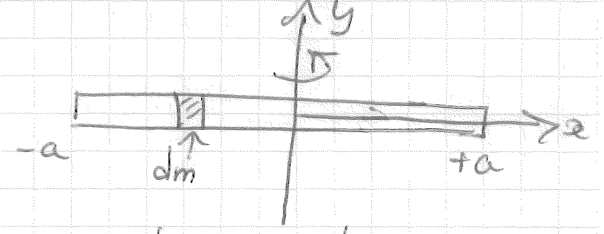
1dim rod: ρ_0 = constant



$$dm = \rho_0\, dx$$

$$I = \int r^2\, dm$$

$$= \rho_0 \int_{-a}^{a} x^2\, dx = \frac{\rho_0}{3} x^3 \Big|_{-a}^{a}$$

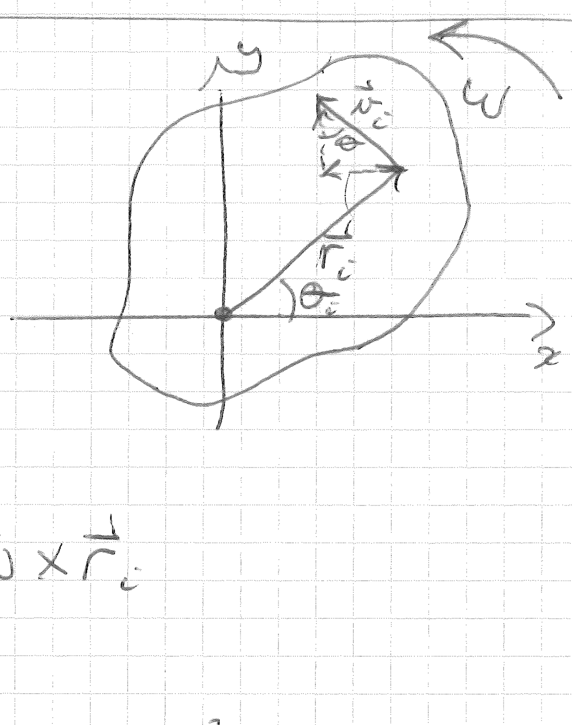
$$\rho_0 = \frac{M}{2a}$$

$$= \frac{2\rho_0 a^3}{3}$$

$$\boxed{I = \frac{Ma^2}{3}}$$

Rigid Body Rotation and I

$$v_i = r_i \omega = (x_i^2 + y_i^2)^{1/2} \omega$$



$$\dot{x}_i = -v_i \sin\theta_i = -\omega y_i$$

$$\dot{y}_i = v_i \cos\theta_i = \omega x_i$$

$$\dot{z}_i = 0$$

These are equivalent to $\vec{v} = \vec{\omega} \times \vec{r}_i$ with $\vec{\omega} = \omega\, \hat{e}_z$

$$T = \sum_i \tfrac{1}{2} m_i v_i^2 = \tfrac{1}{2}\Big(\sum_i m_i r_i^2\Big)\omega^2 = \frac{1}{2} I_z\, \omega^2$$

$$I_z \equiv \sum m_i r_i^2 = \sum m_i (x_i^2 + y_i^2)$$

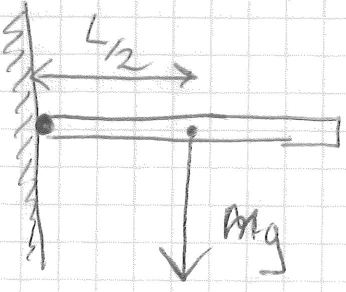
angular momentum $\vec{L} = \sum_i \vec{r}_i \times m_i \vec{v}_i = \sum m_i r_i v_i$

$$= \Big(\sum m_i r_i^2\Big)\omega = I_z\, \omega$$

$$\Rightarrow \vec{M} = \frac{d\vec{L}}{dt} = \frac{d}{dt}\{I_z \omega\} = I_z\, \dot{\omega}$$

Examples: Rotating thin rod:

Rod of length L, mass M.



Released from rest, frictionless pivot.

What is initial angular acceleration of rod?

What is initial linear acceleration of right end of rod?

We have $\vec{M} = I\dot{\vec{\omega}} = I\vec{\alpha}$

$$M = \frac{mgL}{2} = I\alpha = \frac{1}{3}mL^2\alpha$$

$$\Rightarrow \boxed{\alpha = \frac{3}{2}\frac{g}{L}}$$

Initial linear acceleration of end of rod:

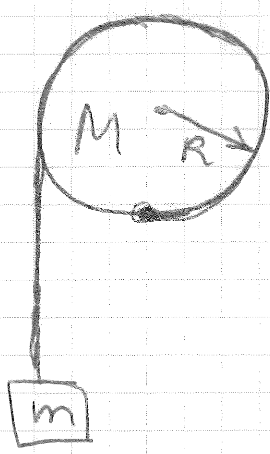
$$\boxed{a = L\alpha = \frac{3}{2}g}$$

Example: Rotating Pulley with torque.

Find: ① Linear acceleration of block.

② angular acceleration of wheel.

③ tension in string.

