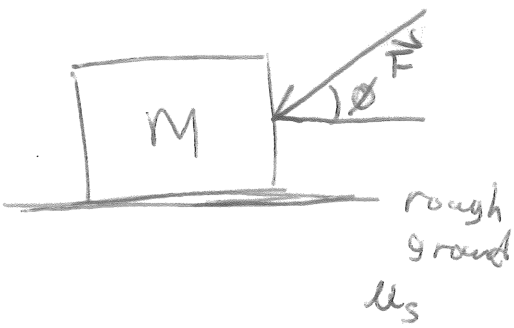
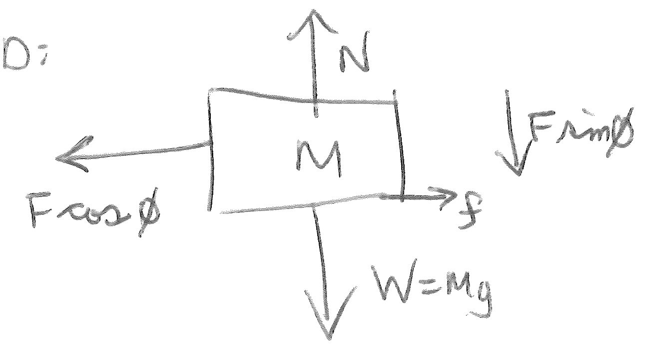


What minimum value of $|\vec{F}|$ is needed to just move the crate?

FBD:



$f = \mu N$

Newton's 2nd law:

$x: \; F\cos\phi - f = ma_x = 0$ (just before $\mu_s \to \mu_k$)

$y: \quad N - W - F\sin\phi = 0$

$\therefore \; N = F\sin\phi + W$ sub into 1st eq'n

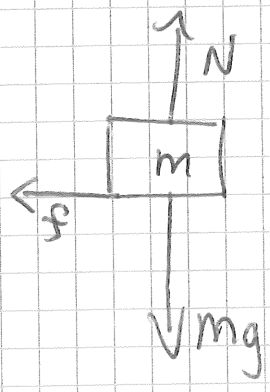
$\Rightarrow \quad F\cos\phi - \mu F\sin\phi - \mu W = 0$

$\Rightarrow F(\cos\phi - \mu\sin\phi) = \mu W$

$$\Rightarrow \boxed{F = \frac{\mu W}{\cos\phi - \mu\sin\phi} = \frac{\mu W\sec\phi}{1 - \mu_s\tan\phi}}$$

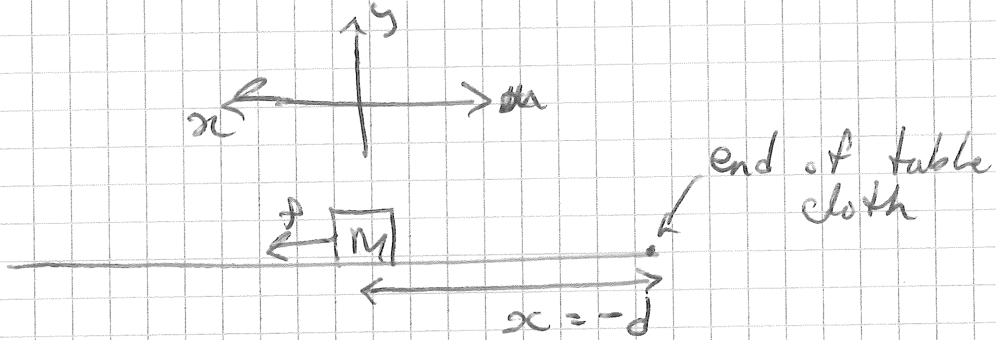
Table Top Magick Trick

Free Body Diagram



$$N = mg$$

$$f = \mu N = \mu m g = m a_m$$



Motion of Cloth:

$$x_c = -d + \cancel{v_c t}^{\,0} + \tfrac{1}{2} a_c t^2$$

$$= -d + \tfrac{1}{2} a_c t^2 \qquad (1)$$

Motion of mass:

$$x_m = \tfrac{1}{2} a_m t^2 = \tfrac{1}{2} \mu g t^2 \qquad (2)$$

Now: Both the cloth & mass share the same time t, and a common x-coordinate at the time when cloth goes out from under the mass.

