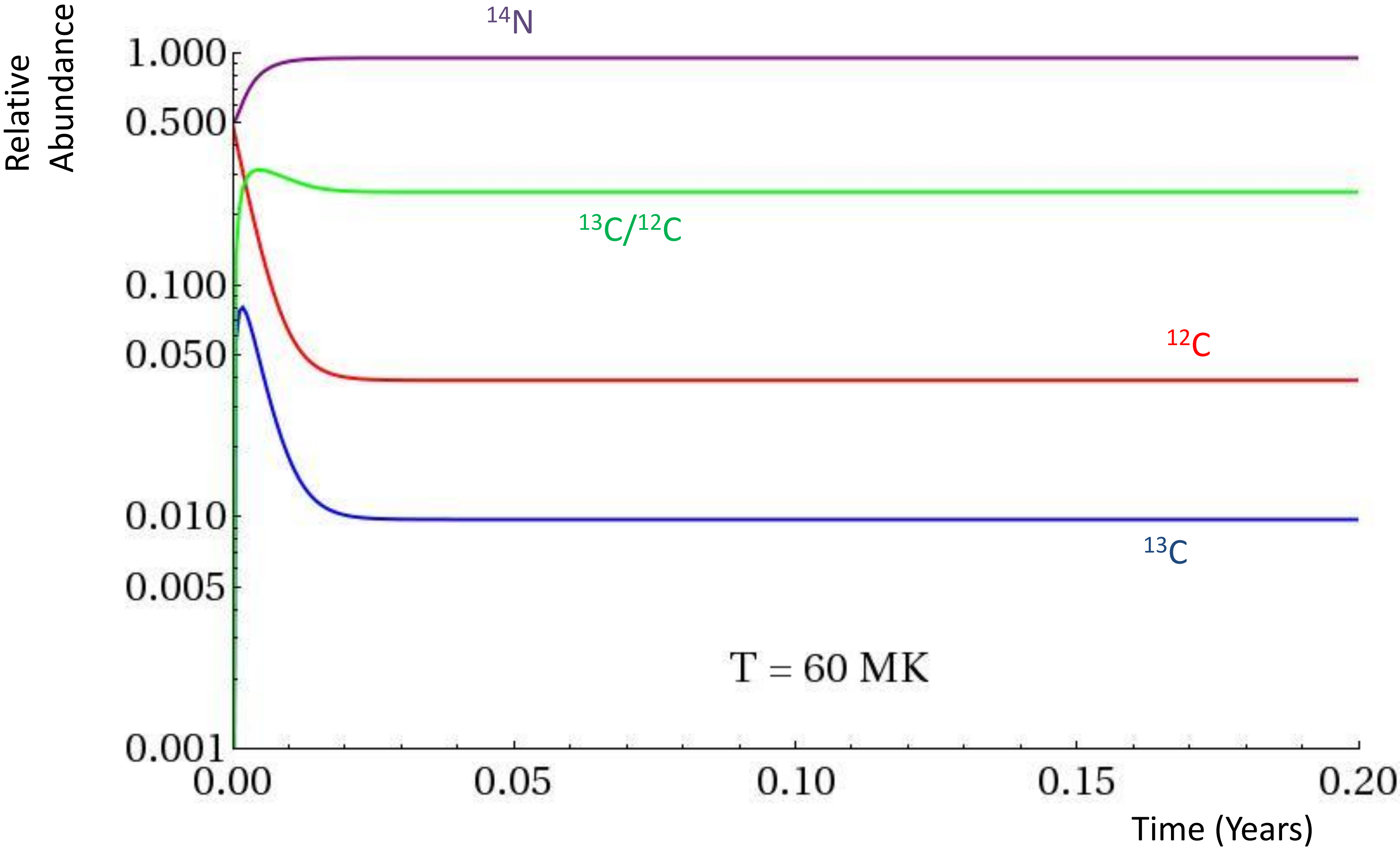
CNO1 Abundances: 15 MK



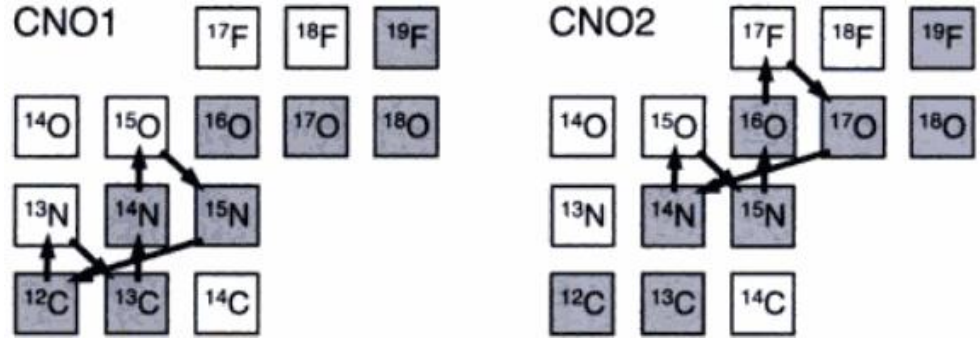
CNO1 Abundances: 30 MK



CNO1 Abundances: 60 MK



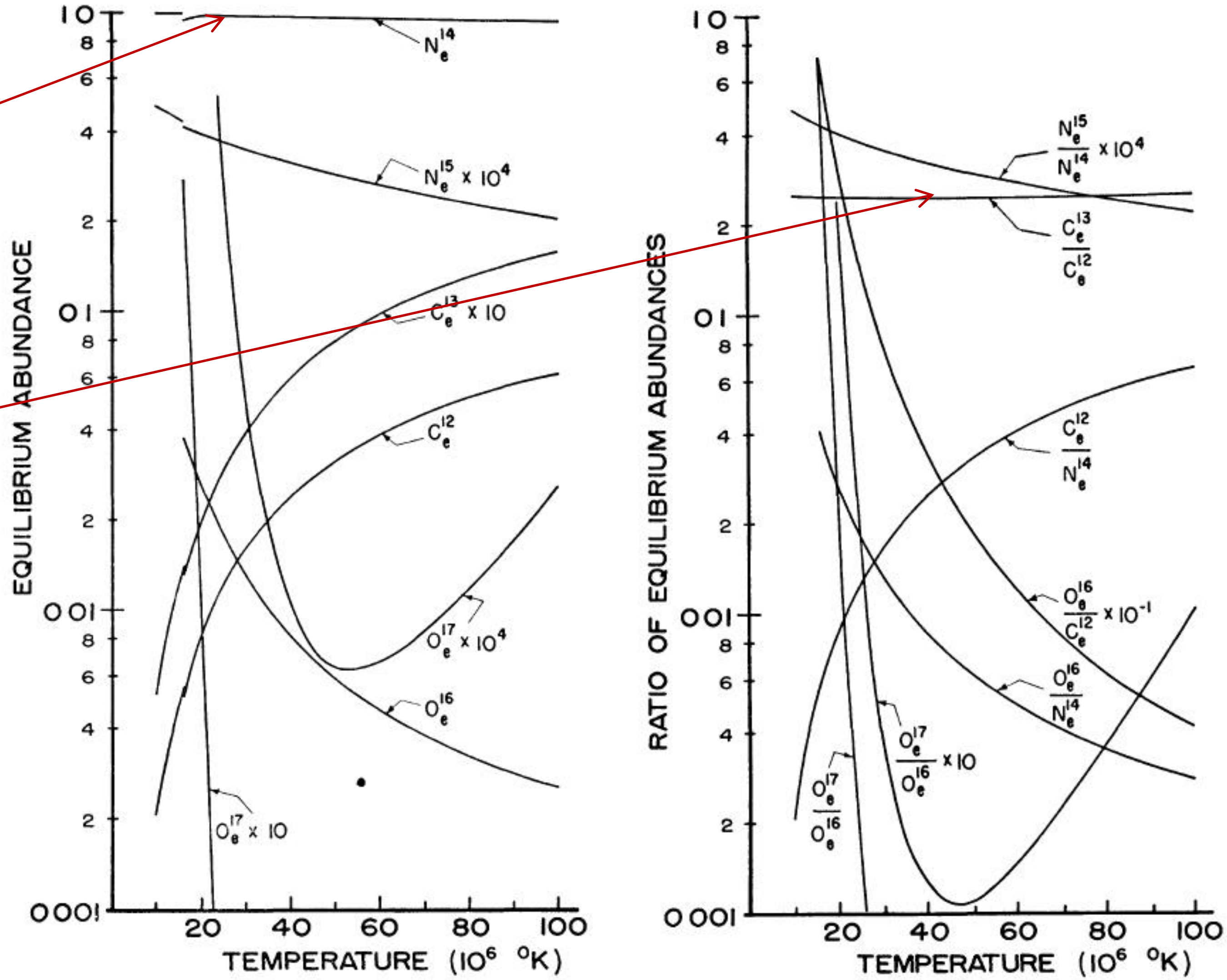
Simultaneous CNO1 and CNO2 Solutions, for Equilibrium abundances as a function of temperature. Solved for X = 0.755, Y = 0.231, Z = rest