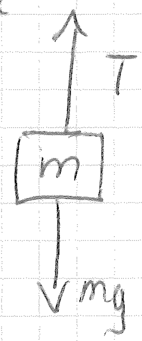


$a = R\alpha$

For block:

$$mg - T = ma$$

FBD: Block



For wheel: $M = I\alpha = TR$

$$\Rightarrow T = \frac{I\alpha}{R} = \frac{I a}{R^2}$$

$$\therefore \quad mg - \frac{Ia}{R^2} = ma$$

$$mg = a\left(I/_{R^2} + m \right)$$

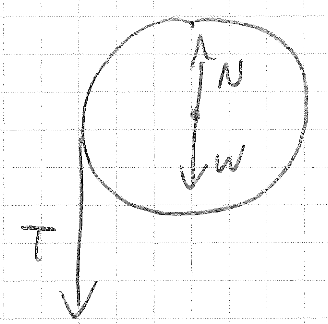
$$a = \frac{mg}{I/_{R^2} + m}$$

$$= \frac{mg}{\frac{1}{2}M + m}$$

$$\Rightarrow \boxed{a = \frac{g}{1 + \frac{M}{2m}}}$$

$$\boxed{\alpha = a/_R = \frac{g}{R + \frac{MR}{2m}}}$$

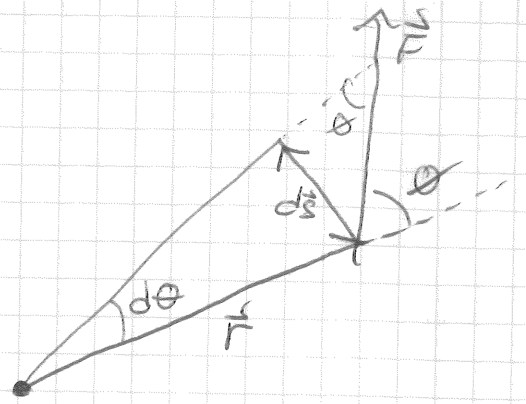
Wheel.



$$W + T - N = 0$$

$$I = \frac{1}{2}MR^2$$

Work in Rotation of Rigid Body



$$dW = \vec{F} \cdot d\vec{s} = F \sin\theta \, ds$$

$$= F r \sin\theta \, d\theta$$

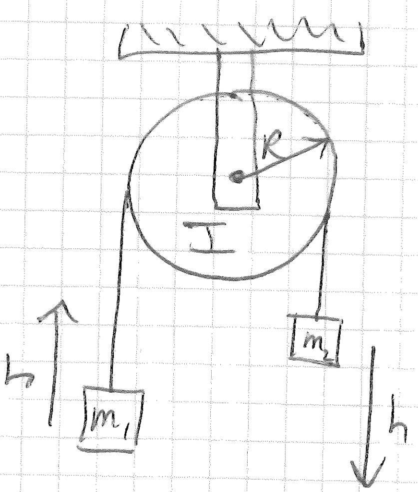
$$= |\vec{r} \times \vec{F}| \, d\theta = M \, d\theta$$

$$\Rightarrow \Delta W = \int_{\theta_1}^{\theta_2} M \, d\theta = \int_{\theta_1}^{\theta_2} I \alpha \, d\theta$$

$$\alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\theta} \frac{d\theta}{dt} = \omega \frac{d\omega}{d\theta}$$

$$= \int_{\omega_1}^{\omega_2} I \omega \, d\omega = \frac{I}{2} (\omega_2^2 - \omega_1^2)$$

Energy Conservation with Linear & Rotational Motions



Rotation is CW, no slipping of rope on wheel. Frictionless axel for wheel. System of blocks moves through displacement h.

Find: linear velocities of m_1, m_2 and ω of pulley after masses move through displacement h.

work energy theorem.

$$\omega = v/R$$

$$E = \text{constant} \Rightarrow \Delta E_k = -\Delta U$$

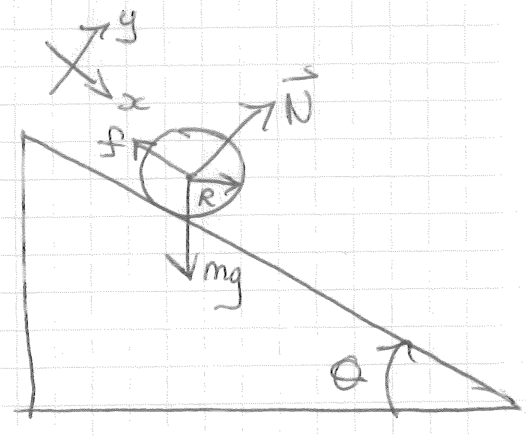
$$\frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2 + \frac{I}{2} \omega^2 = -gh(m_1 - m_2)$$

$$\Rightarrow \frac{1}{2} \left(m_1 + m_2 + \frac{I}{R^2} \right) v^2 = -(m_1 - m_2) g h$$

$$\Rightarrow v = \sqrt{\frac{2(m_2 - m_1) g h}{m_1 + m_2 + I/R^2}}$$

Round object on inclined plane

Solve the equations of motion



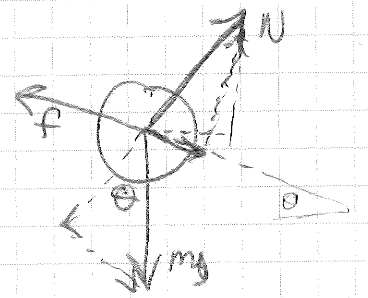
No slipping:

