Experimental Physics I in English

Classical mechanics, Waves and Fluids

Special Relativity

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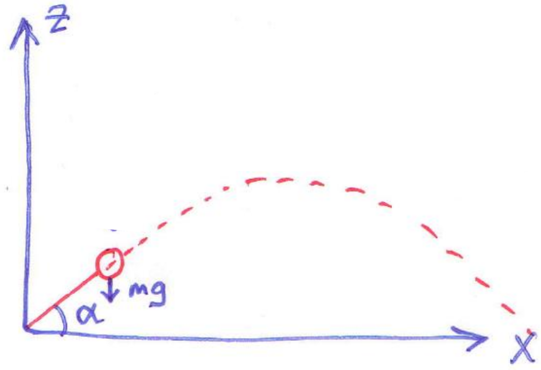
Improve your golf...

- Imagine that a point projectile is thrown with a velocity $\mathbf{v}_0$, making an angle α with the ground and that it moves under the influence of gravity



*First job is to put together an **equation of motion***

$$\vec{F} = m\vec{a} = \vec{F}_g = -m\vec{g}\,\hat{e}_z$$

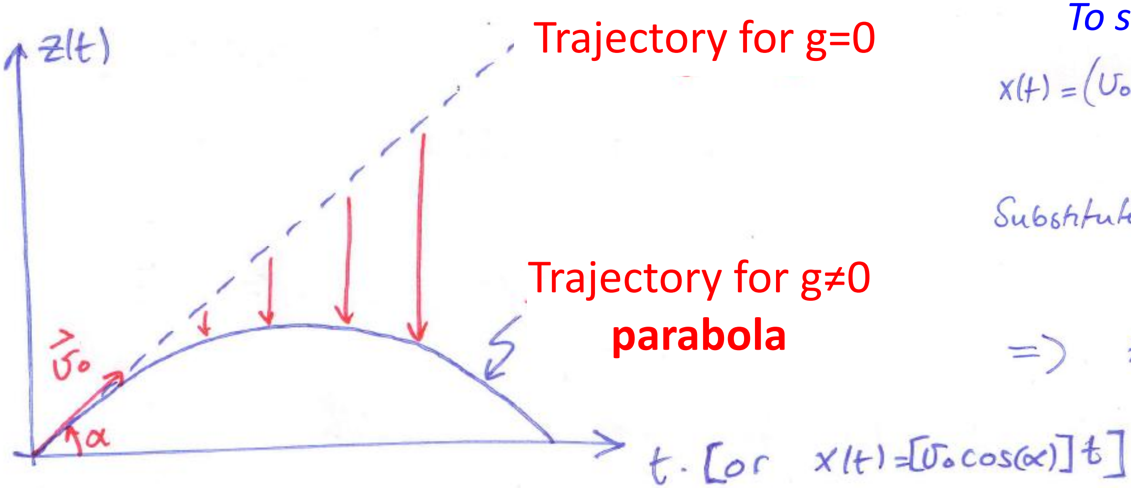
From equation of motion →

$$\vec{r}(t) = \begin{pmatrix} v_0 t \cos(\alpha) \\ 0 \\ v_0 t \sin\alpha - \frac{g t^2}{2} \end{pmatrix}$$

$$x(t) = (v_0 \cos\alpha)\, t$$
$$y(t) = const = 0$$
$$z(t) = -\frac{1}{2} g t^2 + (v_0 \sin\alpha)\, t$$

Now we have x(t) and z(t) we can calculate velocity components by differentiation

$$v_x(t) = \dot{x}(t) = v_0 \cos(\alpha) = v_{0x} = \underline{\underline{Const.}}$$
$$v_z(t) = \dot{z}(t) = -gt + v_0 \sin\alpha = v_{0z} - gt$$



To see that the form is a parabola

$$x(t) = (v_0 \cos\alpha)\, t \implies t = \frac{x(t)}{v_0 \cos\alpha}.$$

Substitute in $z(t) = -\frac{1}{2} g t^2 + (v_0 \sin\alpha)\, t$

$$\implies z(x) = \left(\frac{\sin\alpha}{\cos\alpha}\right) x - \frac{1}{2} \frac{g}{v_0^2 \cos^2\alpha} x^2 \qquad \underline{\underline{Parabola.}}$$

Maximum range?

Applications

(i) Maximum range $X_{max} \implies z(X_{max}) = 0$

$$\implies X_{max} = \frac{2 v_0^2}{g} \cos(\alpha)\sin(\alpha) = \sin(2\alpha) \frac{v_0}{g}$$

MAXIMUM FOR $\alpha = 45°$ $(2\alpha = 90°)$.