So $\boxed{x_m = x_c}$ and from (2) $\boxed{\dfrac{t^2}{2} = \dfrac{x_m}{\mu g}}$ sub into (1)

②

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Magick Trick

$$x_c = x_m = -d + a_c \frac{x_m}{\mu g}$$

$$\Rightarrow \quad d = x_m \left(\frac{a_c}{\mu g} - 1 \right) = \frac{x_m}{\mu g} \left(a_c - \mu g \right)$$

$$\Rightarrow \boxed{x_m = d \frac{\mu g}{a_c - \mu g}}$$

$$= \frac{d}{g} \frac{\mu}{a_c/g - \mu}$$

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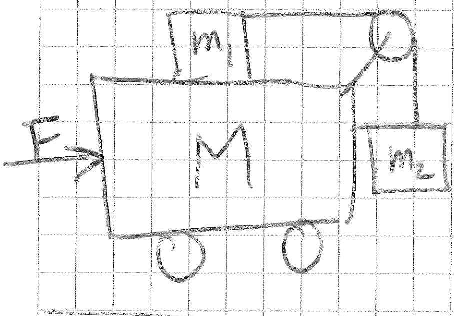
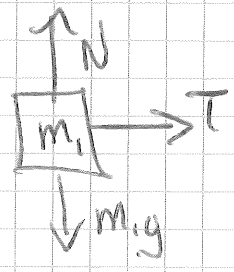
Cart and Masses I



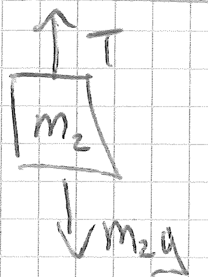
What force F must be applied to M so that m_1's position doesn't change wrt M? (No friction)

FBD



$$N - m_1 g = 0$$

$$T = m_1 a_1 \quad (1)$$



(2) $T - m_2 g = 0$ (since m_1 doesn't move)

$$\Rightarrow T = m_2 g \quad (3)$$

From (1) $\Rightarrow$ $a_1 = \dfrac{m_2}{m_1} g$

For the system of 3 masses:

$$\boxed{F = (M + m_1 + m_2) a_1 = (M + m_1 + m_2) \frac{m_2}{m_1} g}$$

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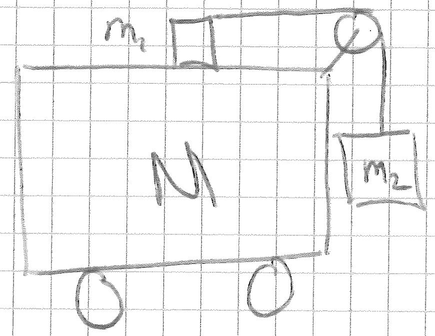
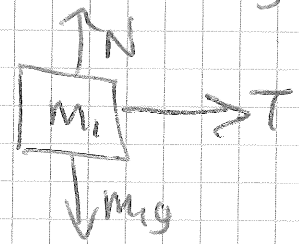
Cart and Masses II



Question: Does M accelerate when m_2 is released?
② What is the acceleration of M?

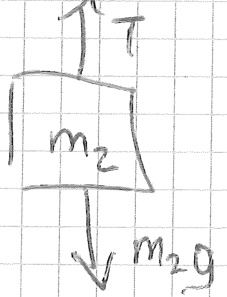
No friction

Free body diagrams!



