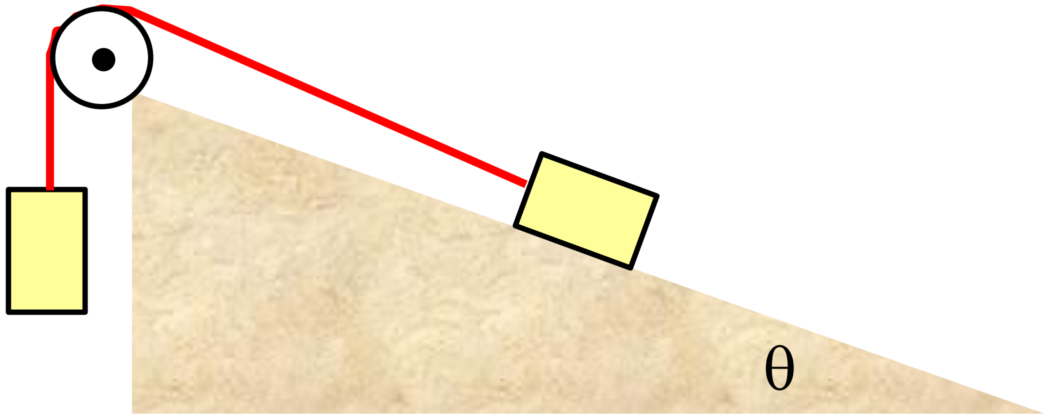
Lecture – 3

Energy, Work and Momentum

Experimentalphysik I in Englischer Sprache

6-11-08

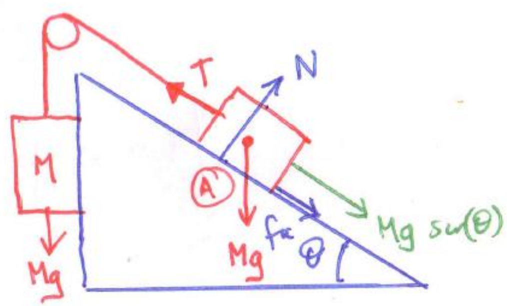
Solution to Homework 2



- Two blocks of equal mass M are connected by a string which passes over a frictionless pulley. If the coefficient of dynamic friction is μ_K, what angle θ must the plane make with the horizontal so that each block will move with constant velocity once in motion ?

Thanks to those of you who submitted answers (almost all correct) !

Solution to Homework 2



Two methods to solve problem. (i) Each block considered as separate systems
(ii) Two blocks connected considered as 1 system

Use method (i)

Forces on block A are TENSION (T) in string, $Mg\sin(\theta)$ - weight down slope
N - normal force, f_k - kinetic friction.

Resolving forces ∥ and ⊥r to slope.

∥ slope $\quad T - Mg\sin(\theta) - f_k = 0$

⊥r $\quad N - Mg\cos(\theta) = 0 \quad$ (since $\vec{a} = 0$ Newton II)

$\Rightarrow N = Mg\cos(\theta)$.

Since $f_k = \mu_k N \quad T - Mg\sin(\theta) - \mu_k Mg\cos(\theta) = 0$

$T = Mg$

$$\Rightarrow \quad 1 - \sin(\theta) - \mu_k\cos(\theta) = 0$$

Let $\cos(\theta) = +\sqrt{1-\sin^2(\theta)}$

$$\Rightarrow \quad 1 - \sin(\theta) - \mu_k\sqrt{1-\sin^2\theta} = 0$$

$$(1-\sin\theta)^2 = \mu_k^2(1-\sin^2\theta)$$

Expand

$$1 + \sin^2(\theta) - 2\sin\theta = \mu_k^2 - \mu_k^2\sin^2(\theta)$$

$$(1+\mu_k^2)\sin^2(\theta) - 2\sin(\theta) + (1-\mu_k^2) = 0$$

Solve quadratic in $\sin(\theta)$

$$\sin(\theta) = \frac{2 \pm \sqrt{4 - 4(1-\mu_k^2)(1+\mu_k^2)}}{2(1+\mu_k^2)}$$

Re-arrange to give $\sin(\theta) = \dfrac{1 \pm \sqrt{\mu_k^4}}{(1+\mu_k^2)} = \dfrac{1 \pm \mu_k^2}{1+\mu_k^2}$

+ sign gives sin(θ)=1, i.e. θ=90deg → vertical slope- just 2 masses on a pulley

- sign gives the wanted solution

$$\underline{\underline{\theta = \arcsin\left(\frac{1-\mu_k^2}{1+\mu_k^2}\right)}} \quad \underline{\underline{\text{ANS}}}$$

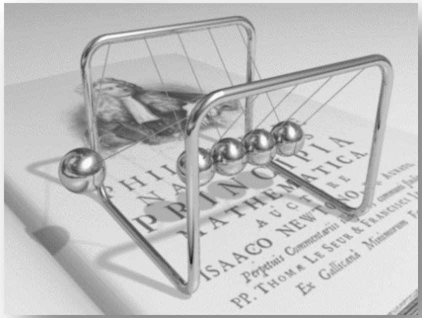
Lecture 3 - Contents

M3.1 Energy fundamentals

– *Work and Energy...*
– *Forms of energy...*

M3.2 Force and Potential Energy

– *Conservative and dissipative forces...*
– *Equilibrium and harmonic oscillations...*



M3.3 Momentum, Impulse and Energy

– *Momentum and impulse...*
– *It’s all a question of velocity...*
– *Conservation of momentum...*

3.1 Work and Energy

Arbeit und Energie

*Calculate the **speed** of the arrow ?*

Need F(t) applied by the string of the bow to the arrow?

→ $F(t)=m_j a(t)$, integrate over time to find v(t) at the point when the arrow looses contact with the string

Varying force as a function of position

→ Equation of motion approach rather complicated

3.1 Work (Arbeit)

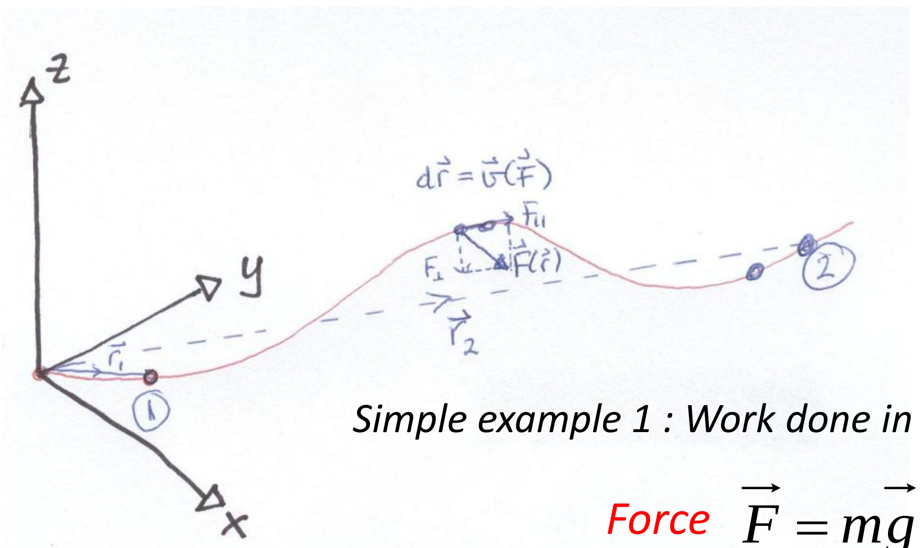
- The idea that to "move something" you have to "expend energy", or do work, is familiar to all of us...

Work is defined by $W = \vec{F}.\vec{s} \quad = [MLT^{-2}][L]$

UNIT
Joule=1N m
=1kg m^2 / s^2

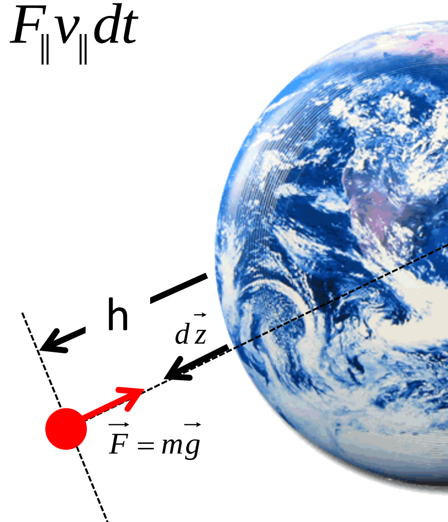
*W – SCALAR, defined by dot product of force (**F**) and displacement (**s**) vectors*

- For a 3D trajectory the work required to move a particle from point 1 to point 2 ?



$$dW = \vec{F}.d\vec{r} = F_{\parallel}dr \quad = F_{\parallel}v_{\parallel}dt$$

$$\boxed{W_{12} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}(\vec{r})d\vec{r}}$$



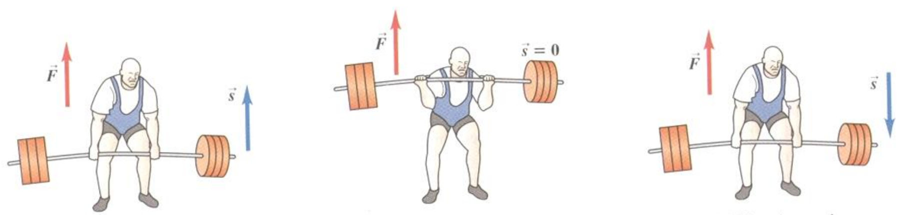
Simple example 1 : Work done in gravitational field of earth

