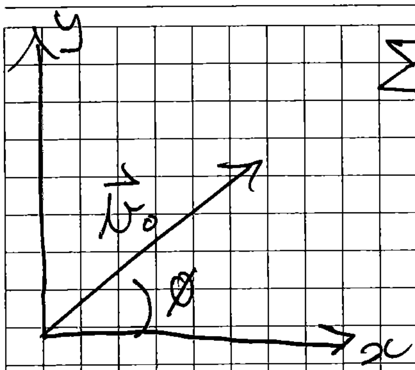
The First EuroGENESIS Workshop

24–26 November 2010

Dubrovnik, Croatia

Projectile Problem



$$\sum \vec{F} = m\vec{a}$$

$$x: \quad m a_x = 0$$

$$y: \quad m a_y = -mg$$

$$\Rightarrow a_y = -g = \frac{dv_y}{dt}$$

$$\Rightarrow \int_{v_0}^{v_y} dv_y = -\int_0^t g\, dt$$

$$v_y - v_{y0} = -gt \Rightarrow v_y = v_{y0} - gt = \frac{dy}{dt}$$

$$\int_0^y dy = \int_0^t (v_{y0} - gt)\, dt$$

$$\boxed{y = v_{y0} t - \frac{1}{2} g t^2}$$

Horizontal:

$$a_x = \frac{dv_x}{dt} = 0$$

$$\Rightarrow v_x - v_{x0} = 0 \Rightarrow v_x = v_{x0}$$

$$v_x = \frac{dx}{dt} = v_{x0}$$

$$\Rightarrow x - x_0 = v_{x0} t \quad , \quad x_0 = 0$$

$$\boxed{x = v_{x0} t}$$

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From initial conditions:

$v_{oy} = v_o \sin \emptyset$, $v_{ox} = v_o \cos \emptyset$

Finally:

$$\boxed{\begin{aligned} y &= v_o t \sin \emptyset - \frac{1}{2} g t^2 \quad (1) \\ x &= v_o t \cos \emptyset \quad (2) \end{aligned}}$$

$$\boxed{\begin{aligned} v_y &= v_o \sin \emptyset - g t \quad (3) \\ v_x &= v_o \cos \emptyset \quad (4) \end{aligned}}$$

What is the shape of the curve?

From (2): $t = \dfrac{x}{v_o \cos \emptyset}$ sub into (1)

$$\boxed{y = x \tan \emptyset - \frac{g}{2 v_o^2 \cos^2 \emptyset} x^2} \quad \underline{\underline{\text{Parabola}}} \quad (5)$$

Max. Height: When $v_y = 0 \Rightarrow t = \dfrac{v_o \sin \emptyset}{g}$

From (1)

$$y_{max} = \frac{v_o^2 \sin^2 \emptyset}{g} - \frac{1}{2g} v_o^2 \sin^2 \emptyset$$

$$\boxed{y_{max} = \frac{1}{2g} v_o^2 \sin^2 \emptyset}$$

Range: $y = x\tan\phi - \dfrac{g x^2}{2 v_0^2 \cos^2\phi}$

When $y = 0 \Rightarrow x = 0$ (no surprise)

or

$$x_{m} = \frac{2 v_0^2 \cos^2\phi \tan\phi}{g} = \text{Range}$$

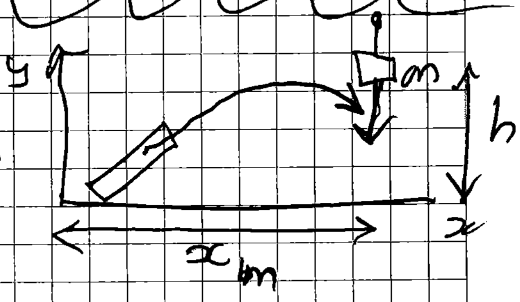
$$= \frac{2 v_0^2}{g} \sin\phi \cos\phi$$

$$\boxed{R = \frac{v_0^2}{g} \sin 2\phi}$$ Range

Observe: When $\phi = 45^\circ$, $\sin 2\phi = 1$ (max value of sin function)

$\boxed{\therefore \text{ Range is maximum when } \phi = 45^\circ.}$

Example: We want to hit the block mass m. We fire gun and cut string at same time.