Maximum height?

When $v_z(t) = 0$ we have reached maximum height

$$\implies gt = v_0 \sin\alpha \implies t_{max} = \frac{v_0 \sin\alpha}{g}$$

$$\implies z_{max} = z(t_{max}) = -\frac{1}{2} g \left(\frac{v_0 \sin\alpha}{g}\right)^2 + \frac{(v_0 \sin\alpha)^2}{g} = \frac{1}{2}\frac{(v_0 \sin\alpha)^2}{g}$$

↳ Max for $\sin\alpha = 1$, ie $\alpha = 90°$ Throwing straight up!

Generalized Circular Motion

The magnitude of any vector, u, is simply: $$u = |\vec{u}| = \sqrt{\vec{u}\cdot\vec{u}} = \sqrt{u_x^2 + u_y^2 + u_z^2}$$

Suppose we have a particle moving on a path r(t) such that: $r = C$, where C is a constant. (Magnitude is constant in time, so motion on a circle or sphere.)

Then we also have: $$C^2 = r^2 = |\vec{r}|^2 = \vec{r}(t)\cdot\vec{r}(t)$$

Taking derivative with respect to (wrt) time:

$$0 = \frac{d}{dt}\left[\vec{r}(t)\cdot\vec{r}(t)\right] = 2\frac{d\vec{r}(t)}{dt}\cdot\vec{r}(t)$$

Or, that the velocity vector and position vector are orthogonal:

$$\boxed{\vec{v}(t)\cdot\vec{r}(t) = 0}$$ (**always** true for **any** circ. motion)

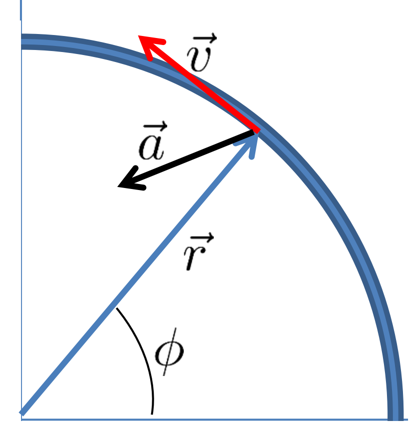
No surprises here, since we did this last week. Now, consider the **special case** when the magnitude of the velocity vector is also constant in time. By the same steps as above, we will have:

$$0 = \frac{d}{dt}\left[\vec{v}(t)\cdot\vec{v}(t)\right] = 2\frac{d\vec{v}(t)}{dt}\cdot\vec{v}(t)$$

Or, in other words: $$\boxed{\vec{a}(t)\cdot\vec{v}(t) = 0}$$ (accel. orthogonal to vel.)

What happens when velocity magnitude is not constant?

Start with: $\vec{v}(t)\cdot\vec{r}(t) = 0$ (**always true** for **<u>any</u>** circular motion)



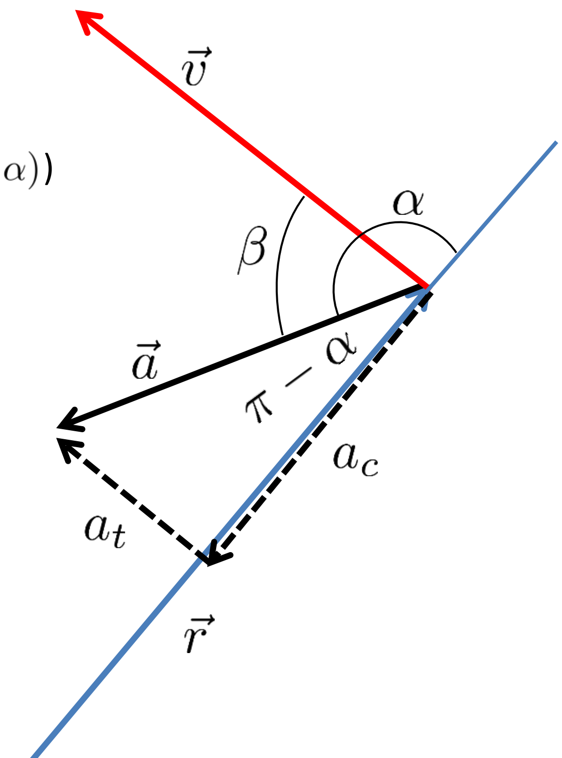
$$\Rightarrow \frac{d}{dt}\left[\vec{v}(t)\cdot\vec{r}(t)\right] = 0$$

$$\Rightarrow \frac{d\vec{v}(t)}{dt}\cdot\vec{r}(t) + \vec{v}\cdot\frac{d\vec{r}(t)}{dt} = 0$$

$$\vec{a}(t)\cdot\vec{r}(t) = -v^2 = ar\cos\alpha$$

$$\Rightarrow a\cos(\pi-\alpha) = +\frac{v^2}{r} \equiv a_c$$

(+ sign because $\cos\alpha = -\cos(\pi-\alpha)$)