$$N - m_1 g = 0 \Rightarrow N = mg$$

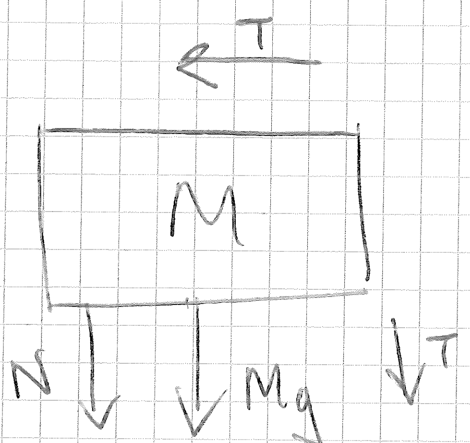
$$T = m_1 a \qquad ①$$



$$-m_2 g + T = -m_2 a \qquad ②$$

Sub ① into ② $\Rightarrow -m_2 g + m_1 a = m_2 a$

$$\Rightarrow \boxed{\frac{+m_2 g}{m_2 + m_1} = a}$$



$$-N - Mg - T = 0$$

$$-T = -M a_m$$

$$a_m = \frac{T}{M} = \frac{m_1 a}{M}$$

$$\boxed{= \frac{m_1 m_2}{M(m_1 + m_2)} g}$$

①

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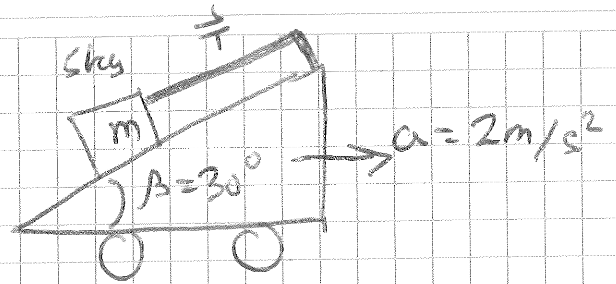
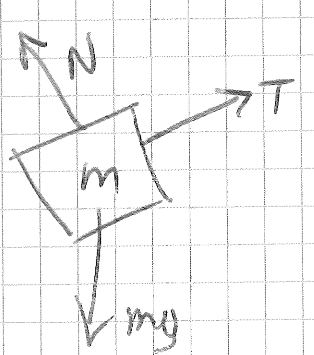
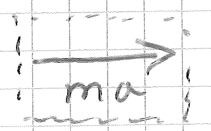


No friction

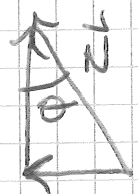
Block doesn't move relative to slope.

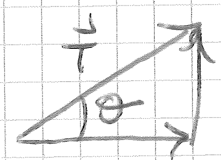
a) Find tension in rope?





$$\boxed{\sum F_x = ma}$$





$\therefore$ ① $T\cos\theta - N\sin\theta = ma$ (x-direction)

② $T\sin\theta + N\cos\theta = mg$

Eliminate N: $N = \dfrac{mg - T\sin\theta}{\cos\theta}$ (from ②)

Sub into ①:

$$T\cos\theta - \left(\frac{mg - T\sin\theta}{\cos\theta}\right)\sin\theta = ma$$

$$\Rightarrow T\left(\cos\theta + \frac{\sin^2\theta}{\cos\theta}\right) - mg\frac{\sin\theta}{\cos\theta} = ma$$

$$\Rightarrow \frac{T}{\cos\theta} = m(a + g\tan\theta)$$

$$\Rightarrow \boxed{T = m\cos\theta\,(a + g\tan\theta) \simeq 33.16\ N}$$

(2)