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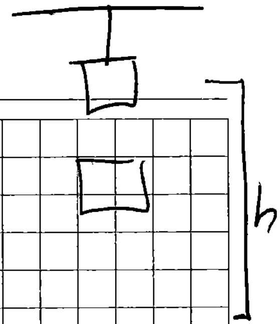

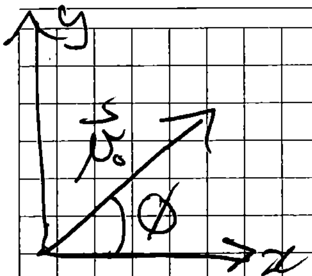

We had: $y=(v_0 \sin\phi)t - \frac{1}{2} g t^2$

$$x = v_0 t \cos\phi$$

For block of mass m, only gravity force.

$$\Rightarrow \quad m a_y = -mg \Rightarrow a_y = \frac{dv_y}{dt} = -g$$

$$\Rightarrow \int_0^{v_y} dv_y = -g\int_0^t dt \Rightarrow v_y = -gt$$

$$v_y = \frac{dy}{dt} = -gt \Rightarrow \int_h^y dy = -g\int_0^t t\,dt$$

$$\Rightarrow \quad y - h = -\frac{gt^2}{2}$$

$$\boxed{y_m = h - g\frac{t^2}{2} \qquad \text{Block}}$$

For collision: (x, y) for block and bullet must be the same.

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So $x_m = v_0 t \cos\phi$ (*)

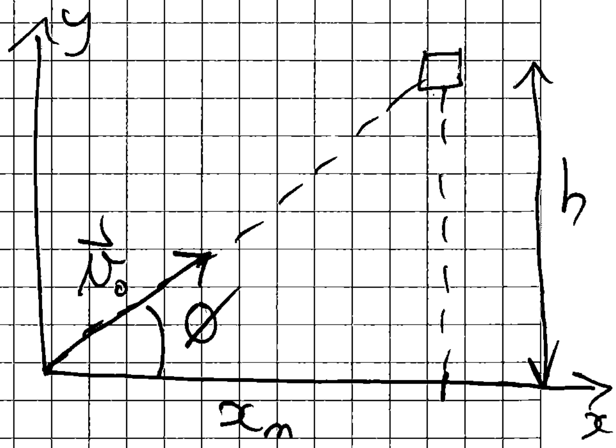
and $y_m = h - g\frac{t^2}{2} = v_0 t \sin\phi - \frac{1}{2} g t^2$

$\Rightarrow h = v_0 t \sin\phi$ (**)

So we have: $\boxed{\frac{h}{x_m} = \tan\phi}$

~~But this is exactly~~

So gun should point directly at the mass m to ensure collision.



Independent of masses

Independent of $|\vec{v}_0|$!!

Independent of g!

1

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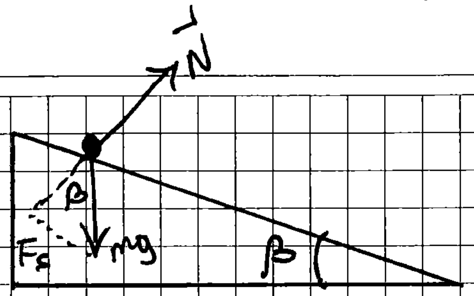
$90 = 180 - \theta + x$