Next, consider time derivative of velocity **magnitude**:

$$\frac{d\mid\vec{v}\mid}{dt} = \frac{d}{dt}\sqrt{\vec{v}(t)\cdot\vec{v}(t)} = \frac{\vec{a}(t)\cdot\vec{v}(t)}{v}$$

$$\Rightarrow \frac{dv}{dt} = a\cos\beta \equiv a_t$$

Circular Motion Summary

For **any** motion on a sphere or circle, the fundamental result $\vec{v}(t) \cdot \vec{r}(t) = 0$ **always** holds.

For non-uniform (meaning $|\vec{v}| \neq C$, C a constant) motion, the total acceleration **vector** $\vec{a}$ can be described by a centripital acceleration plus a tangential acceleration:

$$\vec{a} = a_c \hat{r} + a_t \hat{v}$$

Where:

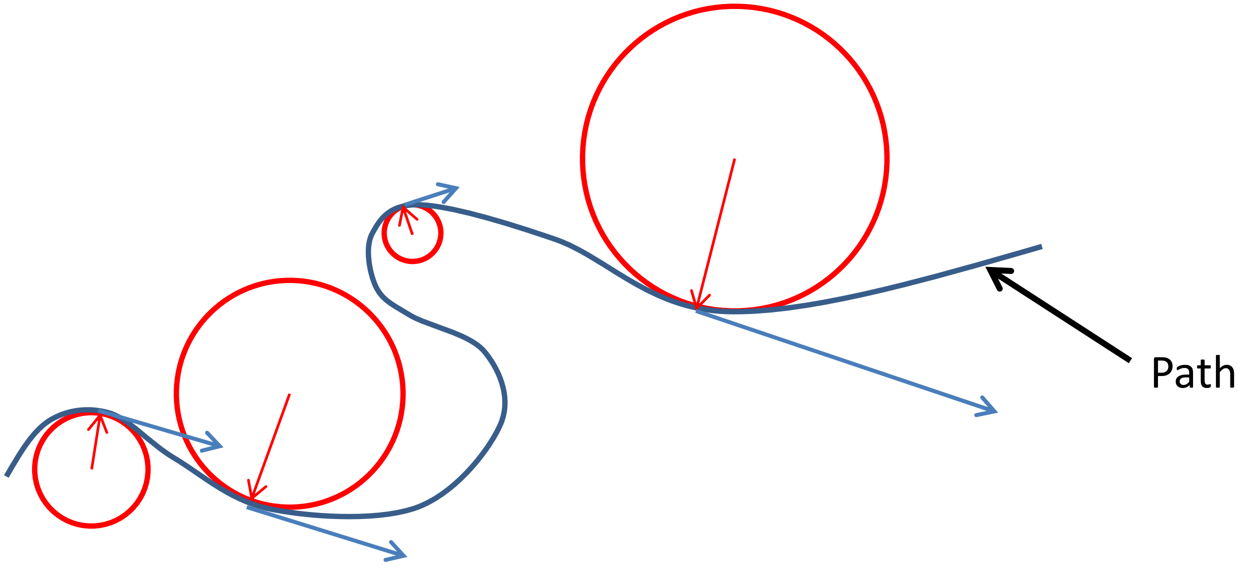
$$a_c = \frac{v^2}{r}$$

And,

$$a_t = \frac{dv}{dt}$$

$$v \equiv |\vec{v}| = \sqrt{\vec{v} \cdot \vec{v}}$$

Beauty of this result: Any smoothly curved path can be "locally" represented by a circular approximation. This approximation becomes exact in the limit that $\Delta t \rightarrow 0$.



This lets us solve for the equations of motion in terms of the radial and tangential components of the path.

The we will see this in an upcoming example with an inclined plane.

Examples of Circular Motion

Simple Pendulum (non-uniform)

"Wall of Death" (crazy)

Some Fun Examples!