All CNO isotopes processed mostly to ^{14}N.

Ratio of $^{13}C/^{12}C$ is a constant for all temperatures. Value is ≈ 0.25

Make note of the other correlations between the isotopes, using the figure on the right.



G. R. Caughlan, ApJ (1962)

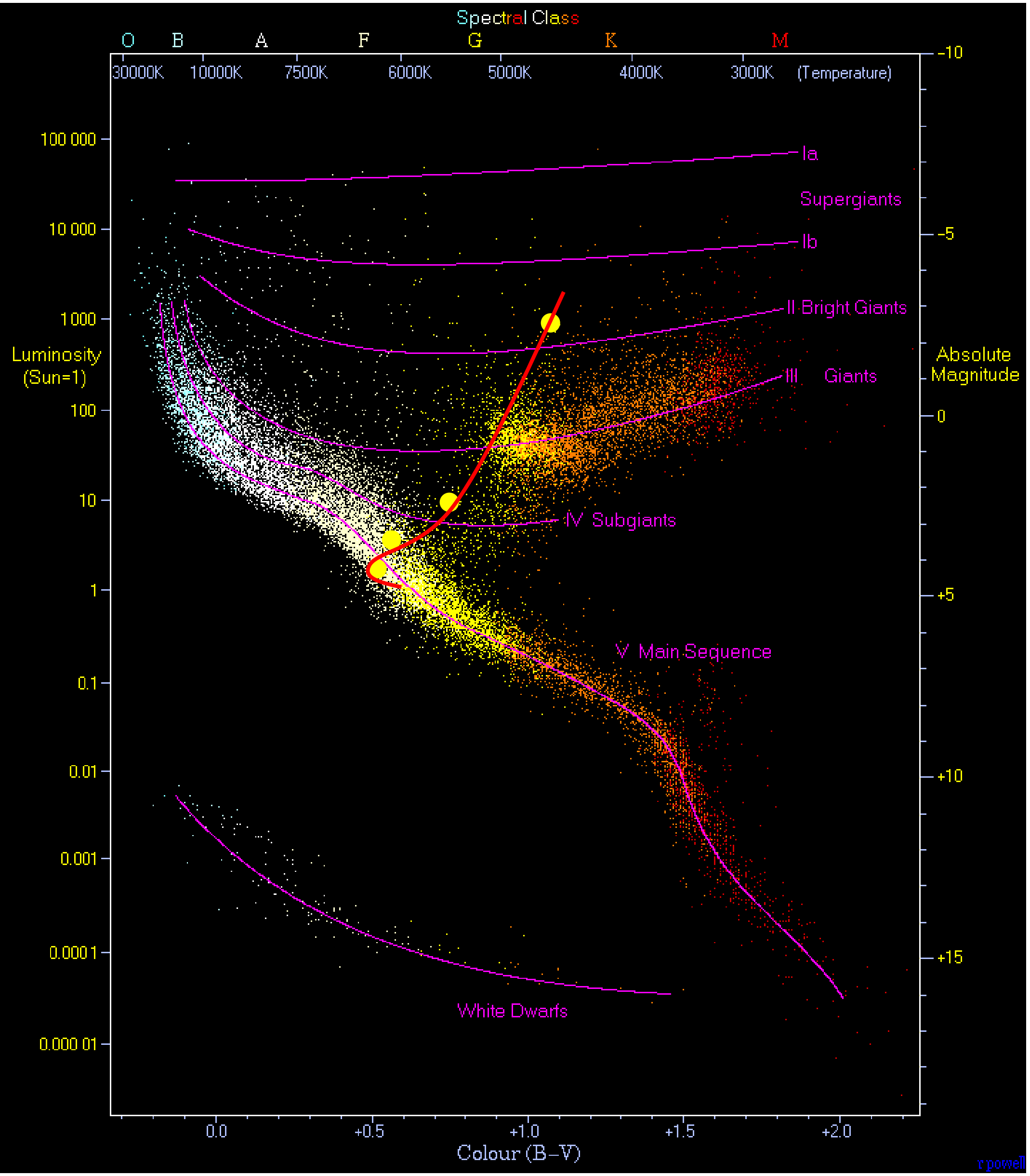
That was the Nuclear Side of the Story. What have the stars themselves have to say?