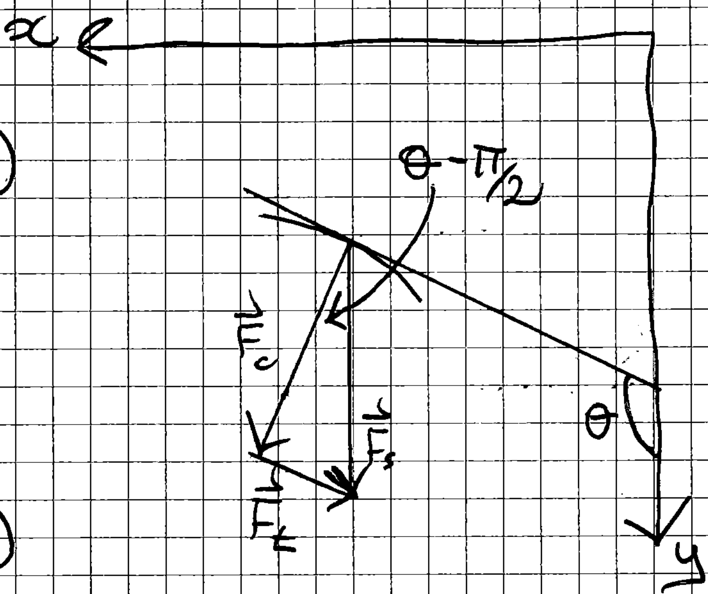
$\boxed{\text{Inclined Plane}}$

a)



1. $mg\cos\beta = N$
2. $F_s = mg\sin\beta$

b)



$$F_c = m\frac{v^2}{R} = mg\sin\beta\cos(\theta - \pi/2)$$

$$\cancel{= mg\sin\beta\ \theta}$$

$$\boxed{F_c = mg\sin\beta\sin\theta} = m\frac{v^2}{R}$$

$$F_t = m\frac{dv}{dt} = mg\,\cancel{\cos\beta}\,\sin\beta\,\sin(\theta - \pi/2)$$

$$= -mg\,\cancel{\cos\beta}\,\sin\beta\cos\theta$$

So, we have:

$$\frac{v^2}{R} = g\sin\beta\sin\theta \qquad (1)$$

$$\frac{dv}{dt} = -g\,\cancel{\cos\beta}\,\sin\beta\cos\theta \qquad (2)$$

We don't know what R is. We need to get rid of it!

On a circle, small path element ds:

$$ds = R\,d\theta \Rightarrow \boxed{R = \frac{ds}{d\theta}}$$

②

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Also, $\frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = v\frac{dv}{ds}$, $v \equiv \frac{ds}{dt}$

So, ① & ② become:

$$v^2 \cdot \frac{d\theta}{ds} = g \sin\beta \sin\theta \quad ③$$

$$v\frac{dv}{ds} = -g \sin\beta \cos\theta \quad ④$$

$$④ \div ③ \Rightarrow \frac{1}{v}\frac{dv}{d\theta} = -\frac{\cos\theta}{\sin\theta}$$

$$\Rightarrow \int \frac{dv}{v} = -\int \frac{\cos\theta}{\sin\theta}\, d\theta$$