A *more tricky* problem

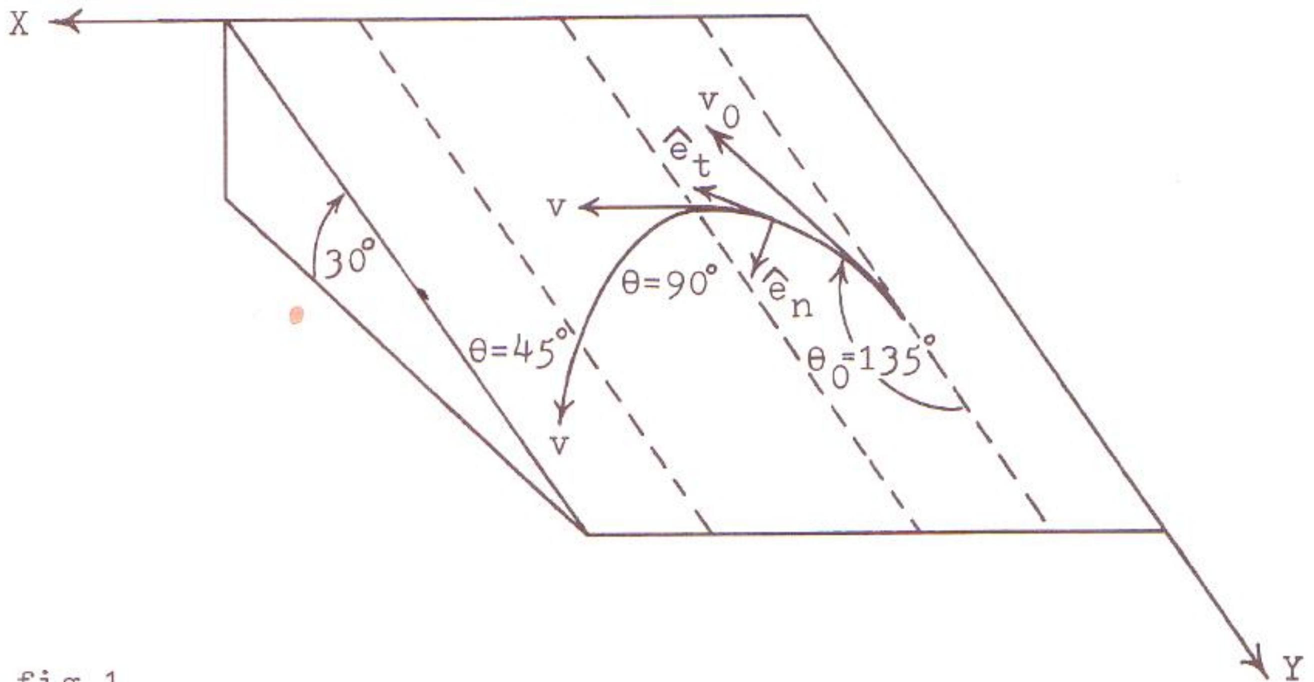


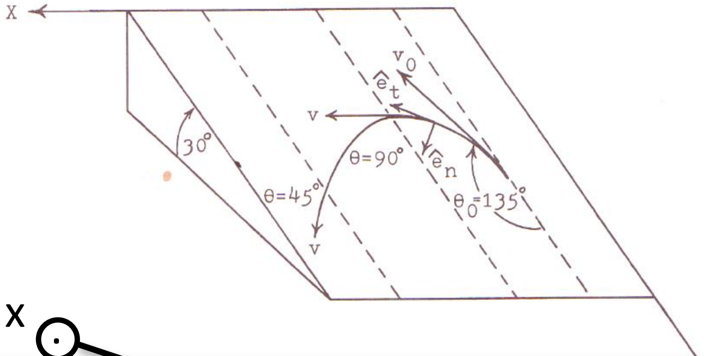
fig 1

Example : as shown on the figure a particle is projected up an inclined plane with an initial velocity $|v_o|$=100cm/s at an angle θ_0=135 degrees from the y-axis

a) Calculate the force with which the particle presses on the plane...

b) Neglecting frictional effects use Newton's laws to calculate the particle velocity when θ has the values 90, 45 and 0 degrees...

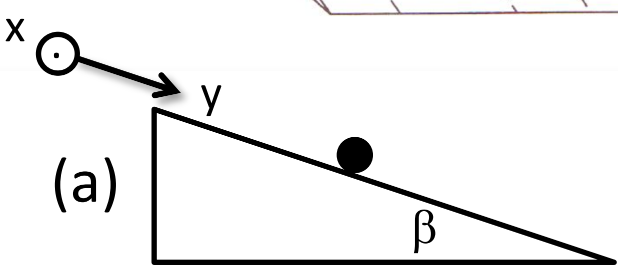
c) If you took the experiment to the moon, would your results be different ?