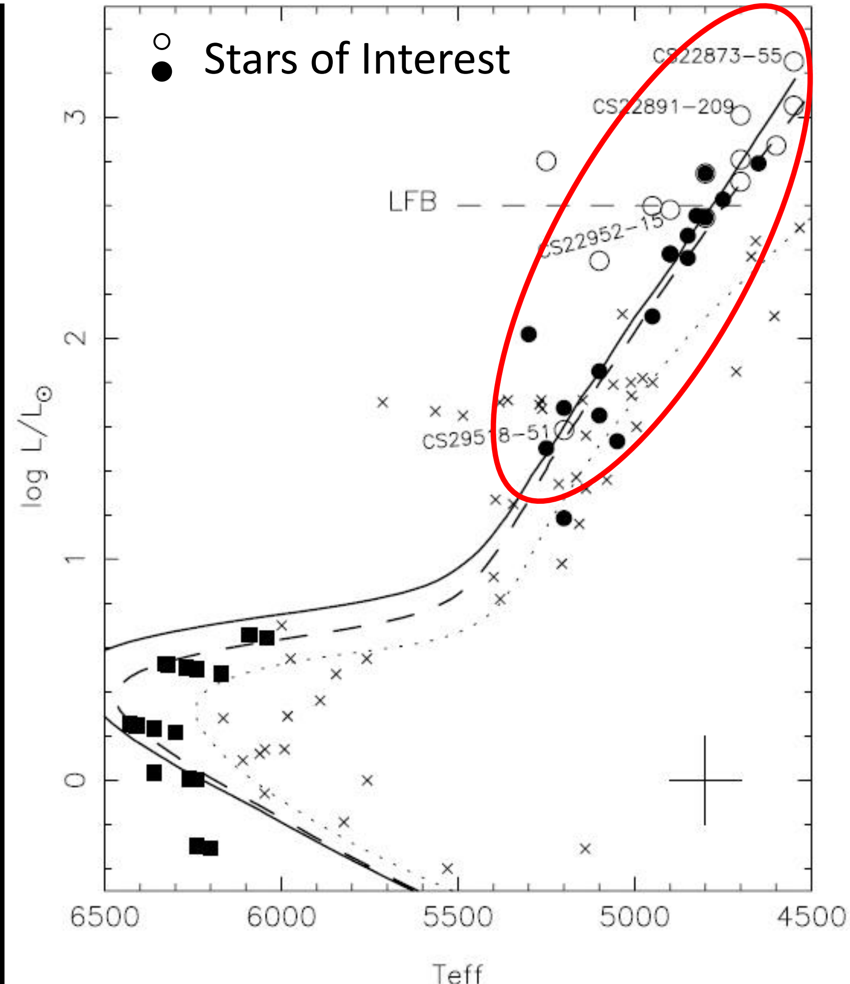
CONNECTION WITH OBSERVATIONS

Very Large Telescope: ESO

- Optical Telescope Array, 4 x 8.2 meter mirrors
- High resolution spectrograph
  - Studies of accretion disk compositions in CV’s
  - Spectroscopic studies
    - AGBs, Red Giants, Planetary Nebula
    - Supernova: Type 1a, Core Collapse
      - Nucleosynthesis yields → determine if Type 1a or CC
        - » Type 1a tell us about Cosmic Expansion (Standard Candle)
      - Absorption lines can be used to determine expansion velocities of ejecta

Connection with Astronomy





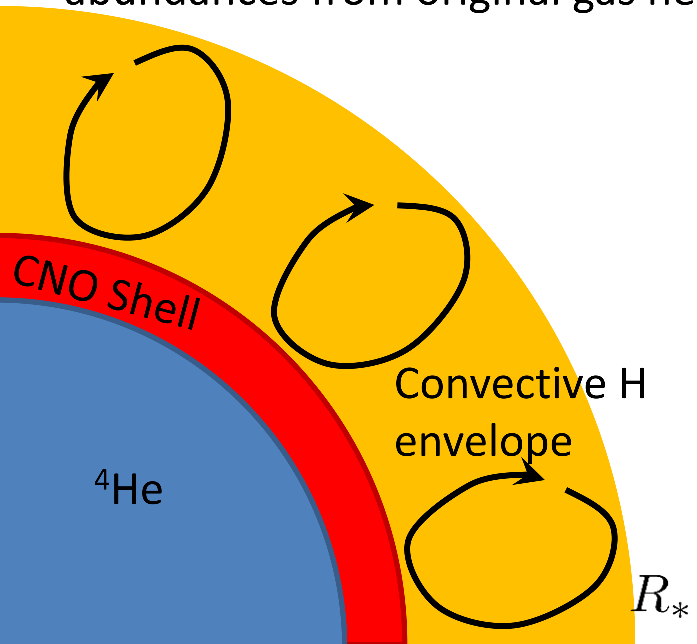
M. Spite et al., A&A **455**, 291 (2006)

Structure of Giants

This is a quick summary of a particular type of Giant structure; this is not exhaustive, nor does it describe all Giant types.

Let's consider this a working "hypothesis" for now. And let us see if it makes sense when we consider what we've just learned from the CNO1,2 formalism along with what the astronomical observations reveal..

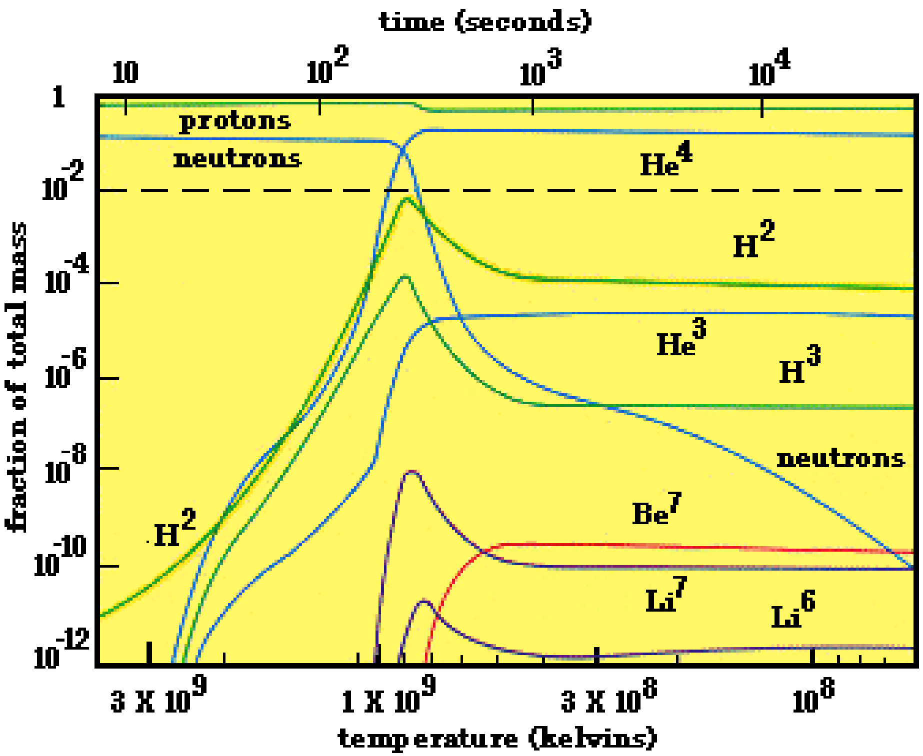
- Helium (alpha) burning core.
- Shell outside core burning CNO-cycle
- Convective outer envelope composed of (original) hydrogen and all other nascent abundances from original gas nebula



The temperature is high enough in the CNO shell that the CNO1,2 cycles can, in principle, both function.

Something to wonder about: Could the convective shell dredge (at some point in star's life) the CNO abundances to surface for eventual observation?

The Case of ^{7}Li: A Stellar Diagnostic



Primordial BBNS predicts that the lithium abundance is dominated by ^{7}Li, with ^{6}Li about one order of magnitude lower.

Lithium is **not** produced in stars; it is only destroyed. Why: Because the (p,α) lifetime is so short compared to potential production reactions such as $\alpha(^3\text{He},\gamma)^7\text{Li}$. Li lifetime is the order of 1/10 – 1/100 of a second at temperatures > 10 MK.

The point: if envelope of Giant shows **no** lithium, convective mixing into hot proton- and CNO-burning regions has **potentially** occurred.