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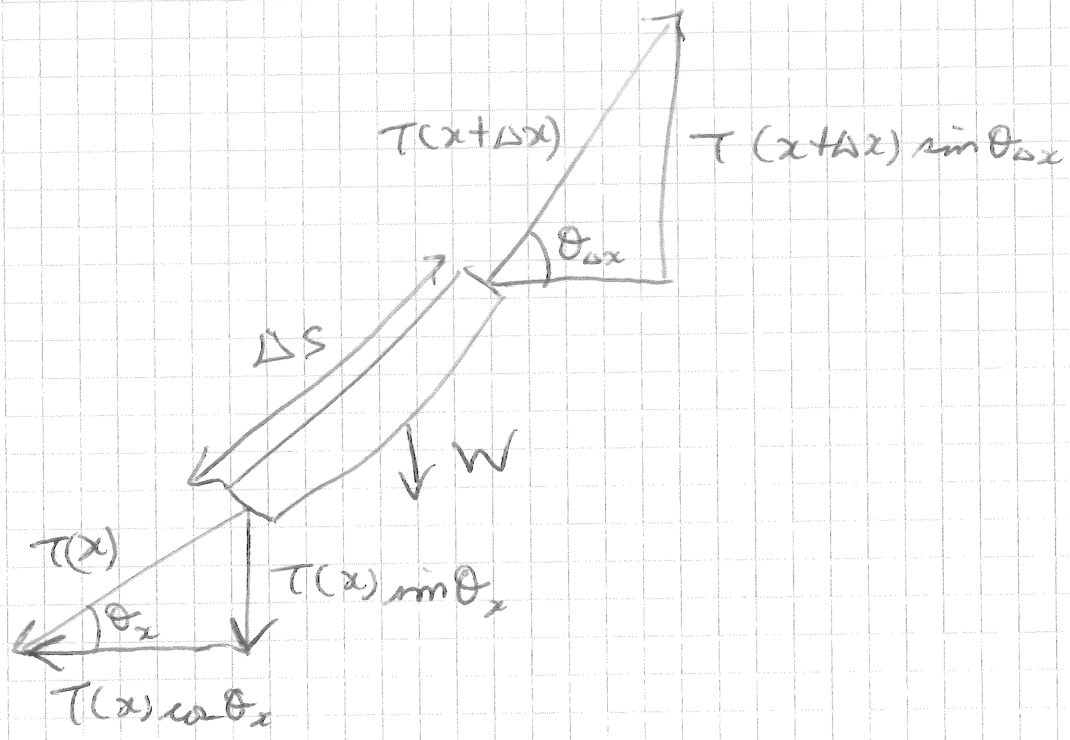
b) What is normal force?

We had $N = \dfrac{mg - T\sin\theta}{\cos\theta}$

$= \dfrac{5 \times 9.8 - 33.16 \sin 30^\circ}{\cos 30^\circ}$

$\boxed{N = 37.43\ N}$

The Catenary



Horizontal: $T_h(x+\Delta x) - T_h(x) = 0$

$$\Rightarrow \lim_{\Delta x \to 0} \frac{T_h(x+\Delta x) - T(x)}{\Delta x} = \frac{dT_h}{dx} = 0$$

$$\boxed{\begin{aligned} &\Rightarrow T_h = \text{constant} = T_0 \\ &\Rightarrow T(x)\cos\theta_x = T(x+\Delta x)\cos\theta_{\Delta x} = T_0 \end{aligned}}$$

Vertical:

$$T(x+\Delta x)\sin\theta_{\Delta x} - T(x)\sin\theta_x = W$$

$\Rightarrow$ LHS: $T(x+\Delta x)\tan\theta_{\Delta x}\cos\theta_{\Delta x} - T(x)\tan\theta_x\cos\theta_x$

$$= T(x)\cos\theta_x\left[\tan\theta_{\Delta x} - \tan\theta_x\right]$$

$$= T_0\left[\tan\theta_{\Delta x} - \tan\theta_x\right] = W$$

$$\Rightarrow \boxed{\frac{dy}{dx}(x+\Delta x) - \frac{dy}{dx}(x) = \frac{W}{T_0}} \qquad (***)$$

$$W = \rho g \Delta s = \rho g \sqrt{1+\left(\frac{\Delta y}{\Delta x}\right)^2} \Delta x$$

$\therefore$

$$\frac{dy}{dx}(x+\Delta x) - \frac{dy}{dx}(x) = \frac{\rho g}{T_0}\sqrt{1+\left(\frac{\Delta y}{\Delta x}\right)^2} \Delta x$$

$$\Rightarrow \lim_{\Delta x \to 0} \frac{\frac{dy}{dx}(x+\Delta x) - \frac{dy}{dx}(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\rho g}{T_0}\sqrt{1+\left(\frac{\Delta y}{\Delta x}\right)^2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{\rho g}{T_0}\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$$\Rightarrow y''^2 - \left(\frac{\rho g}{T_0}\right)^2 y'^2 - \left(\frac{\rho g}{T_0}\right)^2 = 0$$