Along x: $-f + mg\sin\theta = m\ddot{x}_{cm}$

$$N - mg\cos\theta = m\ddot{y}_{cm}$$

Contact with plane $\Rightarrow$ $\ddot{y}_{cm} = 0$

$$\Rightarrow \boxed{N = mg\cos\theta}$$



Rotation:

$$\vec{M}_{cm} = fR = I_{cm}\dot{\omega} = I_{cm}\alpha$$

With no slipping:

$v_{cm} = r\omega$

$a_{cm} = r\alpha$

$$fR = I_{cm}\frac{a_{cm}}{R} = \frac{I_{cm}}{R}\ddot{x}_{cm}$$

$$\Rightarrow f = \frac{I_{cm}}{R^2}\ddot{x}_{cm}$$ sub in top equation

$$-\frac{I_{cm}}{R^2}\ddot{x} + mg\sin\theta = m\ddot{x}_{cm}$$

$$\ddot{x}_{cm} = \frac{mg\sin\theta}{m + \frac{I_{cm}}{R^2}} = \frac{g\sin\theta}{1 + \frac{I_{cm}}{mR^2}}$$

What if we have slipping?

Slipping $\Rightarrow$ $\vec{f} = \mu N = \mu mg\cos\theta$

Then equation of motion for x_{cm} is:

$$m\ddot{x}_{cm} = mg\sin\theta - \mu mg\cos\theta \qquad (1)$$

And for rotation: $I_{cm}\dot{\omega} = fR = \mu mgR\cos\theta \qquad (2)$

From (1) $\ddot{x}_{cm} = g(\sin\theta - \mu\cos\theta)$ (constant).

and from (2): $\alpha = \dot{\omega} = \dfrac{\mu mgR\cos\theta}{I_{cm}}$ (constant).

Integrate

$$\Rightarrow \dot{x}_{cm} = g(\sin\theta - \mu\cos\theta)t$$

$$\omega = \frac{\mu mgR\cos\theta}{I_{cm}} t$$

Notice: $\dfrac{\dot{x}_{cm}}{\omega} = \left(\dfrac{\sin\theta - \mu\cos\theta}{\mu mR\cos\theta}\right)I_{cm} = \dfrac{I_{cm}}{mR}\left(\dfrac{\tan\theta}{\mu} - 1\right)$

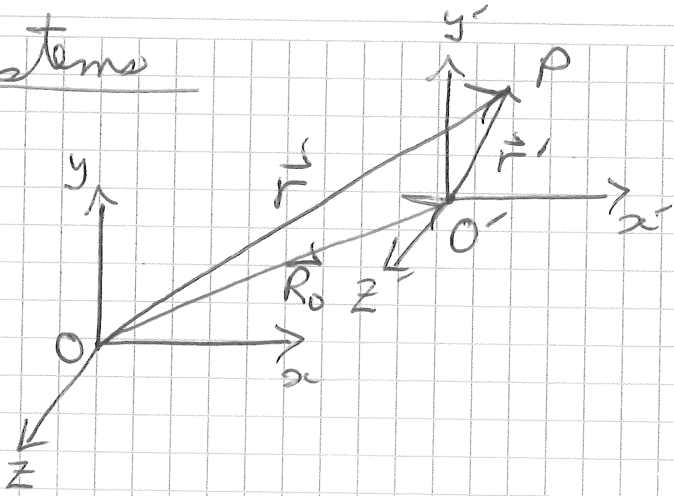
This is also a constant.

When pure rolling occurs $\dot{x}_{cm} = R\omega$

$$\Rightarrow R = \frac{I_{cm}}{mR}\left(\frac{\tan\theta}{\mu} - 1\right) \Rightarrow \left(\frac{mR^2}{I_{cm}} + 1\right)\mu = \tan\theta$$

Or: $$\boxed{\mu_{crit} = \frac{\tan\theta}{1 + mR^2/I_{cm}}}$$

Noninertial Reference Systems



$\vec{r} = \vec{r}' + \vec{R}_0$

$\Rightarrow \vec{v} = \vec{v}' + \vec{V}_0$

$\vec{a} = \vec{a}' + \vec{A}_0$

If the system O' is not accelerating, then $\vec{A}_0 = 0$ and we have:

$$\vec{a} = \vec{a}'$$

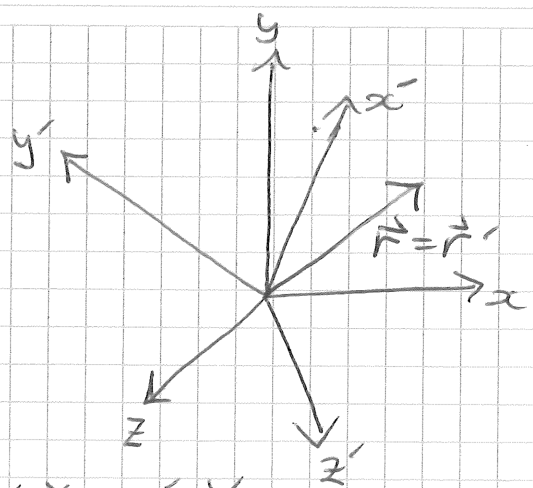
$$\Rightarrow \quad \vec{F} = m\vec{a} = m\vec{a}' = \vec{F}' \qquad \text{(This is inertial).}$$