Measured Giant Abundances: A Story

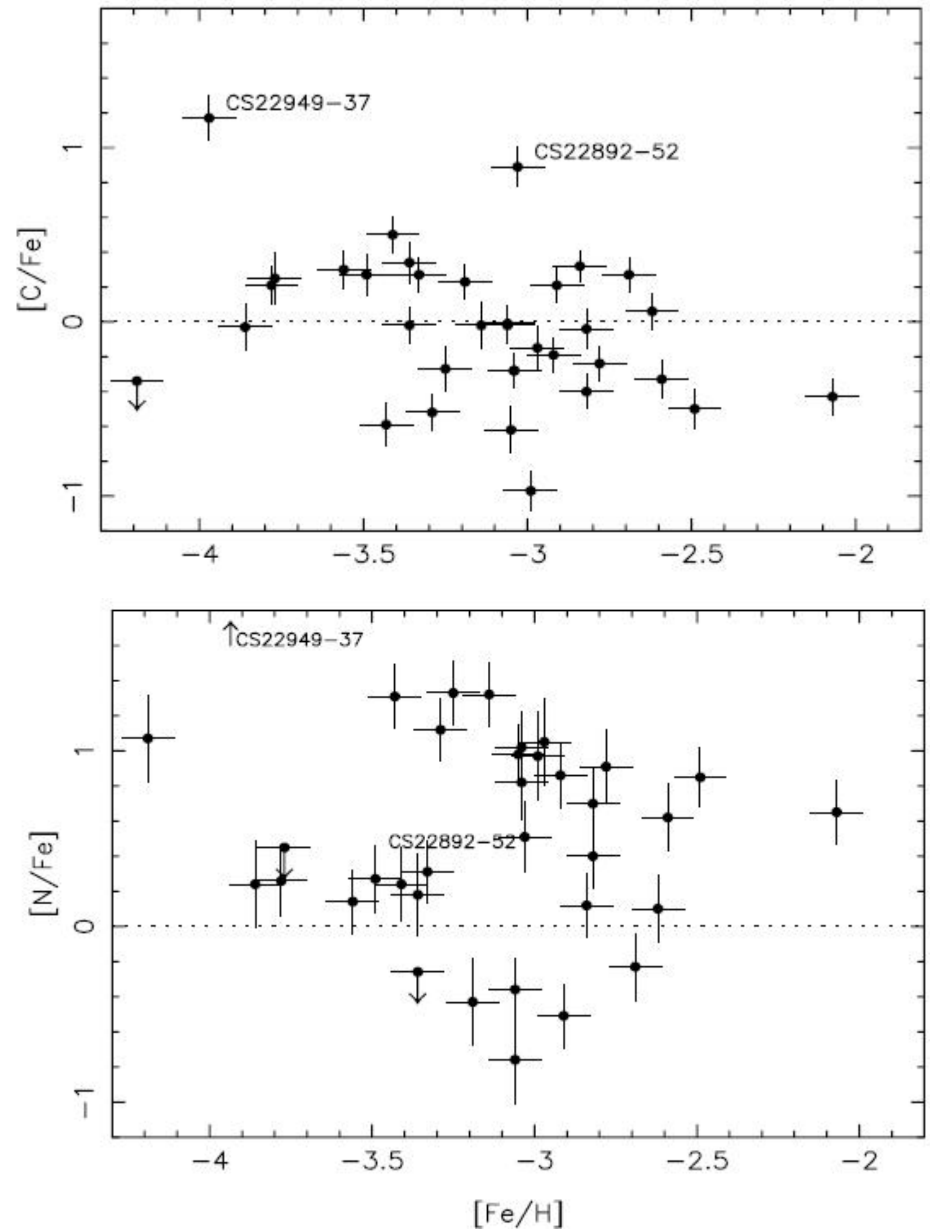
First observations of these stars is the following:

1. Their nitrogen abundance is largely scattered, across ~ 2 orders.

2. They are very "metal poor". The amount of Fe is much lower than the Sun. This means the stars are formed from gas that did not undergo much supernova processing.

3. Note, by definition:

$$[x] \equiv \log_{10}(X) - \log_{10}(X_{\odot})$$



Data from: M. Spite et al., A&A **430**, 655 (2005)

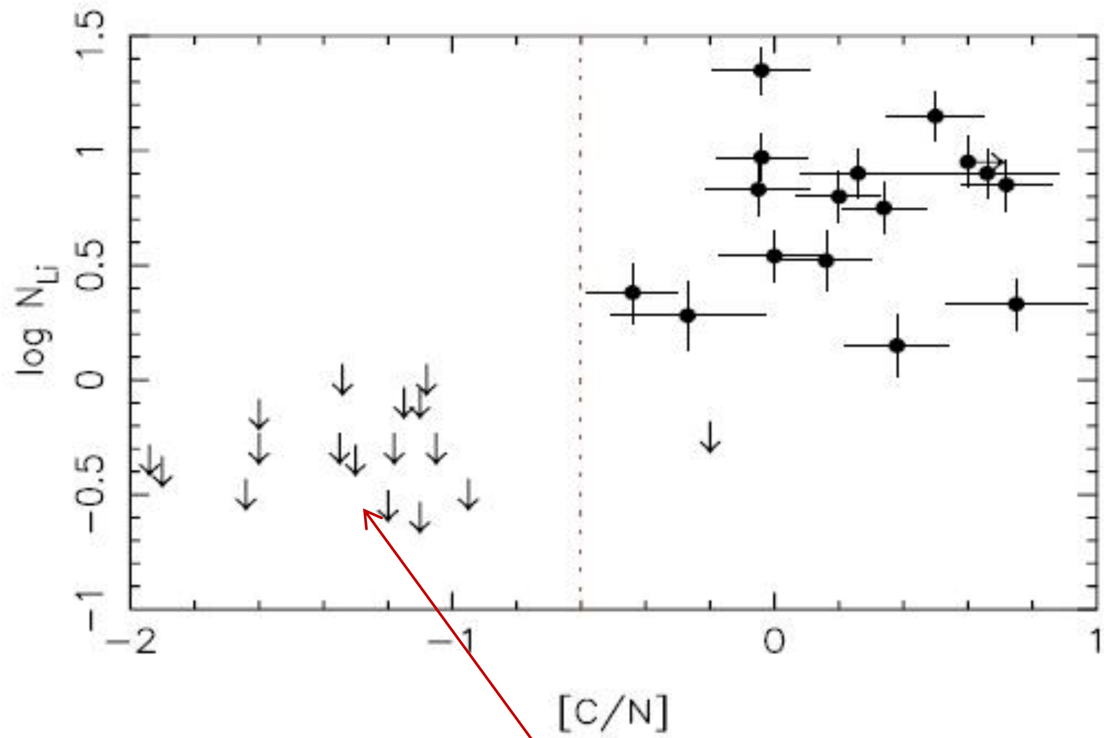
Abundance Correlations: Li

We have seen that the CNO1 cycle, from example, converts any C and O into N. We expect, therefore, that any stars that have burned CNO1 to have low C/N ratios.

We also know that the lifetime of Li to proton destruction is fractions of a second at T > 20 MK.