Solution: $\boxed{y = \frac{T_0}{\rho g} \cosh \frac{\rho g}{T_0} x}$

$$\cosh x = \frac{e^x + e^{-x}}{2}, \quad \sinh x = \frac{e^x - e^{-x}}{2}$$

$$\frac{d}{dx}\cosh x = \sinh x, \quad \frac{d}{dx}\sinh x = \cosh x$$

$$\cosh^2 x - \sinh^2 x = 1$$

Note: $y'(0) = 0$ ✓

The 1D Wave Equation

We had (***)

$$\boxed{\frac{dy}{dx}(x+\Delta x) - \frac{dy}{dx}(x) = \frac{W}{T_0}}$$

1. Let's allow string to move only in vertical direction.
2. Let's also assume vertical displacement is very small compared to length of string.
3. Let's assume that $W \ll m\ddot{y}$

Then, (***) $\Rightarrow$ $\frac{\partial y}{\partial x}(x+\Delta x) - \frac{\partial y}{\partial x}(x) = \frac{\rho}{T_0} \Delta s \frac{\partial^2 y}{\partial t^2}$

$$\Rightarrow \frac{\partial y}{\partial x}(x+\Delta x) - \frac{\partial y}{\partial x}(x) = \frac{\rho}{T_0} \frac{\partial^2 y}{\partial t^2} \sqrt{1+\left(\frac{\Delta y}{\Delta x}\right)^2}\, \Delta x$$

$$\Rightarrow \frac{\partial^2 y}{\partial x^2} = \frac{\rho}{T_0} \frac{\partial^2 y}{\partial t^2} \sqrt{1+\left(\frac{\partial y}{\partial x}\right)^2} \qquad \frac{\partial y}{\partial x} \simeq 0 \ (\text{by } 2)$$

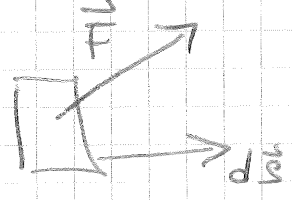
$$\simeq \frac{\rho}{T_0} \frac{\partial^2 y}{\partial t^2}$$

1D Wave equation: $\boxed{\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}, \quad v^2 = \frac{\rho}{T_0}}$

Work:

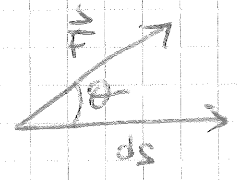
Definition: Work is the product of a force $\vec{F}$ acting along the displacement of the object on which it acts.

Eg



$$dW = \vec{F} \cdot d\vec{s}$$

$$\Rightarrow \Delta W = \int_{s_1}^{s_2} \vec{F} \cdot d\vec{s} \equiv W_{12}$$



Work is related to the change in kinetic energy of the body. To see this:

$$\vec{F} = \frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt}$$

$$\Rightarrow \vec{F} \cdot \vec{v} = m\frac{d\vec{v}}{dt} \cdot \vec{v} = \frac{m}{2}\frac{dv^2}{dt}$$

$$= \frac{d}{dt}\left\{\frac{mv^2}{2}\right\} = \frac{dT}{dt}$$

Also, $\frac{d\vec{r}}{dt} = \vec{v}$, and so:

$$\vec{F} \cdot d\vec{r} = dT \qquad (*)$$

$$\Rightarrow \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r} = \Delta T = \Delta W$$

Now we had in (*) that

$$\vec{F}\cdot d\vec{r} = dT. \quad (*)$$

Suppose that $\vec{F}(r)\cdot d\vec{r}$ can itself be written as the ~~exact~~ differential of another function $V(r)$. In other words:

$$-dV(r) = \vec{F}(r)\cdot d\vec{r}$$

$$\Rightarrow -\int_{r_1}^{r_2} dV(r) = \int_{r_1}^{r_2} \vec{F}(r)\cdot d\vec{r} = \int_{r_1}^{r_2} dT$$

$$\Rightarrow -(V(r) - V_1) = T(r) - T_1$$

$$\Rightarrow T(r) + V(r) = T_1 + V_1 = \text{constant} = E$$

This is the "energy equation":

$$T + V = E = \text{constant} \quad (\text{for conservative forces})$$

From energy equation, we can get velocity:

$$\frac{1}{2} m v^2 = E - V(r)$$

$$\Rightarrow v = \pm\sqrt{\tfrac{2}{m}(E - V(r))} = \frac{dx}{dt}$$

What if non-conservative forces present:

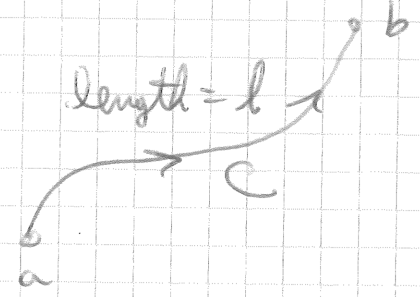
$$\vec{F} = \vec{F}_{cons} + \vec{F}_{non\text{-}cons} = \vec{F}_c + \vec{F}_{nc}$$

$$\Rightarrow \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r} = \int_{r_1}^{r_2} \vec{F}_c \cdot d\vec{r} + \int_{r_1}^{r_2} \vec{F}_{nc} \cdot d\vec{r}$$

$$\Rightarrow \Delta T = -\Delta V + Q_{12}$$

$$\Rightarrow \Delta T + \Delta V = Q_{12} = \Delta (T + V)$$

Friction is Path Dependent



$$\vec{F}_f = -f \frac{\vec{v}}{v}$$

$$W_f = \int_C \vec{F}_f \cdot d\vec{r}$$

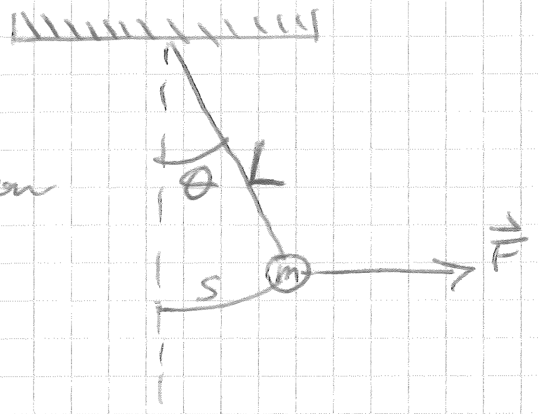
$$= -f\int_C \frac{\vec{v}}{v} \cdot d\vec{r} \quad , \text{ but } d\vec{r} = \vec{v} \cdot dt$$

$$= -f\int_C \frac{v^2}{v}\, dt = -f\int_C v\, dt = -f\int_C dr$$

$$= -f\,l$$

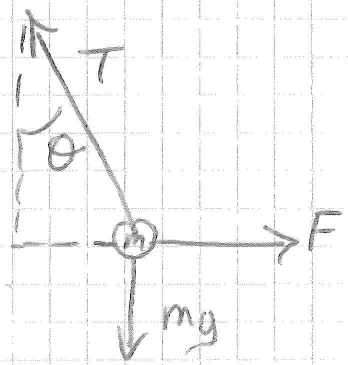
Examples

What is the work done moving mass from bottom to angle θ; the force $\vec{F}$ acts in a way to keep m in equilibrium.



Solution: Equilibrium $\Rightarrow$ no acceleration. $\Rightarrow ma = 0$

FBD



Horizontal:

$$F - T\sin\theta = 0 \quad (1)$$

Vertical:

$$T\cos\theta - mg = 0 \quad (2)$$

Then: $F = T\sin\theta$ and $T\cos\theta = mg$

$$\Rightarrow \frac{F}{mg} = \tan\theta \quad \Rightarrow F = mg\tan\theta$$

The Work:

$$W = \int_{s_1}^{s_2} \vec{F}\cdot d\vec{s} \qquad \text{We need } d\vec{s}$$

Motion is on circle: $\vec{s} = L(\sin\theta, -\cos\theta)$

$$d\vec{s} = L(\cos\theta, \sin\theta)\,d\theta$$

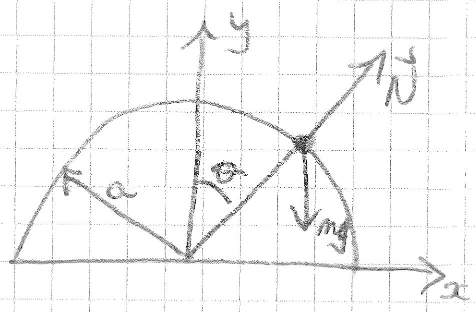
$$\vec{F} = (mg\tan\theta, 0)$$

$$\Rightarrow \vec{F}\cdot d\vec{s} = mgL\tan\theta\cos\theta = mgL\sin\theta\,d\theta$$