However, if O' is accelerating, then:

$$\vec{F} = m\vec{a} = m\vec{a}' + m\vec{A}_0$$

$$\Rightarrow \vec{F}' = m\vec{a}' = m\vec{a} - m\vec{A}_0 = \vec{F} - m\vec{A}_0$$

Rotating Coordinate Systems:



The vector $\vec{r}$ does not change.

$\therefore \ \vec{r} = \vec{r}\,'$

$$\Rightarrow x\vec{e}_x + y\vec{e}_y + z\vec{e}_z = x'\vec{e}_x\,' + y'\vec{e}_y\,' + z'\vec{e}_z\,'$$

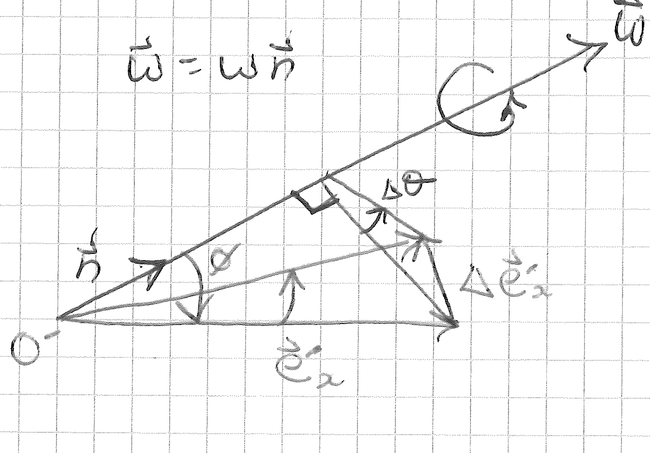
differentiate:

$$\frac{dx}{dt}\vec{e}_x + \frac{dy}{dt}\vec{e}_y + \frac{dz}{dt}\vec{e}_z = \frac{dx'}{dt}\vec{e}_x\,' + \frac{dy'}{dt}\vec{e}_y\,' + \frac{dz'}{dt}\vec{e}_z\,' + x'\frac{d\vec{e}_x\,'}{dt} + y'\frac{d\vec{e}_y\,'}{dt} + z'\frac{d\vec{e}_z\,'}{dt}$$

$$\Rightarrow \vec{v} = \vec{v}\,' + x'\frac{d\vec{e}_x\,'}{dt} + y'\frac{d\vec{e}_y\,'}{dt} + z'\frac{d\vec{e}_z\,'}{dt}$$

What are $\dfrac{d\vec{e}_i\,'}{dt}$?

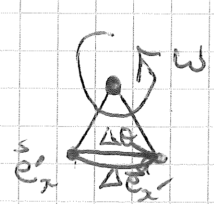
$\vec{\omega} = \omega\vec{n}$



$$|\Delta\vec{e}_x\,'| \simeq |\vec{e}_x\,' \sin\varnothing \ \Delta\theta| = \sin\varnothing \ \Delta\theta$$

$$\Rightarrow \left|\frac{\Delta\vec{e}_x\,'}{\Delta t}\right| = \sin\varnothing \ \frac{\Delta\theta}{\Delta t}$$

$$\Rightarrow \left|\frac{d\vec{e}_x\,'}{dt}\right| = \sin\varnothing \ \frac{d\theta}{dt} = \omega\sin\varnothing$$



Now, $\Delta \vec{e}_x'$ is $\perp$ to both $\vec{\omega}$ and $\vec{e}_x'$, so we can write:

$$\frac{d\vec{e}_x'}{dt} = \vec{\omega} \times \vec{e}_x'$$

Similarly, $\frac{d\vec{e}_y'}{dt} = \vec{\omega} \times \vec{e}_y'$, $\frac{d\vec{e}_z'}{dt} = \vec{\omega} \times \vec{e}_z'$

We have then: ~~$\vec{v} = \vec{v}' + \vec{\omega} \times \vec{r}'$~~

$$x' \frac{d\vec{e}_x'}{dt} + y' \frac{d\vec{e}_y'}{dt} + z' \frac{d\vec{e}_z'}{dt} = x'(\vec{\omega} \times \vec{e}_x') + y'(\vec{\omega} \times \vec{e}_y') + z'(\vec{\omega} \times \vec{e}_z')$$

$$= \vec{\omega} \times (x'\vec{e}_x' + y'\vec{e}_y' + z'\vec{e}_z')$$

$$= \vec{\omega} \times \vec{r}'$$

So, finally, $\boxed{\vec{v} = \vec{v}' + \vec{\omega} \times \vec{r}'}$

or:
$$\left(\frac{d\vec{r}}{dt}\right)_{fixed} = \left(\frac{d\vec{r}'}{dt}\right)_{rot} + \vec{\omega} \times \vec{r}' = \left[\left(\frac{d}{dt}\right)_{rot} + \vec{\omega} \times \right] \vec{r}'$$