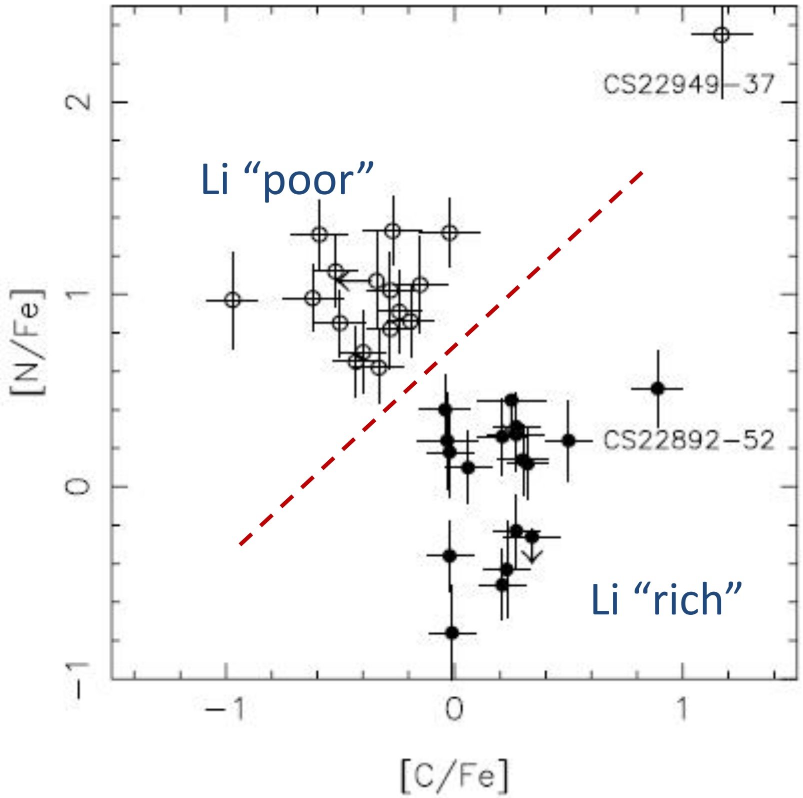
The Li abundance correlation with [C/N] suggests that convective mixing in the stars on the left has taken place.

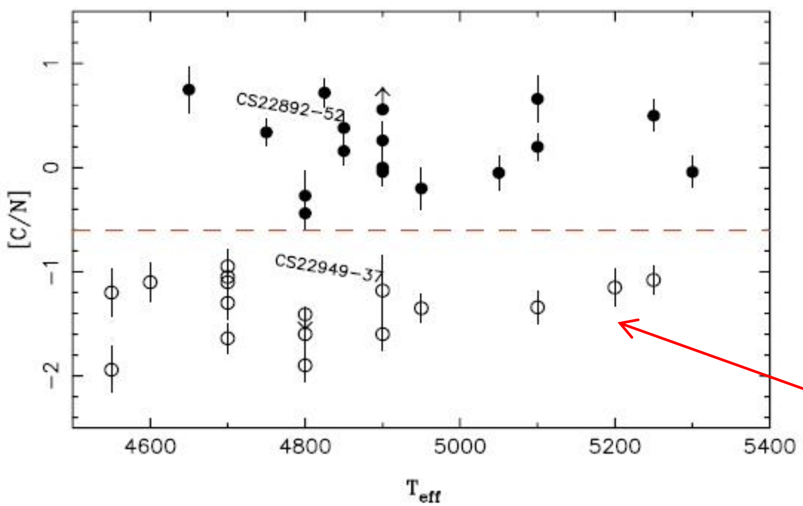
Stars that have mixed Li down into the burning zone will have no Li. Conversely, CNO from burning zone comes up into atmosphere. If CNO1 has burned, C/N should be low. The correlation suggests this.



Upper limits only on Li in these stars.

Abundance Correlations: C & N





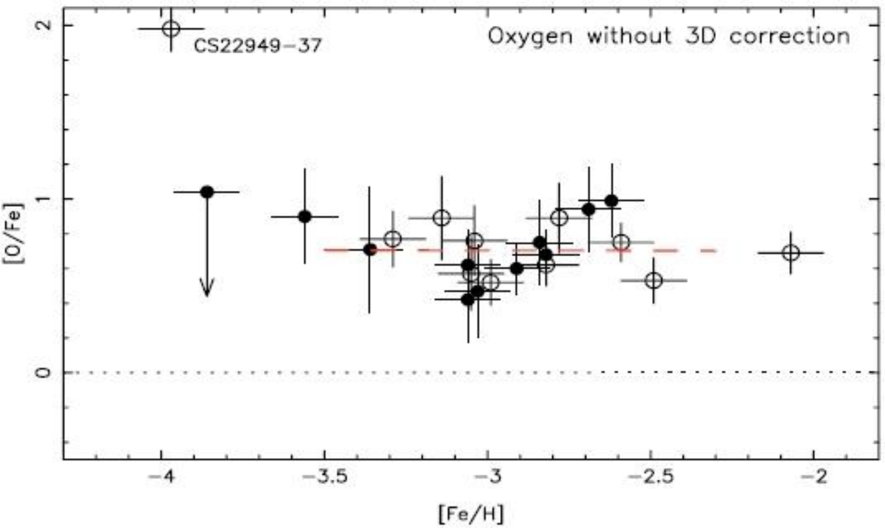
Here, we see the mixing hypothesis confirmed again.

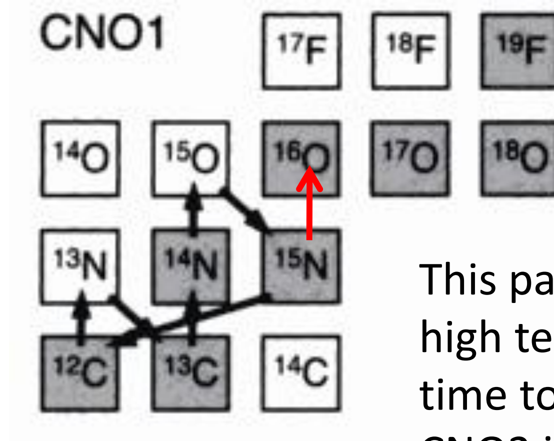
The correlation between the [N/Fe] vs [C/Fe] abundance ratios is what we would expect from CNO1 burning deep down in the star combined with a convective process that is able to dredge CNO processed material up to the atmosphere of the star.

For the Li “rich” stars, C is high and N is low. This is **not** what we expect from CNO1 burning.

The Li “poor” stars show low C and high N, consistent with CNO1 burning.