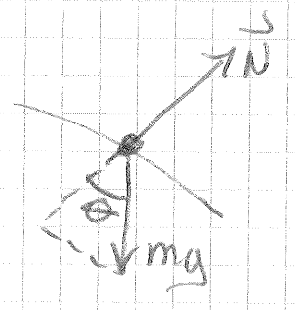
$$W = \int_0^\theta mgL\sin\theta\,d\theta = -mgL\cos\theta\Big|_0^\theta = mgL(1-\cos\theta)$$

Example: Constrained Motion & Energy Conservation

Mass m at rest at top of hemisphere. It falls to one side. At what point will it leave the sphere?



We have the tangential and radial acceleration to consider.



~~In~~ radial direction:

$$\boxed{\sum \vec{F} = m\vec{a}}$$

$$N - mg\cos\theta = -m\frac{v^2}{a} \qquad \text{(radial)} \quad (1)$$

Now, we need v. We know the work energy theorem will get it.

$$W = \int F \cdot ds = \frac{1}{2}mv^2$$

and $\vec{F} = -mg\hat{y}$, $\vec{s} = a(\sin\theta, \cos\theta)$

$$d\vec{s} = a(\cos\theta, -\sin\theta)$$

$$\Rightarrow \vec{F}\cdot d\vec{s} = mga\sin\theta\, d\theta$$

$$\text{So } \int \vec{F}\cdot d\vec{s} = mga\int_0^\theta \sin\theta\, d\theta = -mga\cos\theta\Big|_0^\theta$$

$$= mga(1-\cos\theta) \equiv \frac{1}{2}mv^2$$

Sub into (1)

$$N - mg\cos\theta = -2\frac{mga}{a}(1-\cos\theta)$$
$$= -2mg(1-\cos\theta)$$

$$\Rightarrow N = -2mg + 3mg\cos\theta$$
$$= mg(3\cos\theta - 2)$$

and $\cos\theta = \frac{y}{a}$

$\therefore$ $N = mg\left(\frac{3y}{a} - 2\right)$

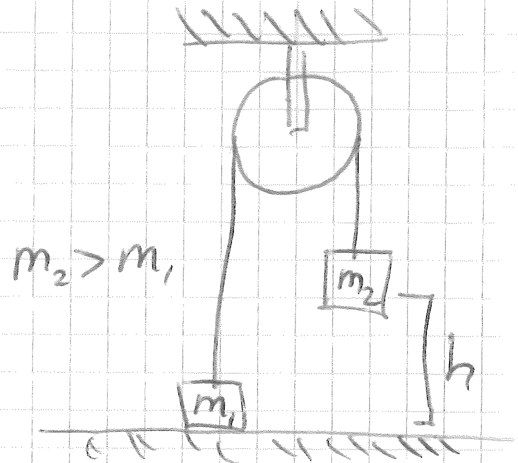
When $N \to 0$, particle leaves surface:

$$\Rightarrow N = 0 = \frac{3y}{a} - 2$$

$$\Rightarrow \boxed{y = \tfrac{2}{3}a}$$

Example (No friction)

What is common velocity just as m_2 hits ground.



The forces acting on each mass are mg down plus a common tension upwards:

$$W = W_1 + W_2 = \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2$$
$$= \frac{1}{2}(m_1 + m_2) v^2$$

$$W_1 = \int_0^h (-m_1 g + T)\, dy \quad , \quad W_2 = \int_h^0 (-m_2 g + T)\, dy$$

$$\Rightarrow W = \int_0^h (T - m_1 g)\, dy + \int_h^0 (T - m_2 g)\, dy$$

$$= \int_0^h (T - m_1 g)\, dy - \int_0^h (T - m_2 g)\, dy$$

$$= \int_0^h (m_2 g - m_1 g)\, dy = (m_2 - m_1) g h$$

$$\Rightarrow v = \sqrt{\frac{2gh(m_2 - m_1)}{m_1 + m_2}}$$