And we had $\vec{r} = \vec{r}'$

$$\Rightarrow \left(\frac{d\vec{r}}{dt}\right)_{fixed} = \left[\left(\frac{d}{dt}\right)_{rot} + \vec{\omega} \times \right] \vec{r}$$

So, for <u>any</u> vector $\vec{Q}$

$$\left(\frac{d\vec{Q}}{dt}\right)_{fixed} = \left(\frac{d\vec{Q}}{dt}\right)_{rot} + \vec{\omega} \times \vec{Q}$$

So, use this for velocity:

$$\left(\frac{d\vec{v}}{dt}\right)_{fixed} = \left(\frac{d\vec{v}}{dt}\right)_{rot} + \vec{\omega}\times\vec{v}$$

but $\vec{v} = \vec{v}' + \vec{\omega}\times\vec{r}'$, so:

$$\left(\frac{d\vec{v}}{dt}\right)_{fixed} = \left(\frac{d}{dt}\right)_{rot}\left(\vec{v}' + \vec{\omega}\times\vec{r}'\right) + \vec{\omega}\times\left(\vec{v}' + \vec{\omega}\times\vec{r}'\right)$$

$$= \left(\frac{d\vec{v}'}{dt}\right)_{rot} + \left(\frac{d}{dt}(\vec{\omega}\times\vec{r}')\right)_{rot} + \vec{\omega}\times\vec{v}' + \vec{\omega}\times(\vec{\omega}\times\vec{r}')$$

$$= \left(\frac{d\vec{v}'}{dt}\right)_{rot} + \left(\frac{d\omega}{dt}\right)_{rot}\times\vec{r}' + \vec{\omega}\times\left(\frac{d\vec{r}'}{dt}\right)_{rot} + \vec{\omega}\times\vec{v}' + \vec{\omega}\times(\vec{\omega}\times\vec{r}')$$

Now $\left(\frac{d\vec{\omega}}{dt}\right)_{fixed} = \left(\frac{d\vec{\omega}}{dt}\right)_{rot} + \vec{\omega}\times\vec{\omega} = \left(\frac{d\vec{\omega}}{dt}\right)_{rot} \Rightarrow \left(\frac{d\vec{\omega}}{dt}\right)_{rot} = \left(\frac{d\vec{\omega}}{dt}\right)_{fixed}$

And $\vec{v}' = \left(\frac{d\vec{r}'}{dt}\right)_{rot}$, and $\left(\frac{d\vec{v}'}{dt}\right)_{rot} = \vec{a}'$

So, $\Rightarrow$ $$\boxed{\vec{a}' + \dot{\vec{\omega}}\times\vec{r}' + 2\vec{\omega}\times\vec{v}' + \vec{\omega}\times(\vec{\omega}\times\vec{r}') = \vec{a}}$$

If we have translation and acceleration, then

$$\boxed{\begin{aligned}\vec{v} &= \vec{v}' + \vec{\omega}\times\vec{r}' + \vec{V}_0 \\ \vec{a} &= \vec{a}' + \dot{\vec{\omega}}\times\vec{r}' + 2\vec{\omega}\times\vec{v}' + \vec{\omega}\times(\vec{\omega}\times\vec{r}') + \vec{A}_0\end{aligned}}$$

$$\Rightarrow \boxed{\vec{F} - m\dot{\vec{\omega}}\times\vec{r}' - 2m\vec{\omega}\times\vec{v}' - m\vec{\omega}\times(\vec{\omega}\times\vec{r}') - m\vec{A}_0 = m\vec{a}'}$$

$$\vec{F} - \vec{F}_{transverse} - \vec{F}_{cor} - \vec{F}_{cent} - m\vec{A}_0 = m\vec{a}'$$

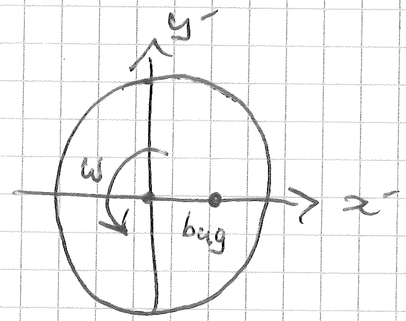
<u>Examples</u>:

Bug crawls radially outward with <u>constant speed</u> v' along spoke of a wheel, which is rotating with constant angular velocity $\vec{\omega}$ about a vertical axis. Find the apparent forces on the bug.

$\vec{r}\,' = x' \vec{e}_x\,'$ ~~(total)~~

$\dot{\vec{r}}\,' = \dot{x}' \vec{e}_x\,' = v' \vec{e}_x\,'$ (no time derivative of $\vec{e}_x\,'$ by design of coordinates)

$\ddot{\vec{r}}\,' = 0$



coordinate system rotates with wheel (fixed to wheel).