Force $\quad \vec{F} = m\vec{g} = -mg\hat{e}_z$

Work $\quad dW = -mgdz \qquad W(h) = \int_0^h -mgdz = -mgh$

- Now, if we push the car the work done ***by us on the car*** is clearly ***positive***
  - *We apply a force (**F**) that is in the same direction as the displacement vector (**s**) of the car*

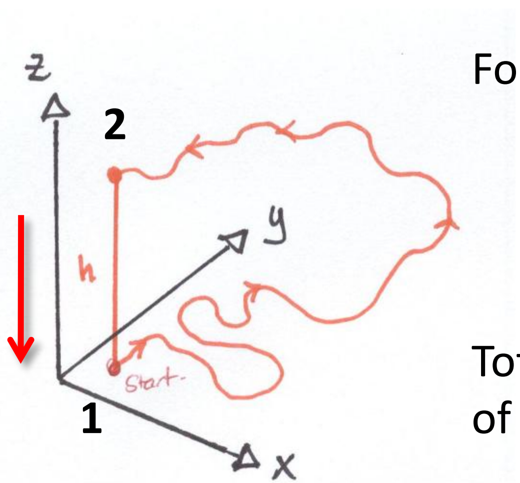
- Work done can also be ***negative*** or ***zero***
  - *Depends on the relative orientation of **F** and **s***



(a) Positive work (b) Zero work (c) Negative work

- In our simple example of a mass moving in a central field (gravity), the work done is independent of the precise trajectory followed

$$\vec{F} = -mg\hat{e}_z$$



For any displacement in the x,y plane $d\vec{r} = dx\hat{e}_x + dy\hat{e}_y$

$$\Rightarrow \quad dW = \vec{F}.d\vec{r} \;\; = 0$$

Since, **F** perpendicular to d**r**
→work done is zero

Total work done 1→2 is $W = -mgh$ <u>independent</u> of the trajectory followed...

Work done by a falling particle 2→1 ? $\Rightarrow$ $W(h) = \int_h^0 -mgdz = +mgh$ gravity **does work**

M3.2 Energy ?

Energy is the capability a body has to do work

- In classical mechanics, energy appears in **three forms**
  - E_{kin} — **Kinetic Energy** – *only* depends on the velocity v and mass m of a body
  - E_{pot} — **Potential Energy** – dependent on the relative position of two bodies ($\mathbf{r}_i$-$\mathbf{r}_j$) that interact with each other via some force
  - Q — **Heat** Energy– internal energy of a body due to the microscopic motion (vibration and rotation) of its constituent atoms

Classical mechanics "arises" since different bodies transfer energy between themselves by doing work W...

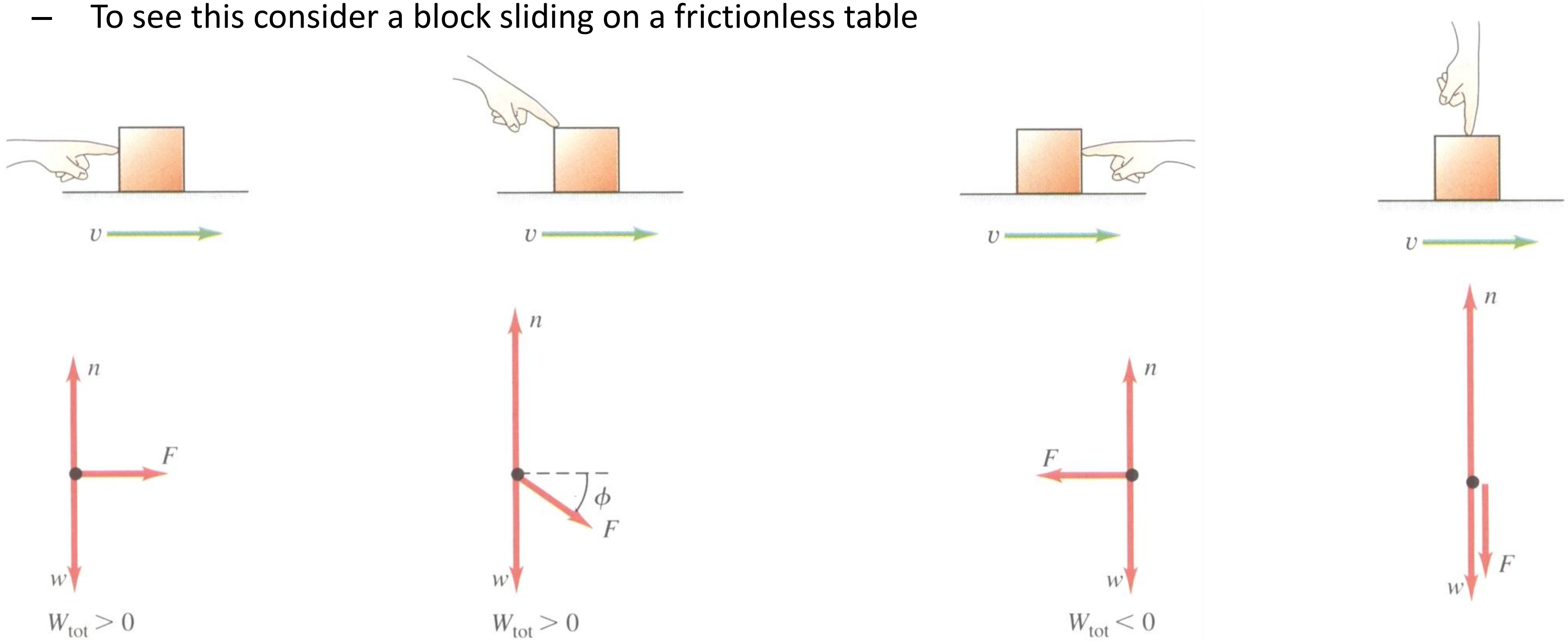
Energy cannot be created or destroyed, simply converted from one form into another form (1st law of thermodynamics)

In any process in classical mechanics ⇨ $\boxed{Q_{12} = \Delta E_{pot} + \Delta E_{kin}}$

Kinetic energy

- The total work done on a body by external forces is related to it's displacement

- BUT - total work is also proportional to the *speed* of the body.
  - To see this consider a block sliding on a frictionless table



F in direction of displacement **s**
→ *block speeds up, work is done* ***on*** *the block (W_{tot}>0)*

F has a component in direction of displacement **s**
→ *block speeds up, work is done* ***on*** *the block*
(W'_{tot}>0, W'_{tot}< W_{tot})

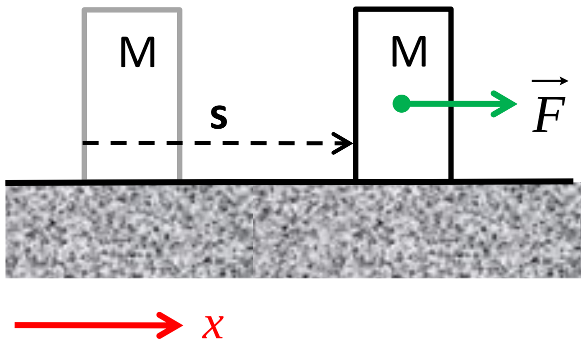
F opposite to displacement **s**
→ *block slows down, work is done* ***by*** *the block*

→ *Negative work done by agent (W_{tot}<0)*

F ⊥ displacement **s**
→ *block maintains speed*

→ *Zero work is done by agent (W_{tot}=0)*

Let’s make these “observations” more quantitative:



Particle subject to constant accn. → $F_x = Ma_x$

Velocity and displacement linked by → $v_2^{\ 2} = v_1^{\ 2} + 2a_x s$

$$\Rightarrow F_x = Ma_x = M\frac{\left(v_2^{\ 2} - v_1^{\ 2}\right)}{2s} \Rightarrow \underbrace{F_x s}_{\text{Net work done by the force F}} = \underbrace{\frac{1}{2}Mv_2^{\ 2} - \frac{1}{2}Mv_1^{\ 2}}$$

Net work done by the force F

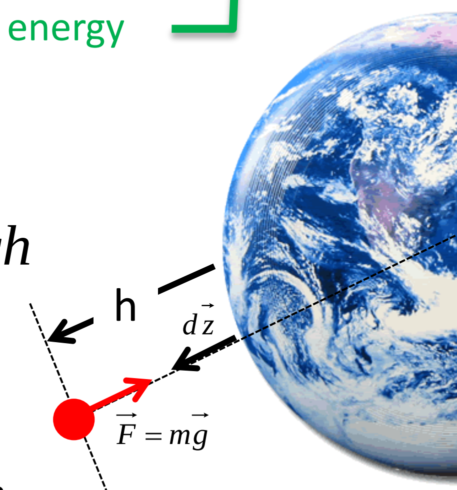
Define **this** as kinetic energy - E_{kin}

$$\boxed{E_{kin} = \frac{1}{2}Mv^2}$$

$$\boxed{W = \Delta E_{kin}}$$

The work done by the force = the change of the body’s kinetic energy

<u>Example of free fall in a central force (gravity)</u>



1) The work done by the body as it is lifted to a height h is: $W = -Mgh$

Work done goes into [potential energy]() of the body in the gravitational field

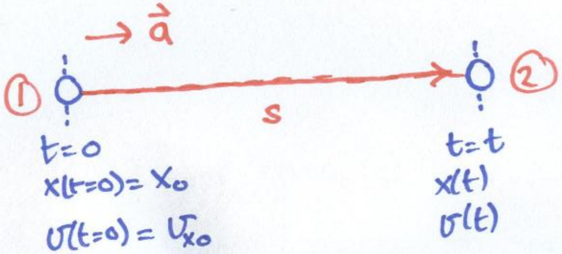
2) Drop the body – the gravitational force changes PE into KE

$$h = \tfrac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}} \Rightarrow v = gt = \sqrt{2hg} \Rightarrow E_{kin} = \tfrac{1}{2}Mv^2 = Mgh \ = -W$$

3) When the body hits the earth, W is converted to heat Q and dissipated in the earth

Just a brief reminder!