$$\Rightarrow \ln v = -\ln[\sin\theta] + C$$

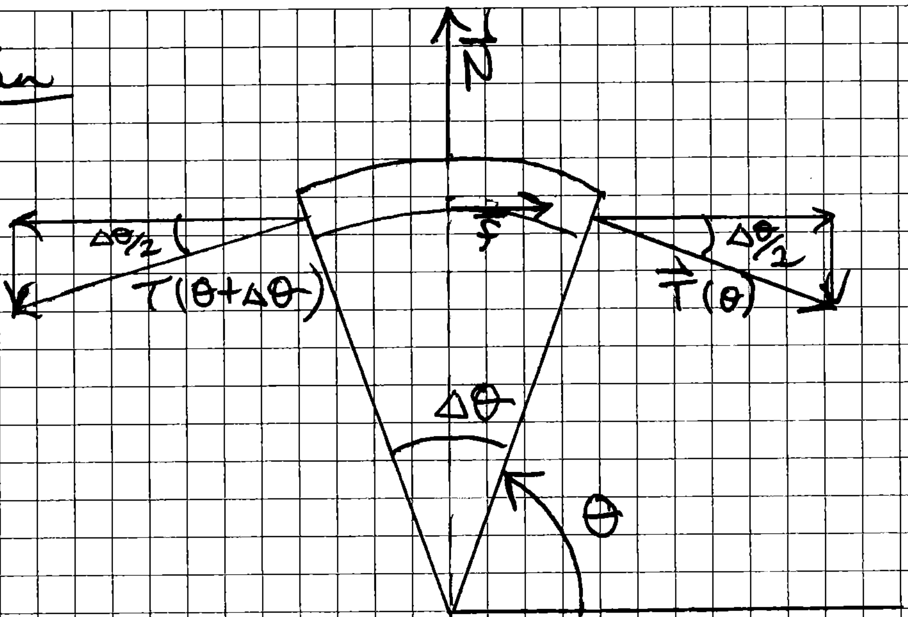
$$\Rightarrow v \sin\theta = C$$

Initial condition: $v = v_0$ @ $\theta = 135^\circ$

$$v_0 \cdot \sin\theta_0 = C$$

$$\boxed{v(\theta) = \frac{v_0 \sin\theta_0}{\sin\theta} = \frac{100 \sin 135^\circ}{\sin\theta} = \frac{70.7}{\sin\theta}}$$

The Capstan



$f = \mu N$

Newton's 2nd Law: $\sum \vec F = m\vec a = 0$

Horizontal: $$T(\theta+\Delta\theta)\cos\Delta\theta/2 - T(\theta)\cos\Delta/2 - \mu N = 0 \quad ①$$

Vertical: $$N - T(\theta+\Delta\theta)\sin\Delta\theta/2 - T(\theta)\sin\Delta\theta/2 = 0 \quad ②$$

Sub N from ② into ①:

$$\cancel{T(\theta+\Delta\theta)\cos} \Rightarrow \cancel{\left[T(\theta+\Delta\theta) - T(\theta)\right]\cos\Delta\theta/2 = \mu\sin\frac{\Delta\theta}{2}}$$

$$\Rightarrow \left[T(\theta+\Delta\theta) - T(\theta)\right]\cos\frac{\Delta\theta}{2} = \mu\left[T(\theta+\Delta\theta) + T(\theta)\right]\sin\frac{\Delta\theta}{2}$$

Divide both sides by $\Delta\theta$:

$$\lim_{\Delta\theta\to 0}\left[\frac{T(\theta+\Delta\theta) - T(\theta)}{\Delta\theta}\right]\cos\frac{\Delta\theta}{2} = \lim_{\Delta\theta\to 0}\mu\left[T(\theta+\Delta\theta) + T(\theta)\right]\frac{\sin(\Delta\theta/2)}{\Delta\theta}$$

Capstan

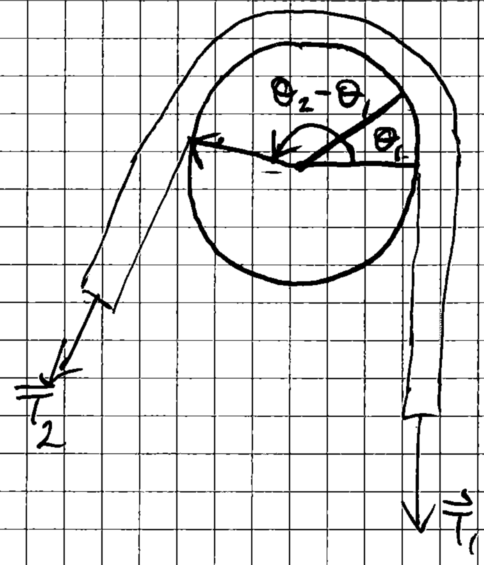
Take limit:

$$\Rightarrow \frac{dT}{d\theta} = 2\mu\, T(\theta) \lim_{\Delta\theta \to 0} \frac{\sin(\Delta\theta/2)}{2(\Delta\theta/2)}$$

$$= \mu\, T(\theta) \lim_{\Delta\theta \to 0} \frac{\sin(\Delta\theta/2)}{(\Delta\theta/2)} \quad (\text{limit is } \underline{1})$$

$$= \mu\, T(\theta)$$

So $$\frac{dT}{d\theta} = \mu\, T(\theta)$$



$$\Rightarrow \int_{T_1}^{T_2} \frac{dT}{T} = \mu \int_{\theta_1}^{\theta_2} d\theta$$

$$\Rightarrow \ln\left(T_2/T_1\right) = \mu(\theta_2 - \theta_1)$$

$$\Rightarrow \boxed{T_2 = T_1\, e^{\mu(\theta_2 - \theta_1)}}$$

(3)