SOLUTION

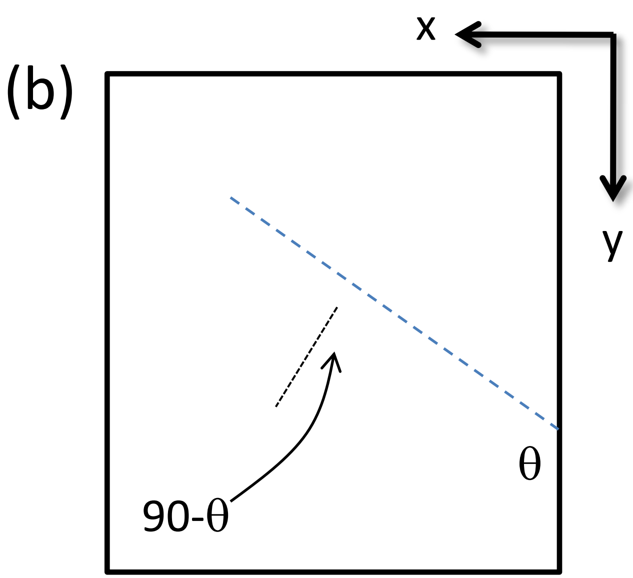
The first thing to notice is that the motion takes place entirely within a single plane (here the x,y plane)

→ Forces can easily be resolved into components normal (F_n) to and tangential (F_t) to the trajectory taken



The solution to (a) is easy:
the particle stays in the plane → no resulting force can act (Newton 3)

$$\rightarrow \quad N = mg\cos\beta$$



The solution to (b) is more difficult:

Start by resolving the forces in the x,y plane into components perpendicular and parallel to the ***TRAJECTORY***

$$F_t = mg\sin(\beta)\cos(\theta)$$

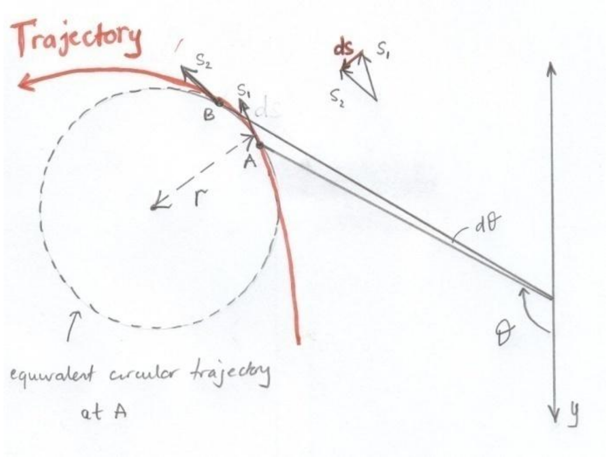
$$F_n = mg\sin(\beta)\cos(90-\theta) = mg\sin(\beta)\sin(\theta)$$

We can now write two equations of motion (i) along the trajectory and (ii) perpendicular to it

*We have used path coordinates **s, r***

$$F_t = mg\sin(\beta)\cos(\theta) = m\frac{dv}{dt} = m\ddot{s}$$

$$F_n = mg\sin(\beta)\sin(\theta) = \frac{mv^2}{r} = \frac{m(\dot{s})^2}{r}$$



The equations of motion in these path co-ordinates become →

$$\frac{dv}{dt} = g\sin(\beta)\cos(\theta)$$

$$\frac{v^2}{r} = g\sin(\beta)\sin(\theta)$$

And since $\frac{dv}{ds}\frac{ds}{dt} = \frac{dv}{ds}v$ and $r = -\frac{ds}{d\theta}$ these become:

$$v\frac{dv}{ds} = g\sin(\beta)\cos(\theta) \quad \text{and} \quad v^2\frac{d\theta}{ds} = -g\sin(\beta)\sin(\theta)$$

$$\Rightarrow \quad \frac{1}{v}\frac{dv}{d\theta} = -\cot(\theta)$$

$$\frac{1}{v}\frac{dv}{d\theta} = -\cot(\theta) \quad \xrightarrow{\textit{Integrate}} \quad \int \frac{dv}{v} = -\int \cot(\theta)\,d\theta + \ln C$$