Displacement from a point x_0, when subject to a constant acceleration a.



We can write $\quad x(t) = x(0) + v(0)t + \frac{1}{2}at^2 \quad (1)$

$$x(t) - x(0) = v(0)t + \frac{1}{2}at^2 = s \quad (2)$$

$$\frac{dx(t)}{dt} = v(t) = v(0) + at \Rightarrow t = \frac{v(t) - v(0)}{a}$$

Substitute in (2)

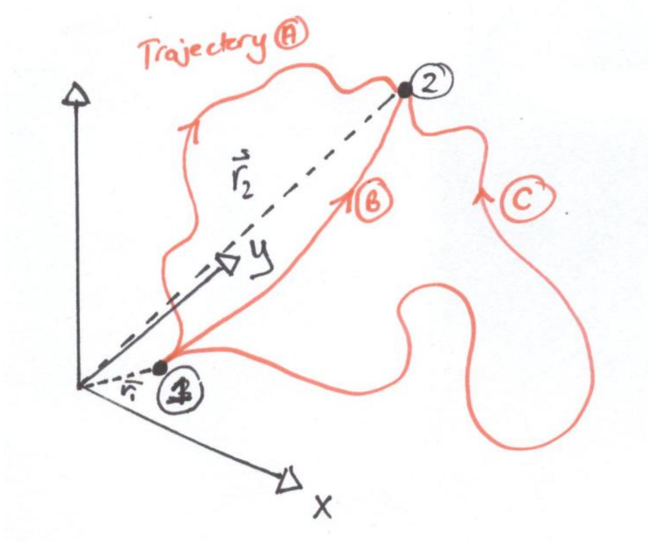
$$s = v(0)\left[\frac{v(t) - v(0)}{a}\right] + \frac{1}{2}a\left[\frac{v(t) - v(0)}{a}\right]^2$$

$$= \frac{v(0)v(t) - v^2(0)}{a} + \frac{1}{2a}\left[v^2(t) - v^2(0) - 2v(t)v(0)\right]$$

$$\boxed{s = \frac{v^2(t) - v^2(0)}{2a}} \quad \text{or} \quad \boxed{a = \frac{v_2^2 - v_1^2}{2s}}$$

Conservative and Non-Conservative forces

- For forces like gravity, the work required to move from point 1 to 2 is <u>independent</u> of the trajectory taken, it depends only on relative position of 1 and 2

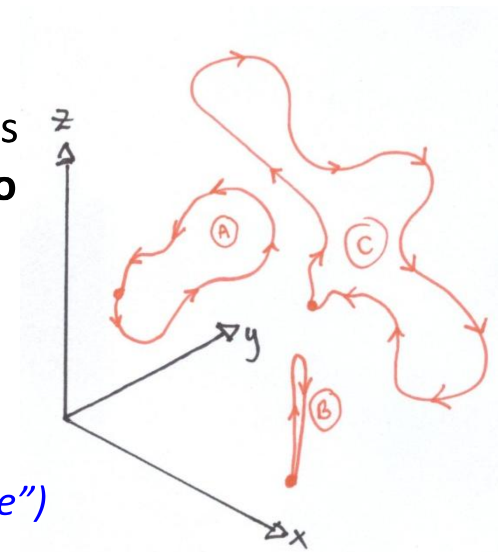


This means that: $W_{12} = W_A + W_B + W_C = -W_{21}$

Work done by external agent to move body from 1 to 2

Work done by external agent to move body from 2 to 1

Furthermore, since the work done is path independent it also follows that the work done moving around any **closed trajectory will be zero**



→Forces that obey this rule are "conservative" or "non-dissipative" (Examples: Gravity, Coulomb interaction, Elastic forces, etc...)

*→Forces that **do not** obey this rule are "non-conservative" or "dissipative" (Examples: Kinetic friction, Fluid resistance – here energy goes "somewhere else")*

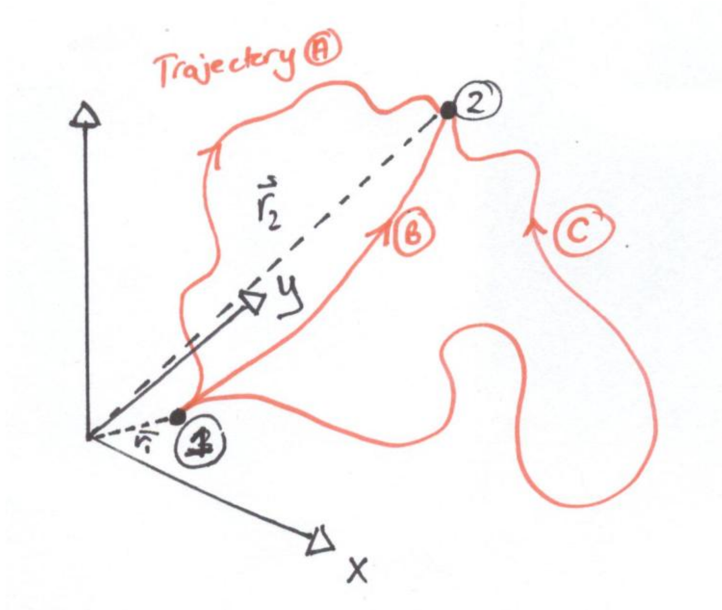
Potential Energy

- We have see that a body can *loose* or *gain* kinetic energy because it *interacts* with other objects that exert *forces* on it...
  - *During such interactions the body's kinetic energy = the work done* ***on it*** *by the force*
  - *If you give a force the "possibility" to do work, the body has* ***POTENTIAL ENERGY***

The "mild" danger associated with storing PE in the earths gravitational field

The "extreme" danger of storing PE in elastic energy (don't try this at home!)

- Unlike **kinetic energy that is associated with motion**, **potential energy is associated with the position** of a particle in the force field of another body



$$\Delta E_{pot}\left(\vec{r}_1,\vec{r}_2\right)\equiv E_{pot}\left(\vec{r}_2\right)-E_{pot}\left(\vec{r}_1\right)\equiv -W_{12}$$

$$\boxed{\Delta E_{pot}\left(\vec{r}_1,\vec{r}_2\right)=-\int_{\vec{r}_1}^{\vec{r}_2}\vec{F}(\vec{r}).d\vec{r}}$$

CHANGE OF POTENTIAL ENERGY
MOVING FROM **r**$_1$ to **r**$_2$ SUBJECT TO A FORCE **F**(**r**)

The potential energy depends on the **relative positions** (**r**$_1$,**r**$_2$), not absolute positions

From the equation above we can write: $dE_{pot}=-\vec{F}.d\vec{r}=-dW$

or equivalently, $\vec{F}\left(\vec{r}\right)=-grad\left(E_{pot}\left(\vec{r}_1,\vec{r}_2\right)\right)=-\begin{pmatrix}\partial E_{pot}/\partial x\\ \partial E_{pot}/\partial y\\ \partial E_{pot}/\partial z\end{pmatrix}$

$$\boxed{\vec{F}(\vec{r})=-\vec{\nabla}E_{pot}\left(\vec{r}\right)}$$

FORCE
= - GRADIENT OF POTENTIAL

NB, in this equation **F**(**r**) is the **measurable quantity**
E_{pot} is obtained from it via integration.

→ It is, therefore, defined only to within an integration constant
→ Only $\Delta E_{pot}=-\Delta W$ has a physical meaning

- For conservative forces we can easily show that energy is a conserved quantity
  - Start from Newton's 2nd law $\vec{F}(\vec{r}) = m\vec{a}(\vec{r})$ and integrate over displacement

$$\int_{\vec{r}_1}^{\vec{r}_2} \vec{F}(\vec{r})d\vec{r} = W_{12} = -\Delta E_{pot}(\vec{r}_1, \vec{r}_2) = m\int_{\vec{r}_1}^{\vec{r}_2} \vec{a}(\vec{r})d\vec{r} = \tfrac{1}{2}mv^2(\vec{r}_2) - \tfrac{1}{2}mv^2(\vec{r}_1) = \Delta E_{kin}(\vec{r}_1, \vec{r}_2)$$

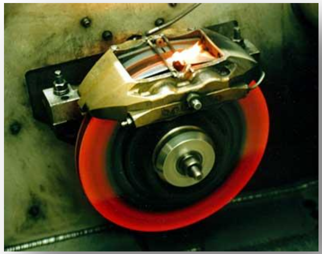
WORK DONE	CHANGE OF PE	NEWTON II	CHANGE OF K.E.

$$\Rightarrow \Delta E_{pot}(\vec{r}_1, \vec{r}_2) + \Delta E_{kin}(\vec{r}_1, \vec{r}_2) = 0 \quad \Rightarrow \quad \boxed{E_{pot}(\vec{r}) + E_{kin}(\vec{r}) = E = const}$$

PRINCIPLE OF CONSERVATION OF ENERGY IN MECHANICS

(Reason why *conservative* forces take their name!)