So, we have: $\vec{\omega} = \omega \vec{e}_z\,'$

Coriolis force: $\vec{F}_{cor} = -2m\, \vec{\omega} \times \vec{v}\,'$

$= -2m\omega v'\, \vec{e}_z\,' \times \vec{e}_x\,' = -2m\omega v' \vec{e}_y\,'$

Transverse force: $\vec{F}_{trans} = -m \dot{\vec{\omega}} \times \vec{r}\,' = 0$ ($\vec{\omega}$ = constant vector)

Centrifugal force: $\vec{F}_{cent} = -m \vec{\omega} \times (\vec{\omega} \times \vec{r}\,')$

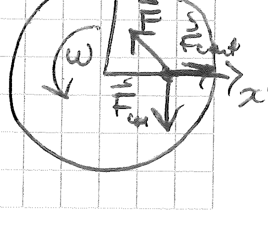
$= -m\omega^2 \vec{e}_z\,' \times (\underbrace{\vec{e}_z\,' \times x' \vec{e}_x\,'}_{\vec{e}_y\,'})$

$= +m\omega^2 x' \vec{e}_x\,'$

So finally:

$$\vec{F} - 2m\omega v' \vec{e}_y\,' + m\omega^2 x' \vec{e}_x\,' = m\vec{a}\,' = 0$$

$\vec{F}$ = real forces acting on bug. (friction, for example)

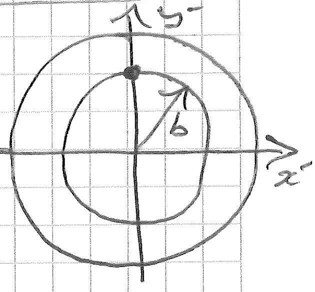


Example: How far can bug crawl before it starts to slip? Given $\mu_s = \mu$.

We have $|\vec F|^2 = (2m\omega v' \vec e_y' - m\omega^2 x' \vec e_x')^2 = \mu^2 m^2 g^2$

We need x': $\mu^2 m^2 g^2 = 4m^2\omega^2 v'^2 + m^2\omega^4 x'^2$

$$\Rightarrow x' = \frac{1}{\omega^2}\left(\mu^2 g^2 - 4\omega^2 v'^2\right)^{1/2}$$



Bug crawls with constant speed in circular path on a record that rotates at constant angular speed ω.
Bug mass $= m$, coeff. friction $= \mu$.
How fast, relative to stationary frame (not rotating) can bug move before slipping if it goes a) direction of rotation and b) opposite to direction of rotation?

Put bug at $\vec r' = b\,\vec e_y'$. Bug doesn't slip, ∴ in rotating frame $\dot{\vec r}' = 0$, $\vec a' = 0$ $\qquad \vec\omega_b =$ bug's $\omega = \omega_b \vec e_z'$

$$\vec F - m\dot{\vec\omega}_b\times\vec r' - 2m\,\vec\omega_b\times\vec v' - m\,\vec\omega_b\times(\vec\omega_b\times\vec r') = m\vec a'$$

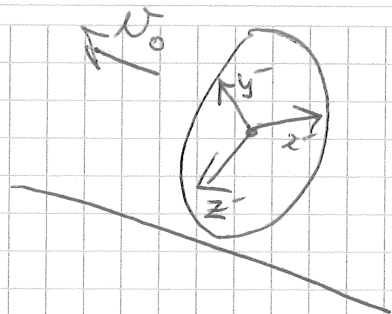
$$\Rightarrow \vec F - m\,\vec\omega_b\times(\vec\omega_b\times\vec r') = 0$$

$$\vec\omega_b\times(\vec\omega_b\times\vec r') = \omega_b^2 b\,\vec e_z'\times(\underbrace{\vec e_z'\times\vec e_y'}_{-\vec e_x'}) = -\omega_b^2 b\,\vec e_y'$$

$$\Rightarrow \vec F = m\omega_b^2 b\,\vec e_y' \quad\Rightarrow \mu m g = m\omega_b^2 b \Rightarrow \omega_b = \pm\sqrt{\frac{\mu g}{b}}$$

direction of rotation: $\Omega = -\omega - \sqrt{\frac{\mu g}{b}}$, opposite: $\Omega = -\omega + \sqrt{\frac{\mu g}{b}}$

Example (Wheels):



Wheel of radius b, rolling with constant v_0.

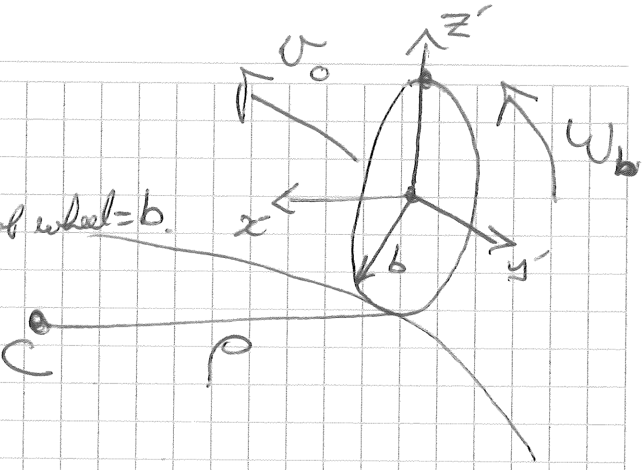
Find: acceleration, relative to ground, of any point on the rim.