$$\ln(v) = -\ln[\sin(\theta)] + \ln C$$

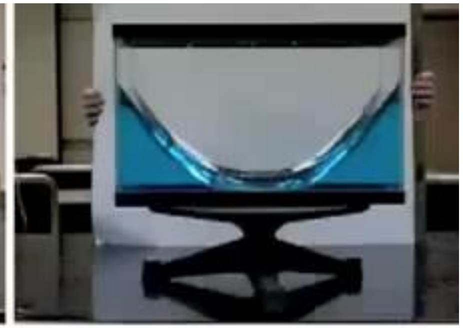
$$v = \frac{C}{\sin\theta}$$

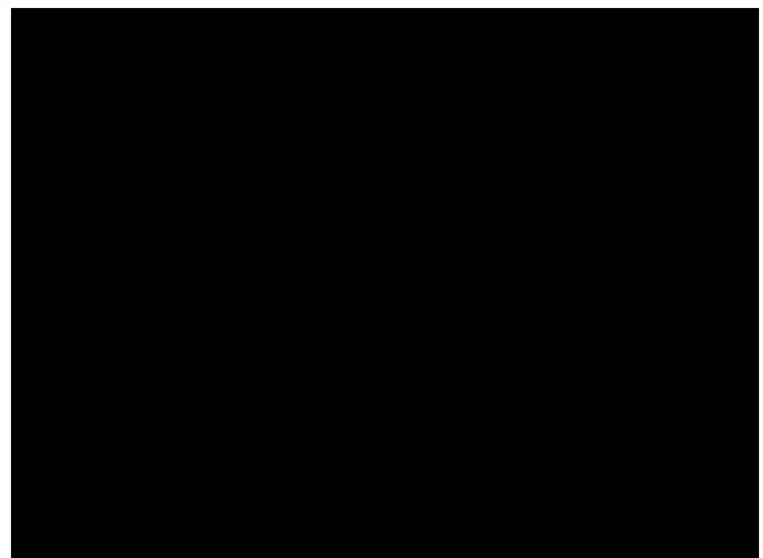
We can find the constant of integration C by inserting the initial conditions, θ=135 degrees when $v=v_0=100$cm/s → C=70.7cm/s

$$v = \frac{70.7}{\sin\theta} cms^{-1}$$

Answer to (c) you would see the same trajectory of motion on the moon → independent of size of g

For HOMEWORK try to do the same problem using Cartesian coordinates !





M2.2 Frames of reference

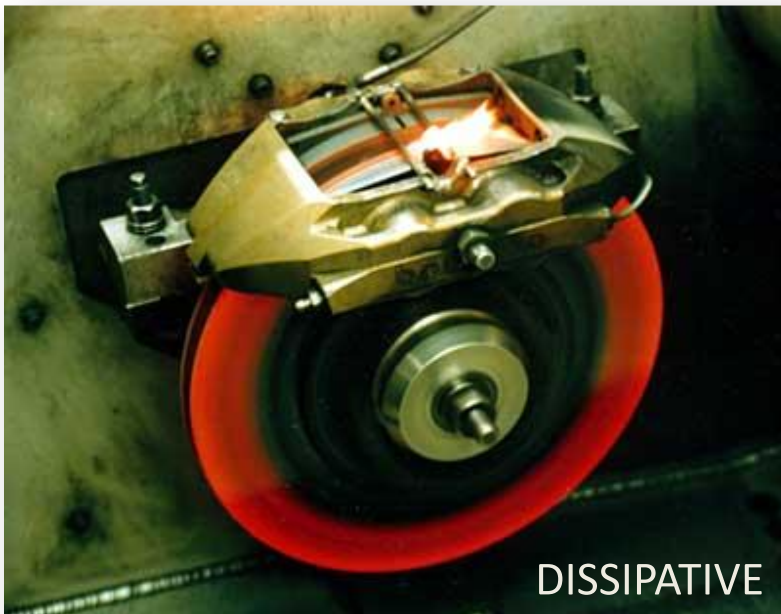
(Bezugssysteme)

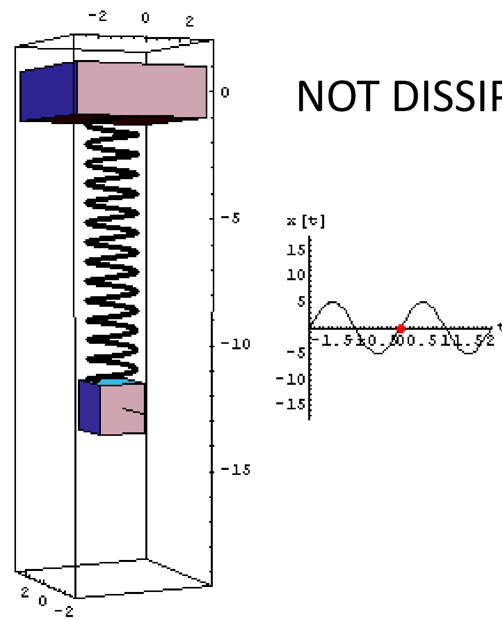
and fictitious forces

(Trägheitskräfte / Scheinkräfte)