What about frictional (non-conservative) forces ?



Total force $\vec{F} = \vec{F}_{cons} + \vec{F}_{fric} \Rightarrow \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}.d\vec{r} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_{cons}.d\vec{r} + \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_{frict}.d\vec{r} = W_{12} + Q_{12}$

conservative

non-conservative

$$Q_{12} - \Delta E_{pot} = \Delta E_{kin} \qquad \boxed{Q_{12} = \Delta E_{pot} + \Delta E_{kin} = \Delta E}$$

ENERGY CONSERVATION with DISSIPATION

Non conservative forces generate heat-Q that is equal to the change of the total energy of the body...

Non conservative forces, Heat and Irreversibility

The idea of non-conservative forces generating heat is very closely linked with the flow of time

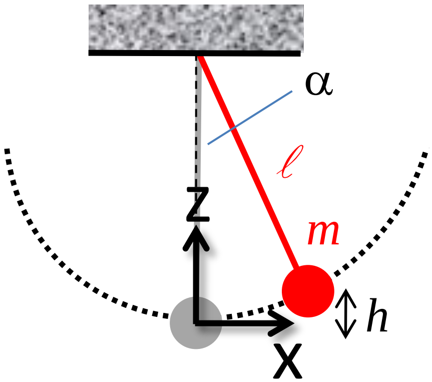
→ *Fundamental principle in nature that systems tend to flow from order → disorder*

Next semester
(Thermodynamics)
Entropy

Dye in water never "unmixes"

Coffee never gets warm by itself and the environment gets cool!

Example - The PE and KE of the simple pendulum



Equation of motion $\ddot{x} = -\frac{g}{\ell}x$

solutions – $x(t) = x_0\cos(\omega t)$ with $\omega = \sqrt{\frac{g}{\ell}}$

Energy ?

P.E. $E_{pot}(h) = mgh = mg\ell(1-\cos(\alpha))$

$h = \ell - \ell\cos(\alpha)$

E_{pot}=0 for x=z=0 ☺

For small α : $\ell^2 = (\ell - h)^2 + x^2$ $\quad \cancel{\ell^2} = \cancel{\ell^2} + \cancel{h^2} - 2\ell h + x^2$ $\quad h \ll \ell \Rightarrow h \approx \frac{x^2}{2\ell}$

Instantaneous PE

$\Rightarrow E_{pot} = mg\frac{x_0^2}{2\ell}\cos^2(\omega t)$

Instantaneous KE

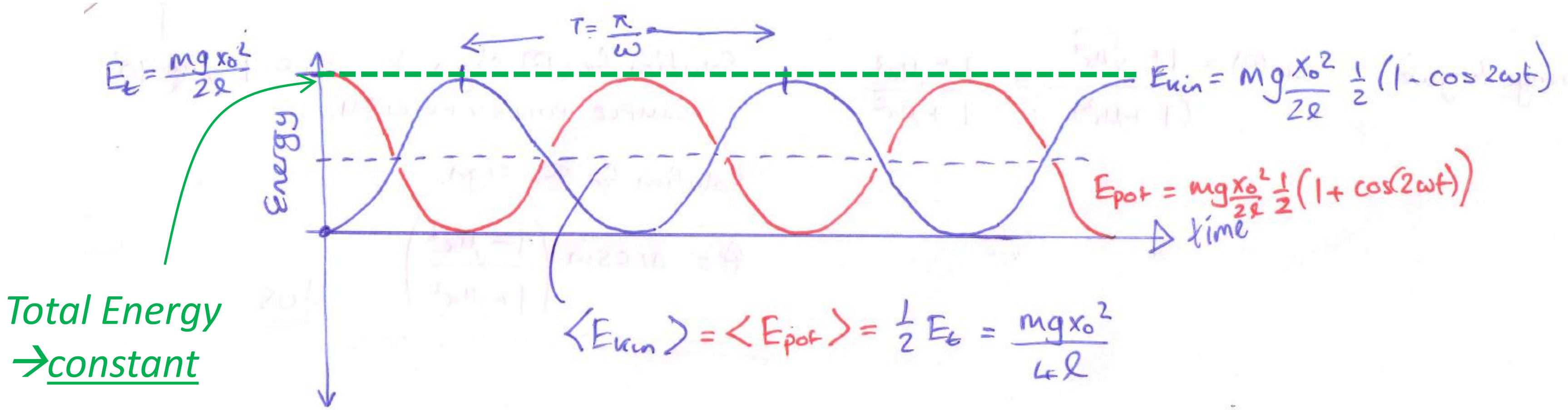
$\Rightarrow E_{kin} = \frac{1}{2}m\dot{x}^2 = \frac{1}{2}mg\omega^2 x_0^2\sin^2(\omega t) = mg\frac{x_0^2}{2\ell}\sin^2(\omega t)$

Total energy $E = E_{kin} + E_{pot} = \frac{mgx_0^2}{2\ell}\left(\cos^2(\omega t) + \sin^2(\omega t)\right) = \frac{mgx_0^2}{2\ell} = const$

The total energy of such an oscillator driven by non-dissipative forces is constant

The solutions for the K.E. and P.E. show that, energy is periodically exchanged between kinetic and potential with a frequency of 2ω

$$E_{kin} = mg\frac{x_0^{\ 2}}{2\ell}\sin^2(\omega t) \ = mg\frac{x_0^{\ 2}}{2\ell}\tfrac{1}{2}\left(1-\cos(2\omega t)\right) \qquad E_{pot} = mg\frac{x_0^{\ 2}}{2\ell}\cos^2(\omega t) \ = mg\frac{x_0^{\ 2}}{2\ell}\tfrac{1}{2}\left(1+\cos(2\omega t)\right)$$

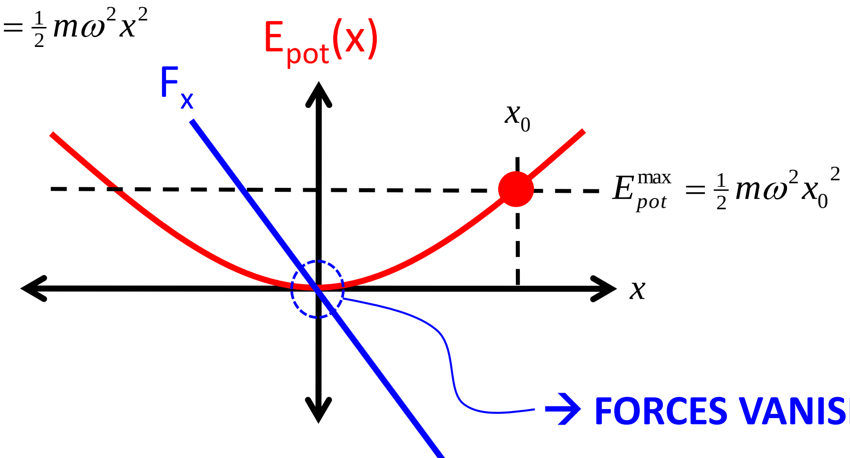


The average kinetic *or* potential energy of this type of simple "harmonic" oscillator over time is :

$$\langle E_{kin}\rangle = \langle E_{pot}\rangle = \tfrac{1}{2}E_{\max} = \frac{mgx_0^{\ 2}}{4\ell}$$

Forces acting on a pendulum ?

$$E_{pot} = \frac{mgx^2}{2\ell} = \tfrac{1}{2}m\omega^2x^2$$



Forces acting $\vec{F} = -\vec{\nabla}E_{pot}(\vec{r})$

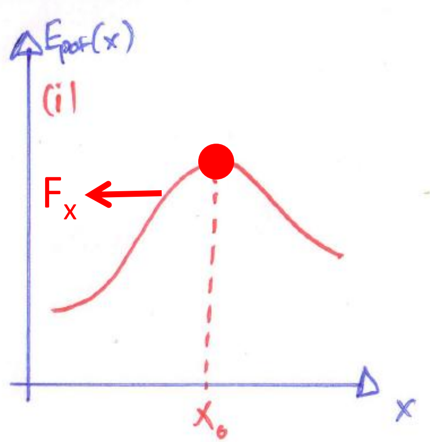
$$\vec{F} = -\begin{pmatrix} \frac{\partial E_{pot}}{\partial x} \\ \frac{\partial E_{pot}}{\partial y} \\ \frac{\partial E_{pot}}{\partial z} \end{pmatrix} = \begin{pmatrix} -\frac{mgx}{\ell} \\ 0 \\ 0 \end{pmatrix} \Rightarrow F_x = -\frac{mgx}{\ell}$$

→ FORCES VANISH AT EXTREMA OF POTENTIAL FUNCTION

Equilibrium and harmonic oscillators

- We've seen that whenever we can define a potential function $E_{pot}(\mathbf{r})$ for <u>all conservative forces</u> and we can rather easily calculate the forces acting on the body from it ...