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$\boxed{\text{Normal Force in Capstan Problem}}$

$$\cancel{T(\theta+\Delta\theta/2)\cos\frac{\Delta\theta}{2} - T(\theta-\Delta\theta/2)\cos(\Delta/2) - \mu N(\theta) = 0} \quad (3)$$

$$\cancel{N(\theta) - T(\theta+\Delta\theta/2)\sin\Delta\theta/2 - T(\theta-\Delta\theta/2)\sin\frac{\Delta\theta}{2} = 0} \quad (4)$$

$$T(\theta+\Delta\theta)\cos(\Delta\theta/2) - T(\theta)\cos\frac{\Delta\theta}{2} - \mu N(\theta+\Delta\theta/2) = 0 \quad (1)$$

$$N(\theta+\Delta\theta/2) - T(\theta+\Delta\theta)\sin\left(\frac{\Delta\theta}{2}\right) - T(\theta)\sin\frac{\Delta\theta}{2} = 0 \quad (2)$$

$$T(\theta)\cos\frac{\Delta\theta}{2} - T(\theta-\Delta\theta)\cos\frac{\Delta\theta}{2} - \mu N\left(\theta-\frac{\Delta\theta}{2}\right) = 0 \quad (3)$$

$$N(\theta-\Delta\theta/2) - T(\theta)\sin\frac{\Delta\theta}{2} - T(\theta-\Delta\theta)\sin\frac{\Delta\theta}{2} = 0 \quad (4)$$

(1) + (3):

$$\left[T(\theta+\Delta\theta) - T(\theta-\Delta\theta)\right]\cos\frac{\Delta\theta}{2} - \mu\left[N\left(\theta+\frac{\Delta\theta}{2}\right) + N\left(\theta-\frac{\Delta\theta}{2}\right)\right] = 0 \quad (5)$$

(2) − (4)

$$N(\theta+\Delta\theta/2) - N(\theta-\Delta\theta/2) + \left[T(\theta-\Delta\theta) - T(\theta+\Delta\theta)\right]\sin\frac{\Delta\theta}{2} = 0$$

$$\Rightarrow N(\theta+\Delta\theta/2) - N(\theta-\Delta\theta/2) = \left[T(\theta+\Delta\theta) - T(\theta-\Delta\theta)\right]\sin\frac{\Delta\theta}{2} \quad (6)$$

(6) → (5):

$$\left[N(\theta+\Delta\theta/2) - N(\theta-\Delta\theta/2)\right]\cot\left(\frac{\Delta\theta}{2}\right) = \mu\left[N\left(\theta+\frac{\Delta\theta}{2}\right) + N\left(\theta-\frac{\Delta\theta}{2}\right)\right]$$

$$N(\theta+\Delta\theta/2) - N\left(\theta-\frac{\Delta\theta}{2}\right) = \mu\left[N\left(\theta+\frac{\Delta\theta}{2}\right) + N\left(\theta-\frac{\Delta\theta}{2}\right)\right]\tan\left(\frac{\Delta\theta}{2}\right)$$

$$\Rightarrow \boxed{\frac{dN}{d\theta} = \mu N} \qquad \cancel{N(\theta) = N_0 e^{\mu\theta}} \qquad \boxed{\cancel{N(\theta) = N_0 e^{\mu\theta}}}$$