2.4 Dissipative Forces

Friction



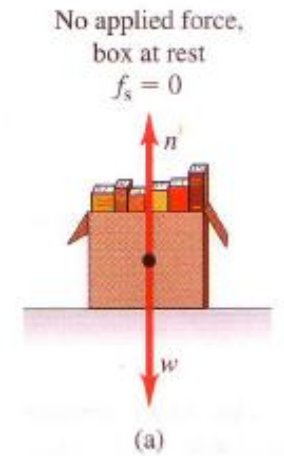


NOT DISSIPATIVE

A **dissipative force** is one that does "work" (work=energy = force x distance) that cannot be recovered later on...

Non-dissipative forces are those that exchange energy between two forms (e.g. Kinetic and potential energy in a mass on a spring...)

Frictional forces
(*Reibungskräfte*)



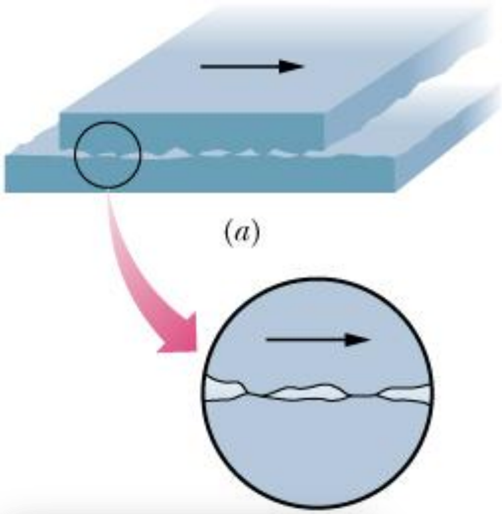
- We are all familiar with the idea of **friction**
  - Whenever two bodies interact by **direct contact** of their surfaces, there is a normal force (Newton III) and also **a *frictional force*** if we try to slide them against each other
  - ***Without*** friction we would not be able to walk, people would fall off bicycles, car engines would If friction did not exist then we could not walk, nails would pull out, bottle-tops would unscrew

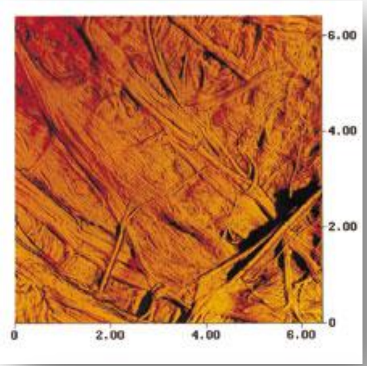
    the world would be a pretty unusual place.
- *Some observations* - Think about sliding a heavy box of books across the floor of your apartment...
  - Depends on the weight of the box (w=mg), i.e. the **normal force** exerted by the floor on the box...
  - If you imagine pulling the box with a rope, but not moving it, there must be a frictional force that acts against you (Newton III), perpendicular to the normal force...
  - Difficult to start the box sliding but, after it is moving, it is comparatively easy to keep it moving...
- ***Where*** does friction actually come from ?

Microscopic origins of friction

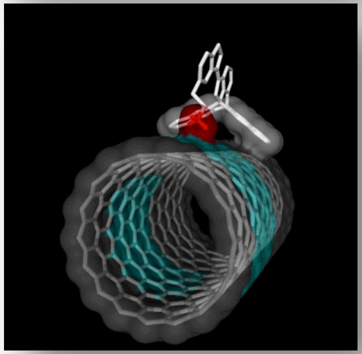
- Fundamentally, the roughness of the two materials results in them "sticking" together and providing a "frictional force" when you try to slide them against each other...