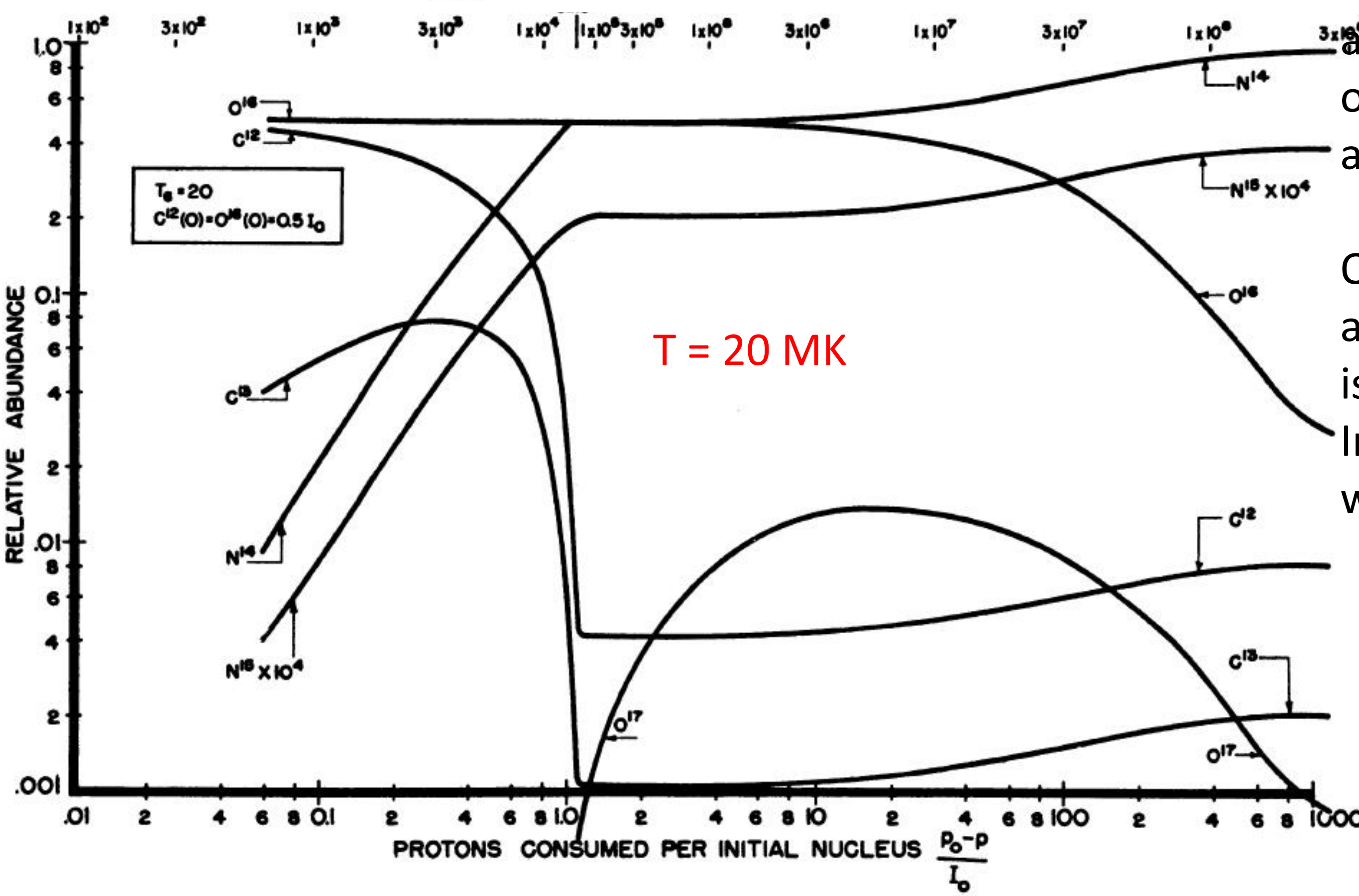
Stars of similar spectral class and masses. But large differences in CNO.



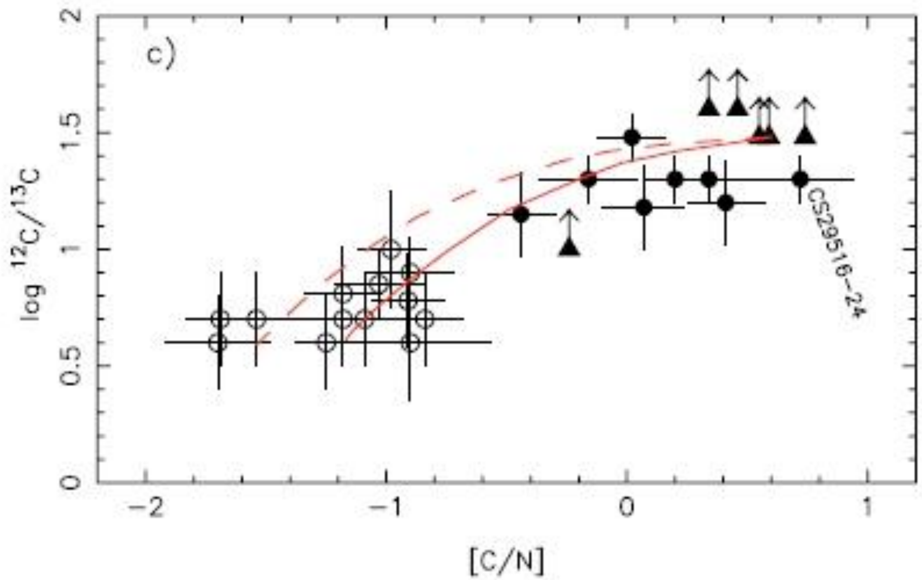


This path is small. Need high temperature and long time to get buildup of CNO2 isotopes. Look at 17O equilibrium abundance as a function of time back on page 15. Only increases after ~ 60 MK.

Observed oxygen abundances for these stars is the same for **all** stars. Implies that CNO2 burning was not significant.



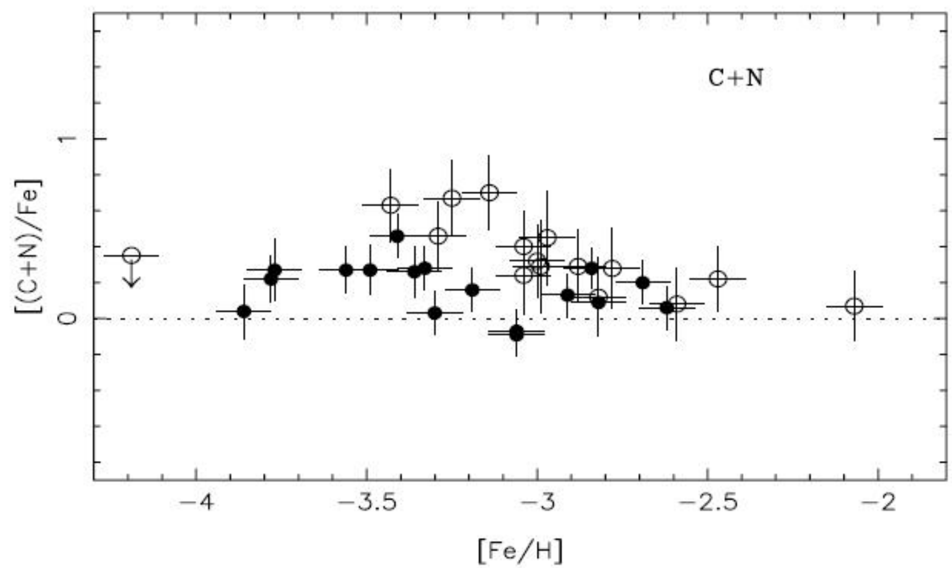
Abundance Correlations: ^{12}C, ^{13}C & N



$^{12}C/^{13}C$ Ratio: Some of the mixed stars show a ratio that is ~$10^{0.6}$ = 4, which is the CNO1 equilibrium ratio! Also, the correlation of $^{12}C/^{13}C$ with C/N is consistent. Mixed stars should have smaller C/N than unmixed stars.