Coordinate system is fixed to wheel, and centered. Have x' axis go through point of interest.

$$\vec{r}' = b\vec{e}_x' \,, \quad \vec{a}' = \vec{v}' = 0 \quad (\text{since } \vec{r}' = \text{constant vector}).$$

angular velocity $\vec{\omega} = \omega \vec{e}_z' = \frac{v_0}{b} \vec{e}_z'$.

We have
$$\vec{a} = \vec{a}' + \dot{\vec{\omega}} \times \vec{r}' + 2\vec{\omega}\times\vec{v}' + \vec{\omega}\times(\vec{\omega}\times\vec{r}')$$
$$= \vec{\omega}\times(\vec{\omega}\times\vec{r}') = \omega^2 \vec{e}_z' \times (\underbrace{\vec{e}_z' \times \vec{e}_x'}_{\vec{e}_y'})$$
$$= -\omega^2 b\, \vec{e}_x' = -\frac{v_0^2}{b} \vec{e}_x'$$



Example (Wheels):

Wheel travels at constant speed v_0. Radius of wheel = b. around track of radius ρ.

What is acceleration of highest point on the wheel?

Let the noninertial frame be rotating about point C. Its axes do not rotate with wheel.

$$\vec{\omega}_\rho = \frac{V_0}{\rho}\vec{e}\,'_z$$

$$\vec{a} = \vec{a}\,' + \dot{\vec{\omega}}_\rho \times \vec{r}\,' + 2\vec{\omega}_\rho \times \vec{v}\,' + \vec{\omega}_\rho \times (\vec{\omega}_\rho \times \vec{r}\,') + \vec{A}_0 \quad (\dot{\vec{\omega}} = 0)$$

$$\vec{r}\,' = b\,\vec{e}\,'_z, \quad \vec{v}\,' = -v_0 \vec{e}\,'_y, \quad \vec{a}\,' = -\frac{v_0^2}{b}\vec{e}\,'_z$$

$$\vec{a} = -\frac{v_0^2}{b}\vec{e}\,'_z + 2\frac{v_0}{\rho}\vec{e}\,'_z \times (-v_0 \vec{e}\,'_y) + \frac{v_0^2}{\rho^2} b\,\vec{e}\,'_z \times (\vec{e}\,'_z \times \vec{e}\,'_z) +$$

$$= -\frac{v_0^2}{b}\vec{e}\,'_z + 2\frac{v_0^2}{\rho}\vec{e}\,'_z + \frac{v_0^2}{\rho}\vec{e}\,'_x$$

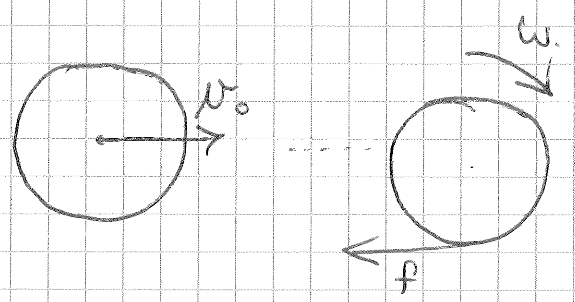
$$= 3\frac{v_0^2}{\rho}\vec{e}\,'_x - \frac{v_0^2}{b}\vec{e}\,'_z$$

Example: Angular momentum and Bowling Ball

Bowling ball released with initial speed v_0 such that it *initially* slides without pure rolling.
Coeffic. of friction is μ. Radius $= R$
Find: When pure rolling occurs.

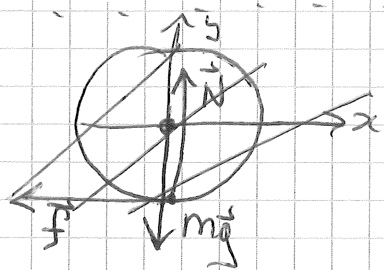
a) velocity of ball's center of mass is $5\,v_0/7$

b) the distance travelled is $12\,v_0^2/49\,\mu g$.

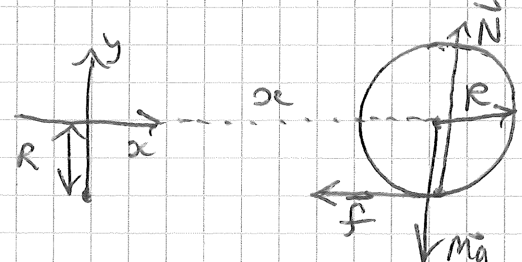


We had $\vec{L} = M\vec{r}_{cm} \times \vec{v}_{cm} + \sum_i m_i \vec{r}_i' \times \vec{v}_i'$

$= M\vec{r}_{cm} \times \vec{v}_{cm} + I\vec{\omega}$



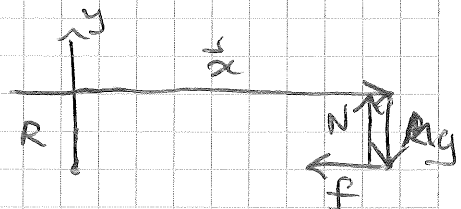
and $\frac{d\vec{L}}{dt} = \vec{M}$.