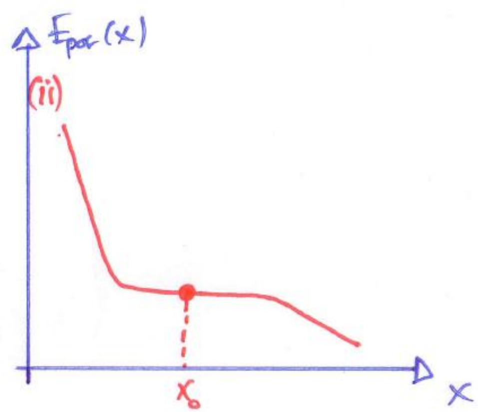
- Whenever $\mathbf{F}(\mathbf{r})=-\mathrm{grad}\,(E_{pot}(\mathbf{r}))$ vanishes, i.e. at maxima and minima of $E_{pot}(\mathbf{r})$, there are no forces acting and the system is in equilibrium
  - *Different "types" of equilibrium exist*



$$\left.\frac{\partial E_{pot}(\vec{r})}{\partial x}\right|_{x_0}=0\,,\ \left.\frac{\partial^2 E_{pot}(\vec{r})}{\partial x^2}\right|_{x_0}<0$$

UNSTABLE EQUILIBRIUM
(labiles Gleichgewicht)

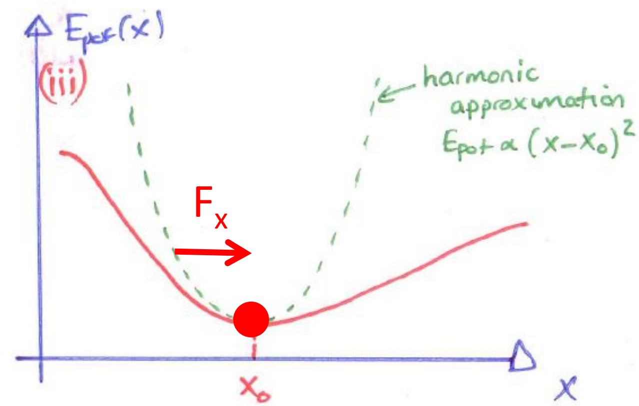
Any small fluctuation of force would result in motion away from x_0



$$\left.\frac{\partial E_{pot}(\vec{r})}{\partial x}\right|_{x_0}=0\,,\ \left.\frac{\partial^2 E_{pot}(\vec{r})}{\partial x^2}\right|_{x_0}=0$$

NEUTRAL EQUILIBRIUM

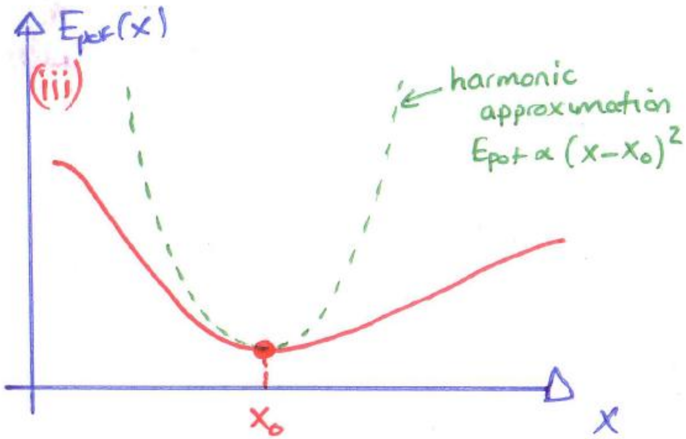
Any small fluctuation would result in a new equilibrium position, close to x_0



$$\left.\frac{\partial E_{pot}(\vec{r})}{\partial x}\right|_{x_0}=0\,,\ \left.\frac{\partial^2 E_{pot}(\vec{r})}{\partial x^2}\right|_{x_0}=0$$

STABLE EQUILIBRIUM

Any small fluctuation would result in oscillations around equilibrium position x_0



Very many "interactions" in physics have a potential function with the form sketched here

EXAMPLES:

→*Elastic forces*

→*Coulomb force between positive and negative charges*

→*Bond force between two atoms in a solid*

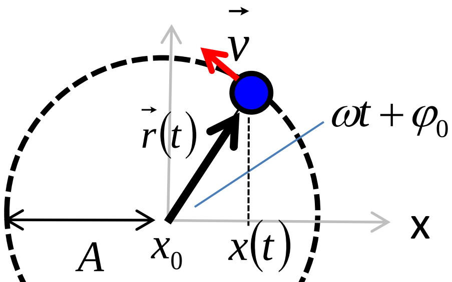
Although these interactions are rather complicated, we very often approximate the minimum in the $E_{pot}(x)$ curve as a parabolic function

We can approximate the potential close to x_0 as $E_{pot}(x) = E_{pot}(x_0) + \frac{1}{2}k(x-x_0)^2$ $\quad \left.\frac{\partial E_{pot}(\vec{r})}{\partial x}\right|_{x_0} = \frac{1}{2}k > 0$

Therefore, the force is given by $F_x = -\frac{\partial E_{pot}(x)}{\partial x} = -k(x-x_0)$

EXACTLY SAME as ELASTIC FORCE WITH SPRING CONSTANT k

Eqn of motion of <u>such an oscillator</u> $m\ddot{x} = -k(x-x_0)$



Solution same as for circular motion with const ω

solutions – $x(t) = x_0 + A\cos(\omega t + \varphi_0)$

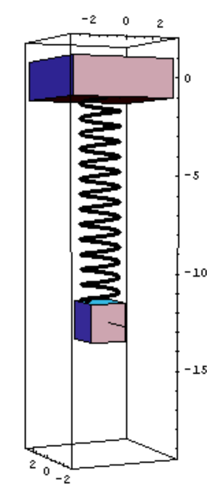
x_0 - equilibrium position

A – amplitude (depends on size of initial displacement)

ω – angular frequency $\omega = \sqrt{\frac{k}{m}}$

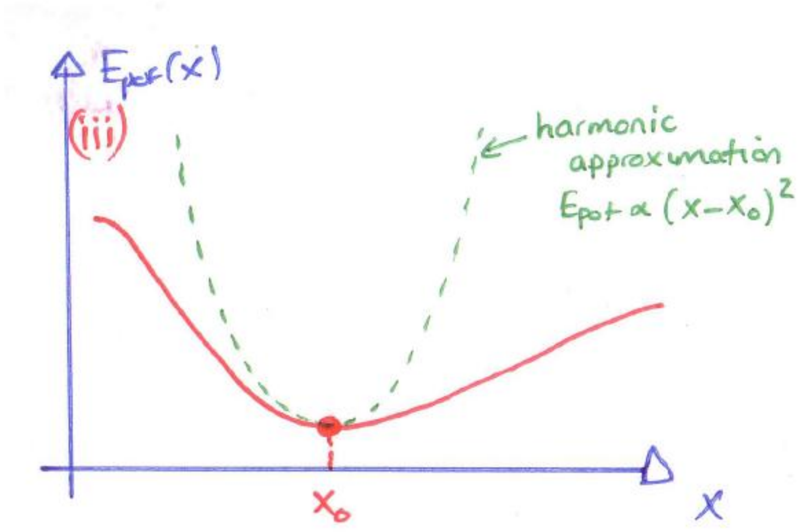
φ_0 – phase

$x(0) = x_0 + A\cos(\varphi_0)$



→CALLED A **<u>SIMPLE HARMONIC OSCILLATOR</u>**

→*Very useful in physics to describe the response of a system to small perturbations*

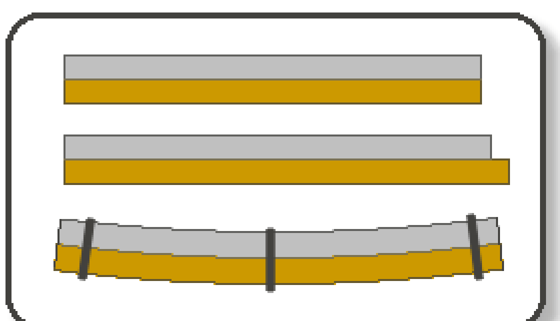


This anharmonicity in the interatomic potential are responsible for the thermal expansion of solids (*Thermische Auslenkung*)

→ *Harmonic approx bad for large amplitude A*

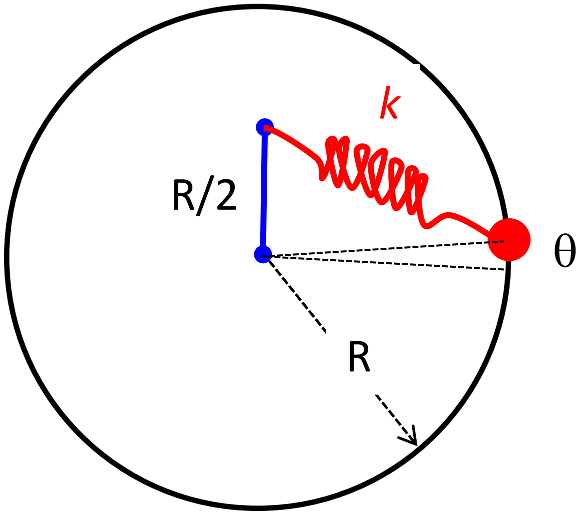
Principle used in many thermometers and thermostat “temperature controllers”

Bimetallic Strip



Example: A bead of mass m is free to move without friction on a vertical hoop of radius R.