M. Spite et al., A&A **455** (2006)

The **sum** of all CNO1 nuclei in the cycle must be conserved. Thus, mixed and unmixed stellar envelopes, for comparable stars, should have summed CNO1 abundances that are compatible with each other. Within the errors, mixed and unmixed stars agree!



Hydrostatic Burning Summary

- PP-chains and CNO cycles work to covert 4 protons into one alpha-particle
- CNO chains operate with C, N and O nuclei as “catalysts”; their total number is not destroyed, and the operation of the CNO-cycles converts almost all C and O into ^{14}N. ($^{14}N(p,\gamma)^{15}O$ is, by far, the slowest reaction in the CNO cycle. Thus, material “piles up” here.)
- PP-chains, when in equilibrium, burn with a temperature dependence like $\sim T^4$; CNO cycle, by contrast, burns in equilibrium with a temperature dependence like $\sim T^{16}$.
- See Appendix of this lecture, and refer back to last lecture, for the equilibrium energy generation rates for each chain.
- The CNO cycle, combined with observational data, has revealed to us something about the structure of Red Giant stars
  - CNO1 cycle operates in such stars, including those around the same mass as our Sun
  - Lithium and CNO abundances show that Red Giants have a structure that allows deep convection from the surface down into the region of the core
  - This property, however, is simultaneous **not** seen in other Red Giant data
- Question: Why do some Red Giant data show the result of mixing between core and envelope, and others do not (yet?) show such an effect?

SUPPLEMENTARY SLIDES FOR CNO1 CYCLE

For temperatures T < 100 MK, the beta decay lifetimes of ^{13}N and ^{15}O are much shorter compared to the lifetimes of ^{12}C and ^{14}N to destruction by protons. In other words, as soon as a ^{12}C + p (or ^{14}N + p) reaction creates ^{13}N (^{15}O) , the newly produced ^{13}N (^{15}O) basically decay before the next new ^{13}N (^{15}O) is made. In this limiting case, the ^{13}N and ^{15}O abundances will quickly reach equilibrium.

Exercise for you: set the time derivatives of ^{13}N (^{15}O) to zero in the set of equations on page 2. Then eliminate ^{13}N (^{15}O) from the system of equations, allowing us to now approximate the CNO cycle with the following system of equations:

$$\frac{d(^{12}C)}{dt} = H(^{15}N)\langle\sigma v\rangle_{^{15}N(p,\alpha)} - H(^{12}C)\langle\sigma v\rangle_{^{12}C(p,\gamma)}$$

$$\frac{d(^{13}C)}{dt} = H(^{12}C)\langle\sigma v\rangle_{^{12}C(p,\gamma)} - H(^{13}C)\langle\sigma v\rangle_{^{13}C(p,\gamma)}$$

$$\frac{d(^{14}N)}{dt} = H(^{13}C)\langle\sigma v\rangle_{^{13}C(p,\gamma)} - H(^{14}N)\langle\sigma v\rangle_{^{14}N(p,\gamma)}$$

$$\frac{d(^{15}N)}{dt} = H(^{14}N)\langle\sigma v\rangle_{^{14}N(p,\gamma)} - H(^{15}N)\langle\sigma v\rangle_{^{15}N(p,\alpha)}$$

Another exercise: Prove the ***sum*** of CNO abundances is constant in time.

Let's simplify even further, and assume the star is old enough that the CNO cycle is operating in steady state (all reactions are in equilibrium).

The equilibrium abundance ratios for any two species is then just given by (check this using the equations on previous page):

$$\left(\frac{^{14}N}{^{12}C}\right)_e = \frac{\langle\sigma v\rangle_{^{12}C(p,\gamma)}}{\langle\sigma v\rangle_{^{14}N(p,\gamma)}}$$

Fractional abundance for a particular species (^{12}C for example) relative to all CNO nuclei:

$$\frac{(^{12}C)_e}{\sum CNO} = \left(1 + \frac{\langle\sigma v\rangle_{^{12}C(p,\gamma)}}{\langle\sigma v\rangle_{^{13}C(p,\gamma)}} + \frac{\langle\sigma v\rangle_{^{12}C(p,\gamma)}}{\langle\sigma v\rangle_{^{14}N(p,\gamma)}} + \frac{\langle\sigma v\rangle_{^{12}C(p,\gamma)}}{\langle\sigma v\rangle_{^{15}N(p,\gamma)}}\right)^{-1}$$

And a reminder: Each $\langle\sigma v\rangle$ is the result of an *experimentally determined* S-factor measurement which is then integrated in the reaction rate integral. In other words, the results shown here can *only* be known from the results of nuclear physics experiments.

CNO Cycle Equilibrium Abundances

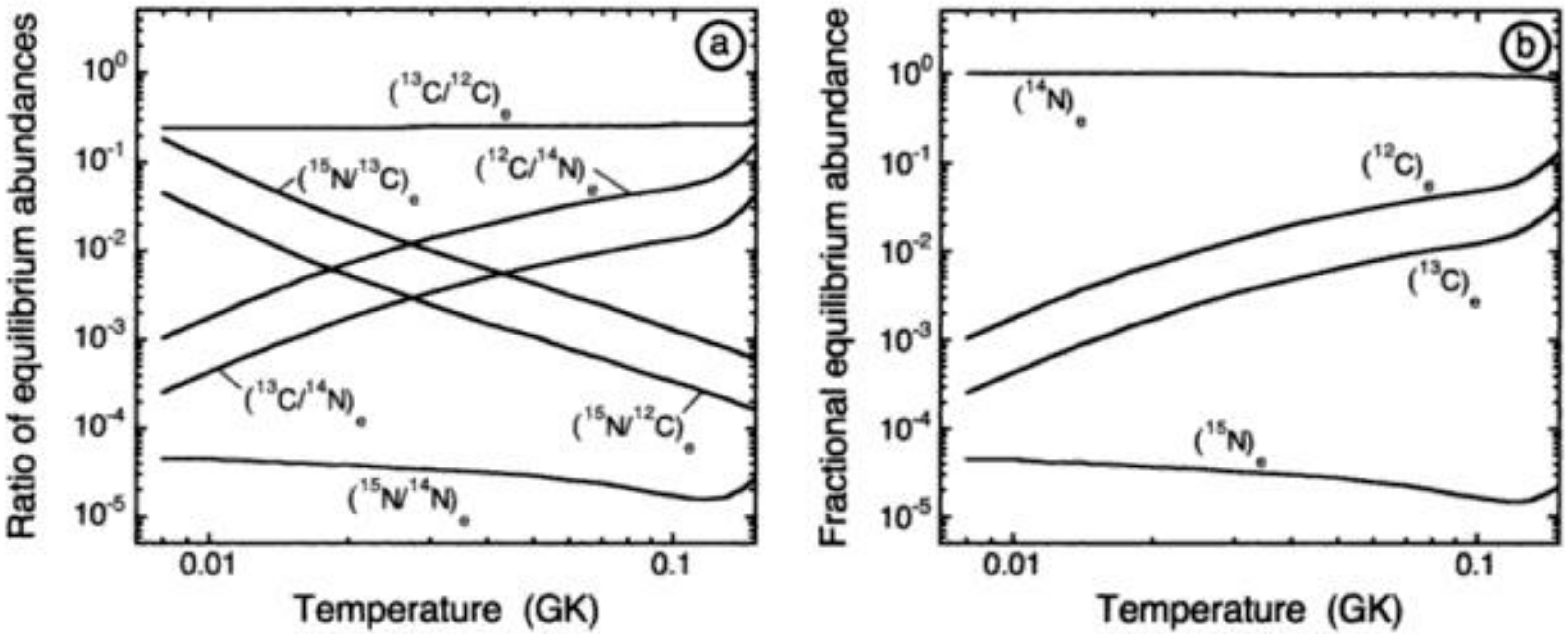


Fig. 5.11 (a) Abundance ratios and (b) fractional abundances versus temperature. The curves are calculated by assuming steady-state operation of a closed CNO1 cycle.