With this coordinate system: $\vec{r}_{cm} \times \vec{v}_{cm}$

$= \vec{x}_{cm} \times \vec{v}_{cm} = 0$

So $\vec{L} = I\vec{\omega} \Rightarrow \frac{d\vec{L}}{dt} = I\dot{\vec{\omega}} = \vec{M}$.



What is $\vec{M}$? $\vec{M}_N = +xMg\,\vec{e}_z$

$\vec{M}_g = -xMg\,\vec{e}_z$

$\vec{M}_f = -fR\,\vec{e}_z \Rightarrow \vec{M} = \vec{M}_N + \vec{M}_g + \vec{M}_f$

$= -Rf\,\vec{e}_z$

$\frac{d\vec{L}}{dt} = -I\dot{\omega}\,\vec{e}_z$

$\Rightarrow \quad I\dot{\omega} = Rf = R\mu mg \Rightarrow \dot{\omega} = \frac{R\mu mg}{I}$ (constant)

$\omega_0 = 0$ (initial condition).

$\Rightarrow \omega_f = \frac{R\mu mg}{I} t \Rightarrow \boxed{\mu g t = \frac{I\omega_f}{mR}}$ (*)

Now, Newton's 2nd law:

in x-direction: $-f\vec{e}_x = m\vec{a} \Rightarrow \boxed{\vec{a} = -\frac{\mu mg}{m} = -\mu g \vec{e}_x}$

When pure rolling occurs $v = R\omega$

$v_f = v_i + at = +at + v_0 = v_0 - \mu g t$ use (*)

$= v_0 - \frac{I\omega_f}{mR}$ use $\omega_f = \frac{v_f}{R}$

$= v_0 - \frac{I v_f}{mR^2}$

$\Rightarrow v_f = \frac{v_0}{1 + I/mR^2}$ $\left(I = \frac{2}{5} mR^2 \right)$

$= v_0 / (1 + 2/5) = \frac{5 v_0}{7}$ ∎

b) $d = d_0 + v_i t + \frac{1}{2} a t^2 = v_i t + \frac{1}{2} a t^2$

$= \left(v_0 - \frac{\mu g}{2} t\right) t$

$= \left(v_0 - \frac{1}{7} v_0\right) \frac{2}{7} \frac{v_0}{\mu g}$

$= \frac{12 v_0^2}{49 \mu g}$

$t = \frac{I\omega_f}{\mu g m R} = \frac{I v_f}{\mu g m R^2}$

$= \frac{\frac{2}{5} mR^2 \cdot 5v_0/7}{\mu g m R^2}$

$= \frac{2 v_0}{7 \mu g}$

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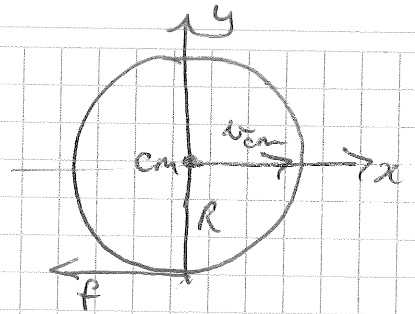
<u>Easier Way:</u> $\frac{d\vec{L}'}{dt} = \vec{M}'$

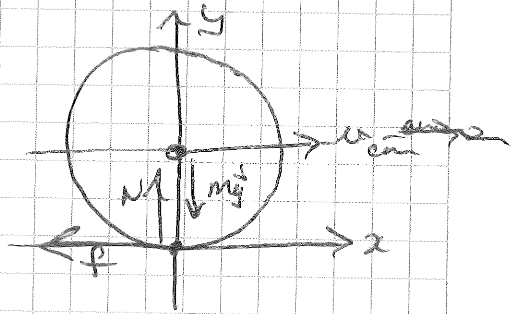


<u>Directly:</u> $\vec{M}' = Rf$

$L' = I\omega$

<u>Easiest Way:</u>

$$\frac{d\vec{L}}{dt} = \vec{M}$$



But, $\vec{M} = 0$!

$$\Rightarrow \frac{d\vec{L}}{dt} = 0 \Rightarrow L_f = L_i$$

$$\vec{L}_i = M\,\vec{r}_{cm} \times \vec{v}_{cm} + I\vec{\omega}_i^{\,0} = -MRv_{cm}\vec{e}_z$$

$$\vec{L}_f = M\,\vec{r}_{cm} \times \vec{v}_{cm\,final} + I\vec{\omega}$$

$$= -MRv_{final}\vec{e}_z + I\vec{\omega}_f \qquad \omega_f = -\frac{v_f}{R}\vec{e}_z$$

$$= -MRv_f\vec{e}_z - \frac{v_f}{R}I\vec{e}_z$$

$$\Rightarrow \quad MRv_0 = MRv_f + \frac{v_f}{R}I = v_f\left(MR + \frac{I}{R}\right)$$

$$v_f = \frac{MRv_0}{MR + I/R} = \frac{v_0}{1 + \frac{I}{MR^2}} = \frac{5v_0}{7}$$