The beam moves on the hoop, experiences gravity and a spring of spring constant k, which has one end attached to a pivot a distance $R/2$ above the center of the hoop.

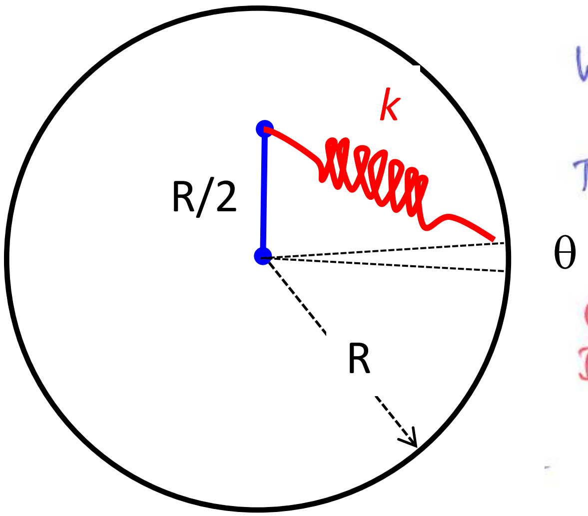


If the spring is *not* extended when the bead is at the top of the circle then:

(a) Find the potential energy of the bead as a function its angular position, measured from center of the circle – draw the potential energy diagram of $V(\theta)$

(b) What minimum K.E. must the bead have at the top to go all the way around the hoop?

(c) If the bead starts from the top with this kinetic energy what force does the hoop exert on it at the top and bottom points of the hoop ?



We can solve this problem using principle of energy conservation:

Total energy (i) Gravitational potential energy

(ii) Spring potential energy

Gravitational PE

Define the top position as the "zero reference" of the gravitational PE.

↳ Gravitational PE at bottom of ring $= -mgh \equiv -mg(2R)$

At any general position around the ring $h = R - R\cos\theta = R(1-\cos\theta)$, so potential energy due to gravity is given by $E_{grav.}(\theta) = -mgR(1-\cos\theta)$

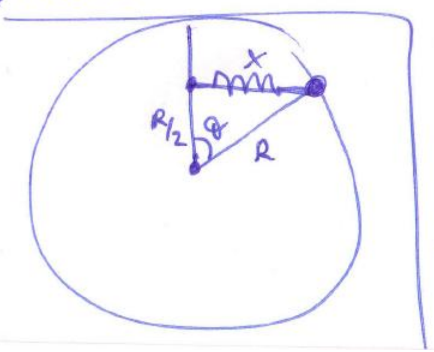
Spring PE

$E_{spring} = \frac{1}{2}k(x-x_0)^2$ (SHO) $\Rightarrow E_{spring} = \frac{1}{2}k(x - R/2)^2$

x - stretched length of spring

$R/2$ - equilibrium (unstretched) length

⇒ Need to find x



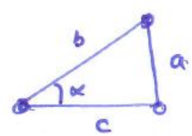
$x^2 = \left(\frac{R}{2}\right)^2 + R^2 - 2\left(\frac{R}{2}\right)R\cos(\theta)$

From law of cosines

$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$

$x^2 = \frac{5}{4}R^2 - R^2\cos(\theta)$

$\Rightarrow x = \frac{1}{2}R\sqrt{5 - 4\cos(\theta)}$



We can write this as: $x = \frac{1}{2}R\sqrt{5 - 4\cos(\theta)} = \frac{1}{2}R\sqrt{1 + 4(1 - \cos(\theta))}$

So, our spring PE function is $E_{spring}(\theta) = \frac{1}{2}K\left(\frac{1}{2}R\sqrt{1 + 4(1-\cos(\theta))} - R/2\right)^2$

$$E_{spring}(\theta) = \frac{1}{2}K\left(\frac{R^2}{4}\right)\left[\sqrt{1+4(1-\cos\theta)} - 1\right]^2$$

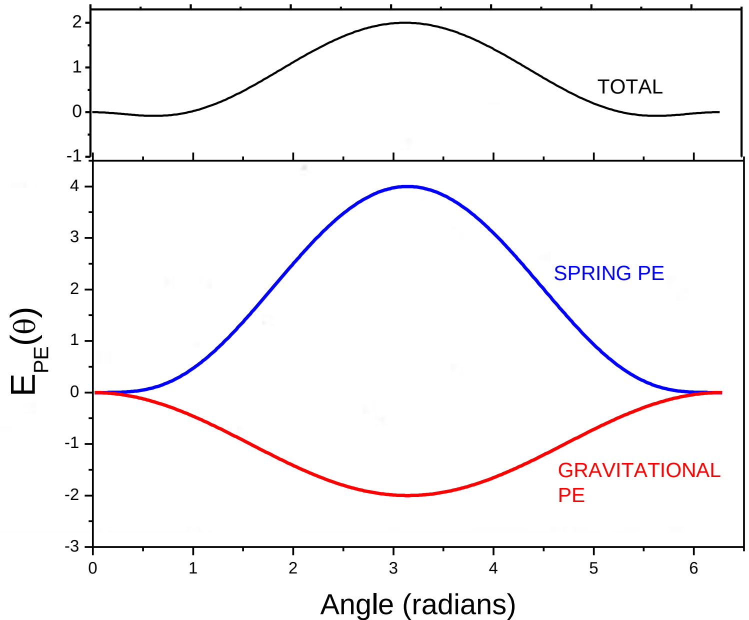
$$E_{spring}(\theta) = \frac{1}{8}KR^2\left[\sqrt{1+4(1-\cos\theta)} - 1\right]^2$$

Total PE is then

$$\underline{E_{PE,tot}(\theta) = E_{spring}(\theta) + E_{grav.}(\theta) = \frac{1}{8}KR^2\left[\sqrt{1+4(1-\cos\theta)} - 1\right]^2 - mgR\left[1 - \cos\theta\right]}$$

ANS (i)

(ii) Max KE for bead to go all around the loop.



$E_{PE,tot}$ is maximum at $\theta = \pi$ (see plot)

Therefore $KE > \frac{1}{2}KR^2 - 2mgR$

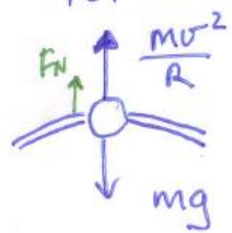
If this is exactly fulfilled then bead has this energy at top of loop

(iii) Forces? TOP + BOTTOM of HOOP

TOP

If the bead has exactly this KE (ie, $\frac{1}{2}KR^2 - 2mgR$) at top of loop then the forces acting on the bead will be CENTREPETAL ACC^n x mass - outwards + Weight acting down + Normal force from ring F_N.

TOP OF RING



① —— $mg - F_N = \frac{mv^2}{R}$ FORCES BALANCE

② — $KE = \frac{1}{2}mv^2 = \frac{1}{2}KR^2 - 2mgR$ ENERGY BALANCE

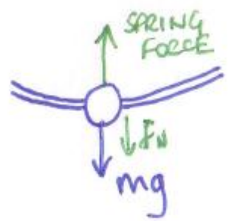
$\Rightarrow \quad mv^2 = KR^2 - 4mgR, \quad \Rightarrow \quad mg - F_N = KR - 4mg$

$\Rightarrow \quad F_N = 5mg - KR$ Answer

BOTTOM

At the bottom of the hoop, the original KE just equals the PE

$E^{tot}_{pot}(\theta) = 0$ when $\theta = \frac{3\pi}{2}$



$F_N + mg = KR \quad \Rightarrow \quad F_N = KR - mg$. Answer

M3.3 Momentum Impulse and Collisions

Many problems are very difficult if you just try to directly apply Newton´s 2nd Law

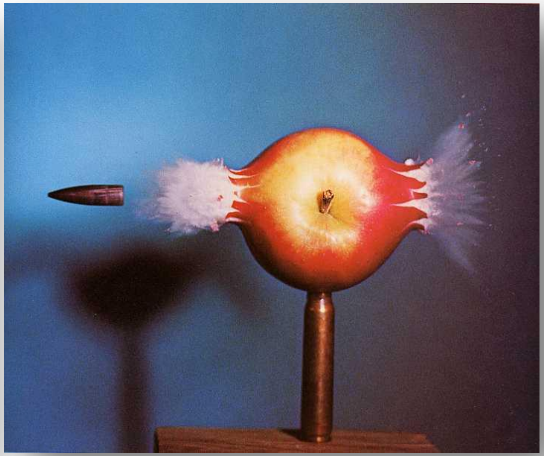
→ A car crashes head on with a truck

→ Playing billiards, snooker and pool

→ A meteorite collides with earth

Momentum and Impulse

- We know from Newton´s 2nd law that force is given by $\vec{F} = m\left(\frac{d\vec{v}}{dt}\right) = \frac{d(m\vec{v})}{dt}$
  - Force is defined by the rate of change of a quantity $\vec{p} = m\vec{v}$, that is defined as the **linear momentum** (***Impuls***)
  - Momentum is a <u>vector quantity</u>
    - Car driving north at 20m/s has *different* momentum from one driving east at 20m/s
  - In every inertial reference frame, we can define the net force acting on a particle as the rate of change of its linear momentum $\sum \vec{F} = \frac{d\vec{p}}{dt}$


- Momentum and kinetic energy both depend on the mass and velocity of the particle...