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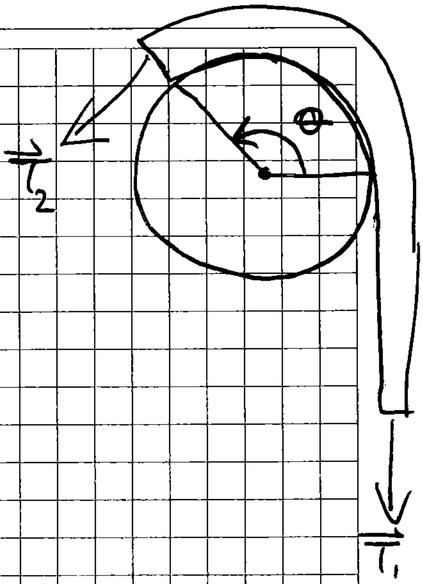
Capstan Normal Force



$$\frac{dN}{d\theta} = \mu N \quad \Rightarrow \quad \int_{N_1}^{N_2} \frac{dN}{N} = \mu \int_{\theta_1}^{\theta_2} d\theta$$

$$\Rightarrow \ln\left(\frac{N_2}{N_1}\right) = \mu(\theta_2 - \theta_1)$$

So ~~that~~ $\boxed{N_2 = N_1 e^{\mu(\theta_2-\theta_1)}}$

It can be shown that $N_1 = T_1$

So $\boxed{N_2 = T_1 e^{\mu(\theta_2-\theta_1)}}$

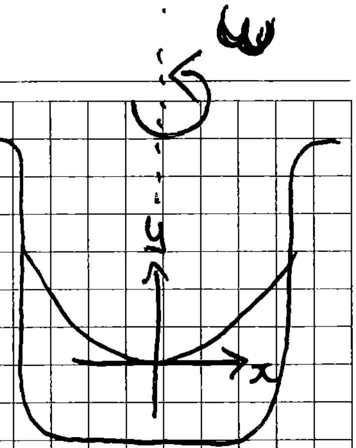
How to show $N_1 = T_1$:

In horizontal direction: $\sum N = \int N(\theta)\cos\theta\, d\theta = \int T(\theta)\cos\theta\, d\theta$ (Newton's 3rd)

$$\Rightarrow N_1 \int_{\theta_1}^{\theta_2} \cos\theta\, e^{\mu\theta}\, d\theta = T_1 \int_{\theta_1}^{\theta_2} \cos\theta\, e^{\mu\theta}\, d\theta$$

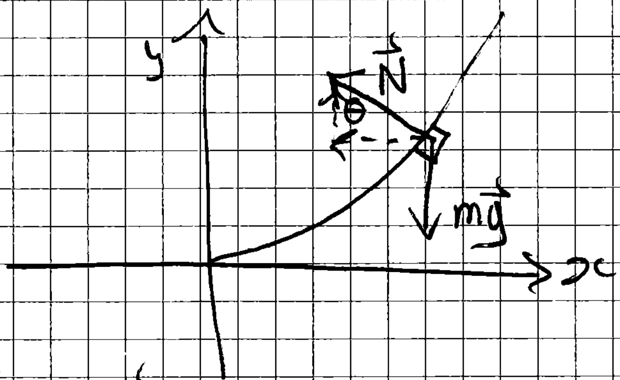
$$\Rightarrow N_1 = T_1$$

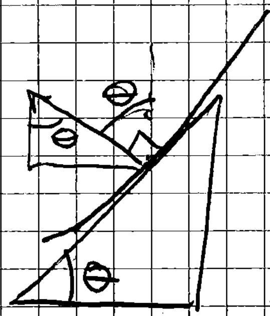
Spinning Water



Glass of H_2O. Spin about y-axis. What shape is the H_2O surface?

FBD of mass-element @ surface:





Newton: $\sum \vec{F} = m\vec{a}$

In vertical direction y: $\quad -mg + N\cos\theta = 0 \qquad (1)$

x: $\quad N\sin\theta = m\omega^2 x \qquad (2)$

From (1):

$$N\cos\theta = mg \qquad (3)$$

$$N\sin\theta = m\omega^2 x \qquad (4)$$

(4) ÷ (3)

$$\tan\theta = \frac{\omega^2}{g}x = \frac{dy}{dx}$$

$$\Rightarrow \boxed{y = \frac{\omega^2}{2g}x^2}$$