Equilibrium abundances of the cycle result in ^{14}N being the dominant nuclide produced, no mater what the temperature the cycle operates at.

For the CNO1 (equilibrium operation) energy generation rate, a similar game can be played as with the PPI chain, on page 25 of Lecture 1. By setting the derivatives of the differential equations on page 28 to zero, and doing back-substitutions for the $\langle\sigma v\rangle$ terms, the energy rate can be expressed in terms of the rate for the ^{14}N + p reaction rate.

Recall, energy gen. rate is: $$\epsilon_{CNO} = \frac{1}{\rho}\sum_{i\to j} Q_{i\to j} H Z_i \langle\sigma v\rangle_{i\to j}$$

Here, $Q_{i\to j}$ is the Q-value for each (p,γ) reaction in the CNO1 cycle, H is the hydrogen number density, and Z_i is a generic symbol for C, N or O. You will need the following:

$Q_{^{12}C(p,\gamma)^{13}N(\beta^+\nu)} = 3.458\ \mathrm{MeV}$ Average neutrino energy has been subtracted out.

$Q_{^{13}C(p,\gamma)} = 7.551\ \mathrm{MeV}$

$Q_{^{14}N(p,\gamma)^{15}O(\beta^+\nu)} = 9.055\ \mathrm{MeV}$ Average neutrino energy has been subtracted out.

$Q_{^{15}N(p,\gamma)} = 4.966\ \mathrm{MeV}$

Continuing, you should be able to derive the following expression for the equilibrium energy generation rate of the CNO1 cycle:

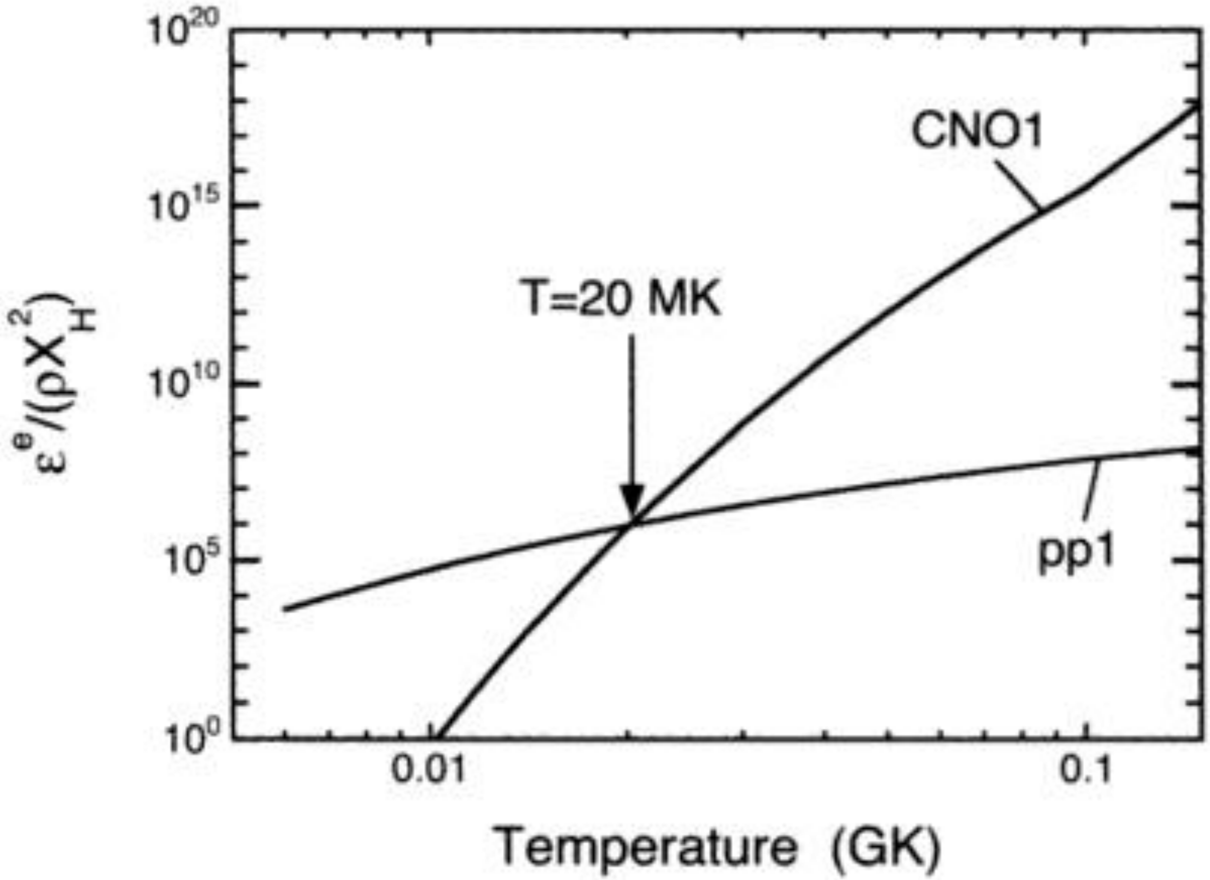
$$\epsilon_{CNO} = \frac{25.03 \text{ MeV}}{\rho} \left(\sum CNO1\right) H \langle \sigma v \rangle_{^{14}N(p,\gamma)}$$

$$= 25.03 \left(\sum_i \frac{Z_i}{M_i}\right) \frac{X_H}{M_H} \rho N_A^2 \langle \sigma v \rangle_{^{14}N(p,\gamma)} \quad \text{MeV g}^{-1}\text{ s}^{-1}$$

This, too, can be written in a power law form, much like the expression for the PPI chain. The exponent, n, is 16.7, which you can confirm from the formulae taught in Lectures 11 and 12, pages 2-5, of last Semester.

$$\epsilon_{CNO} = \epsilon_{CNO}(T_0) \left(\frac{T}{T_0}\right)^{16.7}$$

Equilibrium Energy Generation of PPI Chain and CNO1 Cycle for Solar Composition



PPI Chain has higher energy rate at Solar core temperature (15 MK) than does CNO .

Note: In stars where CNO dominates energy generation rate, the structure of the star will be different from that of our Sun. Why? The exponent in the power law: 16.7 versus 3.9. → radiation transport not enough to get the energy out!