- Besides **p** being a vector and E_{KE} a scalar quantity, to see the physical difference between them we define a new quantity, closely related to the momentum

  - The impulse **J** $\vec{J} = \sum \vec{F}(t_2 - t_1) = \sum \vec{F}.\Delta t$

Vector quantity that is the net force acting on a body x the time that it acts for

Unit N.s = 1kg m /s2 x s= 1 kg m / s

- So, what is impulse **J** good for ?
  - Suppose that the net force is constant , i.e. ΣF=const, then d**p**/dt = const (NEW-II)
  - We can then write ⇨ $\sum \vec{F} = \dfrac{\vec{p}_2 - \vec{p}_1}{t_2 - t_1}$ or $(t_2 - t_1)\sum \vec{F} = \vec{p}_2 - \vec{p}_1 = \vec{J}$

$$\boxed{\vec{J} = \vec{p}_2 - \vec{p}_1}$$

IMPULSE MOMENTUM THEOREM

The change in linear momentum of a particle during a time interval equals the impulse of the net force that acts on the particle during that interval

• The impulse momentum theorem also holds when forces are not constant, to see this integrate both sides of Newton´s 2nd law over time between the limits t_1 and t_2

In this case $$\int_{t_1}^{t_2} \sum \vec{F} dt = \int_{t_1}^{t_2} \left(\frac{d\vec{p}}{dt} \right) dt = \int_{\vec{p}_1}^{\vec{p}_2} d\vec{p} = \vec{p}_2 - \vec{p}_1$$

This is the general definition of the impulse

$$\boxed{\vec{J} = \int_{t_1}^{t_2} \sum \vec{F} dt = \vec{p}_2 - \vec{p}_1}$$ IMPULSE MOMENTUM THEOREM

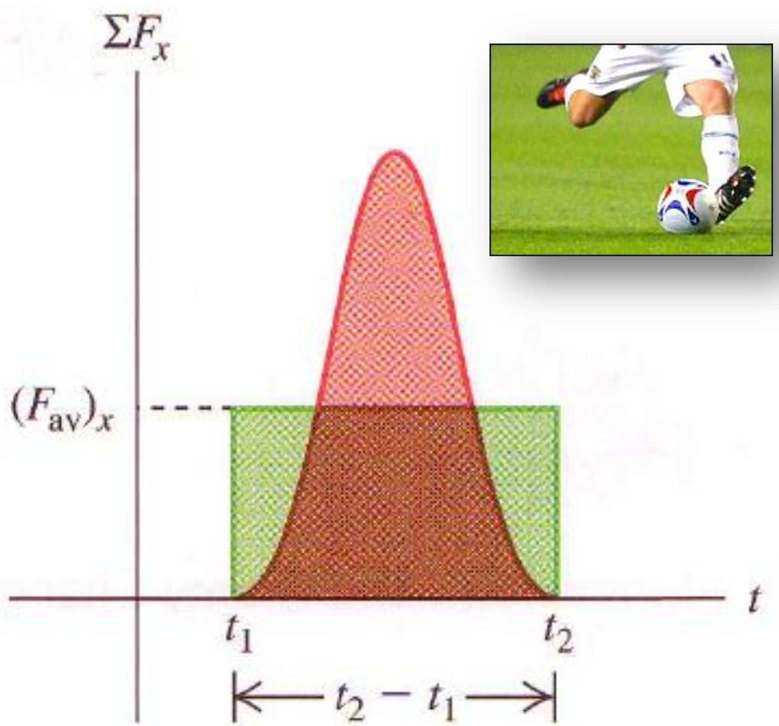


Image shows the typical F(t) when kicking a football

The average force (F_{av}) is such that $\vec{J} = F_{av}(t_2 - t_1)$

such that the area under the $F_{av}(t)$ and F(t) curves are identical

Momentum and Kinetic Energy

The impulse momentum theorem highlights a fundamental difference between momentum, which depends on velocity and kinetic energy, which depends on speed

$\boxed{\vec{J} = \vec{p}_2 - \vec{p}_1}$

"changes of a particles momentum is due to *impulse*"

***Impulse = Force x** Time*

$\boxed{W_{tot} = E_{kin,2} - E_{kin,1}}$

"changes of a particles energy is due to *work*"

*Work**= Force x Displacement***

Consider a particle that starts from rest (initial momentum $\mathbf{p}_1 = m\mathbf{v}_1 = \mathbf{0}$, initial KE = $1/2mv^2=0$)

It is now acted on by a constant force **F** from time t_1 to t_2, and it moves through a displacement **s** in the direction of the force

The particles momentum at time t_2 is $\vec{p}_2 = \vec{p}_1 + \vec{J} = \vec{J}$

The particles KE at time t_2 is $W_{tot} = Fs$

An illustration of the distinction between momentum and KE

Which ball would you rather catch ?

m=0.5kg

$\vec{v} = 4ms^{-1}$

$\vec{v} = 20ms^{-1}$

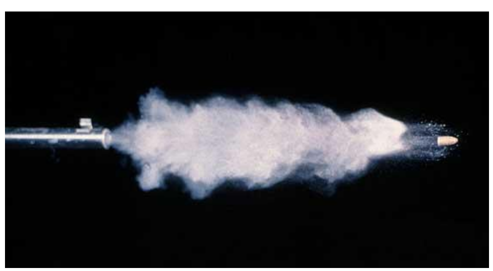
m=0.1kg

MOMENTUM (kgm/s) $p=mv$	K. ENERGY (J) $E_{KE}=1/2m\mathbf{v}^2$
2	8
2	40

Since the change of momentum of both balls is the same, you need to provide the same impulse with your hand to stop the ball → For a given force it takes the <u>same time</u> to stop

But, your hand has to do 5x more work with the golf ball, i.e. Your hand gets pushed back 5 times further c.f. the football.

M=12g
$|v|=130ms^{-1}$

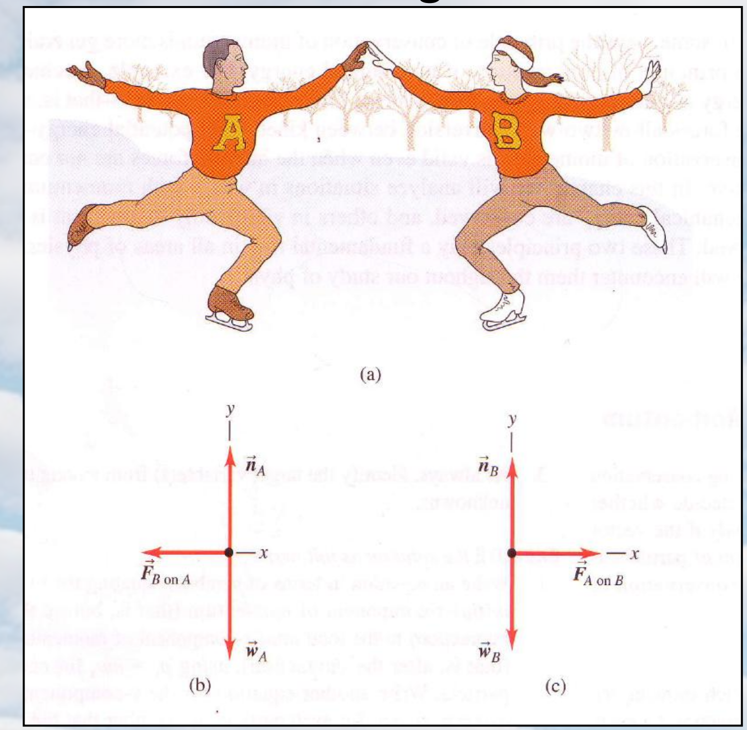


You CHOOSE !

M=142g
$|v|=45ms^{-1}$

Momentum, like energy, is a conserved quantity

- The concept of momentum is especially important when we consider two or more interacting bodies



We differentiate between **internal** and **external** forces

Internal forces

$$\vec{F}_{B-on-A} = \frac{d\vec{p}_A}{dt} = -\vec{F}_{A-on-B} = -\frac{d\vec{p}_B}{dt}$$

$$\frac{d\vec{p}_A}{dt} + \frac{d\vec{p}_B}{dt} = 0 = \frac{d}{dt}\left(\vec{p}_A + \vec{p}_B\right)$$

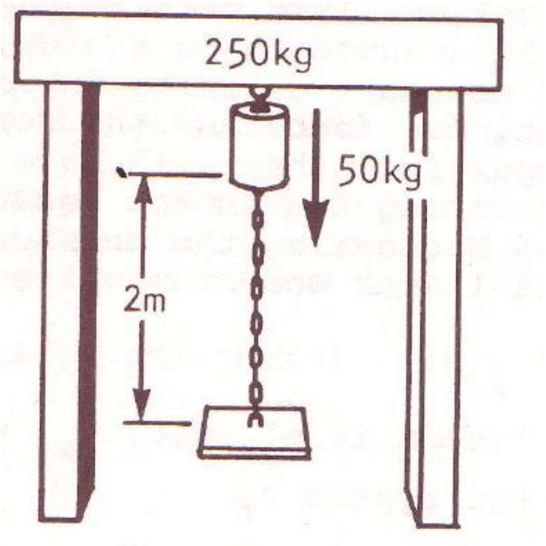
$$\Rightarrow \boxed{\vec{P}_{tot} = \vec{p}_A + \vec{p}_B = const}$$

If the vector sum of external forces acting on a closed system is zero, then the total momentum of that system is a constant of the motion

Direct consequence of Newton-III but useful since it doesn't depend on the precise nature of the internal forces

Example: Elastic Collision and Conservation of Momentum

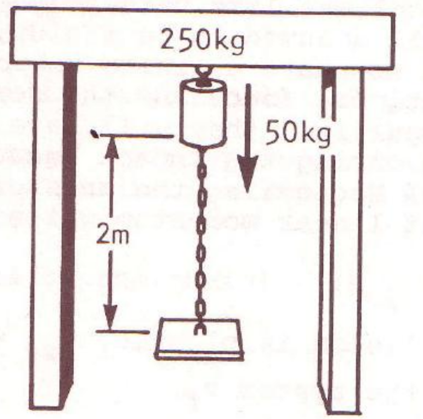
To test the ability of a chain to resist impact it is hung from a 250kg block. The chain also has a metal plate hanging from it´s end as shown below. A 50kg weight is released from a height 2m above the plate and it drops to hit the plate.



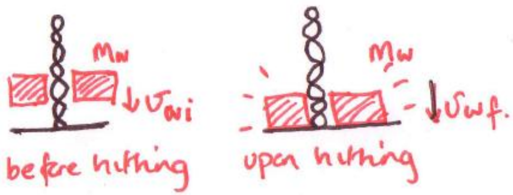
Find the impulse exerted by the weight *if the impact is perfectly elastic and the block is supported by :*

(a) Two perfectly rigid columns
(b) Perfectly elastic springs

Finally, for part (c) of the question find the energy absorbed by the chain in cases (a) and (b) above.



...e impulse momentum relationship states that the impulse on the chain ...ill be equal and opposite to the change of momentum of weight.



⇒ Denote subscripts "b" and "w" to designate block and weight, respectively

⇒ Downward direction (velocities) are +tive.

(a) Columns are perfectly rigid ⇒ Entire system has zero velocity after impact

↳ Change of momentum of weight $\quad P_{wf} - P_{wi} = m_w(v_{wf} - v_{wi}) \xRightarrow{v_{wf}=0} P_{wf} - P_{wi} = -m_w v_{wi}$

↳ Impulse on chain is $\quad -(P_{cf} - P_{ci}) = m_c v_{ci}$ $\quad$ EQUAL + OPPOSITE TO IMPULSE ON WEIGHT ①

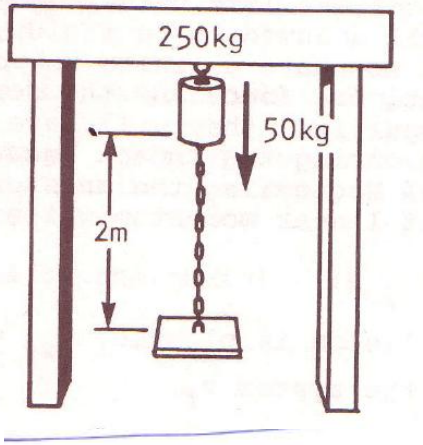
↳ At the point of impact $\quad K_{wi} = \frac{1}{2} m_w v_{wi}^2 = m_w g h \quad \Rightarrow \quad v_{wi} = \sqrt{2gh}$ ②

↳ ① + ② We obtain $\quad m_w v_{wi} = m_w \sqrt{2gh} = 50 \cdot \sqrt{2.(9.81) \times 2} = \underline{\underline{313.21 NS}}$ ANS.

(b) With the columns like perfectly elastic springs, the block-chain weight system will have a finite velocity after collision

(Neglecting mass of chain) $\quad m_w v_{wi} + m_b v_{bi} = m_w V_{wf} + m_b v_{bf}$ $\quad$ Momentum conservation.

Collision now is "plastic" ⇒ $v_{wf} = v_{bf}$ ⇒ call this v_F the final velocity of system.
"weight does not rebound"



Knowing that $U_{wi} = \sqrt{2gh}$ and that $U_{bi} = 0$, we obtain

$$m_w\sqrt{2gh} + m_b(0) = m_w U_f + m_b U_f$$

Solving for U_F => $\boxed{U_F = \frac{m_w\sqrt{2gh}}{m_w + m_b}}$

The impulse exerted on the chain is: $-(P_{wf} - P_{wi}) = -(m_w U_f - m_w U_i) = m_w(U_i - U_f)$

Therefore;

$$-(P_{wf} - P_{wi}) = m_w\left[\sqrt{2gh} - \frac{m_w\sqrt{2gh}}{m_w + m_b}\right] = m_w\left[\sqrt{2gh}\cdot\frac{m_b}{m_b + m_w}\right]$$

$$= 50kg \times \sqrt{2.(9.81 ms^{-2})2m} \times \left(\frac{250kg}{250kg + 50kg}\right) = \underline{\underline{261 NS}} \text{ ANS.}$$

(C) Since no change of the POTENTIAL ENERGY of the block or weight occurs during impact, from the conservation of energy we have.

$$\Delta E = \text{energy absorbed by chain} = (K_{wi} - K_{wf}) + (K_{bi} - K_{bf})$$

CASE(a) $K_{wf} = 0$, $K_{bi} = 0$ and $K_{ci} = m_w g h$, Hence

$$\Delta E = m_w g h = 50kg\,(9.81\, ms^{-2}).\, 2.0m = 981J$$

For case (b)

$K_{bf} = \frac{1}{2} m_b v_F^2$

$K_{bi} = 0$ / block initially stationary)

$K_{wi} = m_w g h$ (as before)

$K_{wf} = \frac{1}{2} m_w v_F^2$

KE terms for block and weight

where $v_F = \frac{m_w \sqrt{2gh}}{(m_w + m_b)}$ deduced in answer to (b).

$\Rightarrow$ Insert in

$$\Delta E = (K_{wi} - K_{wf}) + (K_{bi} - K_{bf})$$

$$\Delta E = m_w g h - \frac{1}{2} m_w \left[\frac{m_w \sqrt{2gh}}{m_w + m_b}\right]^2 + \left(-\frac{1}{2} m_b \left(\frac{m_w \sqrt{2gh}}{m_w + m_b}\right)^2\right)$$

$$= m_w g h - \frac{m_w^2 g h}{(m_w + m_b)^2}\left[m_w + m_b\right]$$

$$= m_w g h \left[1 - \frac{m_w}{(m_w + m_b)}\right] = m_w g h \left[\frac{m_b}{m_w + m_b}\right]$$

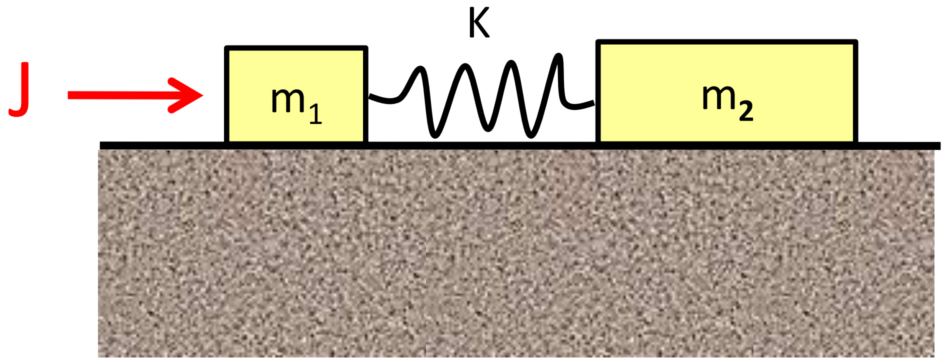
$$\Delta E = \left[\frac{m_w m_b}{m_w + m_b}\right] g h = \frac{(50)(250)}{(300)} \times 9.81 \times 2.0 = \underline{\underline{817.5\,J}}$$

ANS.

As expected, the energy absorbed by the chain is less in the case when the support is damped

Homework 3

Two bodies of masses m_1 and m_2 are free to move along a horizontal straight, frictionless track. They are connected by a spring with constant K.



The system is initially at rest before an instantaneous impulse **J** is give to m_1 along the direction of the track.

Q) Determine the motion of the system and find the energy of oscillation of the bodies

Summary of lecture 3

- **Work and Energy** *(Arbeit und Energie)*
  - Work = Energy = **Force** x **Distance**
  - **Kinetic energy** = work required to accelerate a particle from rest to a velocity v

$$E_{kin} = \frac{1}{2} M v^2$$

  - **Potential energy** = energy defined in the conservative field of a force
    - **Force** defined by gradient of potential energy functional
    - Potential energy stability diagrams

$$\vec{F}(\vec{r}) = -\vec{\nabla} E_{pot}(\vec{r})$$

  - **Conservative forces** = work-kinetic energy relationship is completely reversible, dissipation is negligible

$$Q_{12} = \Delta E_{pot} + \Delta E_{kin}$$

  - Energy in a **simple harmonic oscillator**

- **Momentum and Impulse** *(Impuls und Impuls Übertrag)*
  - **Momentum** of a particle is defined by **p**=m**v**
  - **Impulse Momentum Theorem J = Δp = Force** x Time
  - **Momentum is a conserved quantity** when no external forces act.