- This "sticking together" due to electrons in one body forming "bonds" with atoms in the other material...
  - *The bonds have to be broken to slide the two materials over each other → gives rise to the frictional force*
  - *Once the two materials are sliding, bonds are continually being broken and then reformed.*
  - *More bonds exist when the two materials are not moving , compared to the situation when they slide against each other.*
  - *Smoother surfaces* ***do not necessary*** *give less friction*





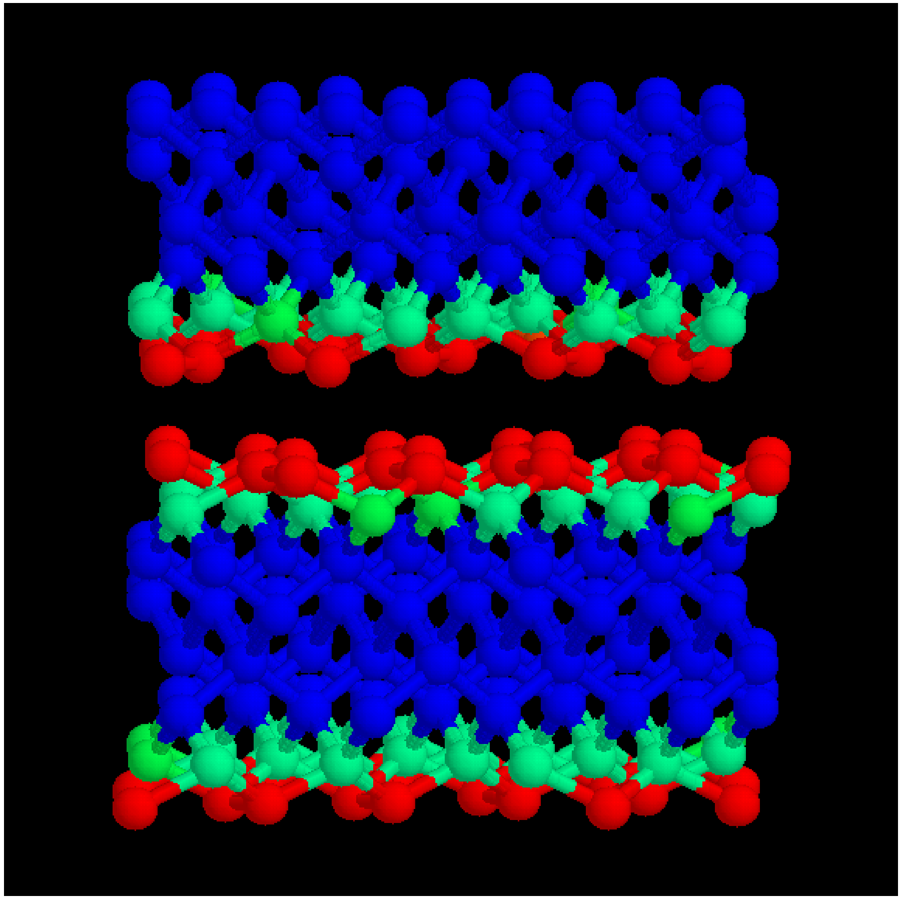
Atomic Force Microscope image of a grain of wood

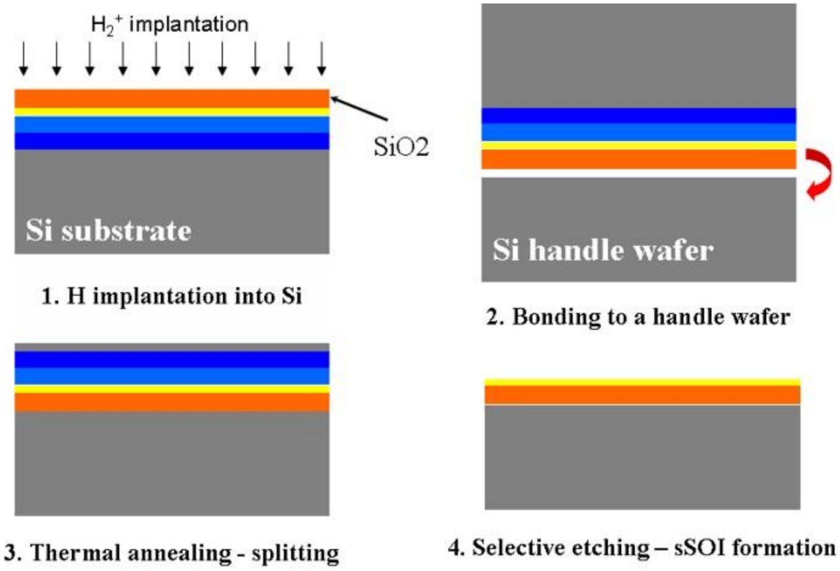


Molecule bonding to a carbon nanotube

When two “atomically smooth” materials come into contact an extremely high number of atoms are in close proximity and **many bonds** can be formed per unit area

→ The two materials “fuse” together is what is known as a “cold weld” (welding = *schweißen*)

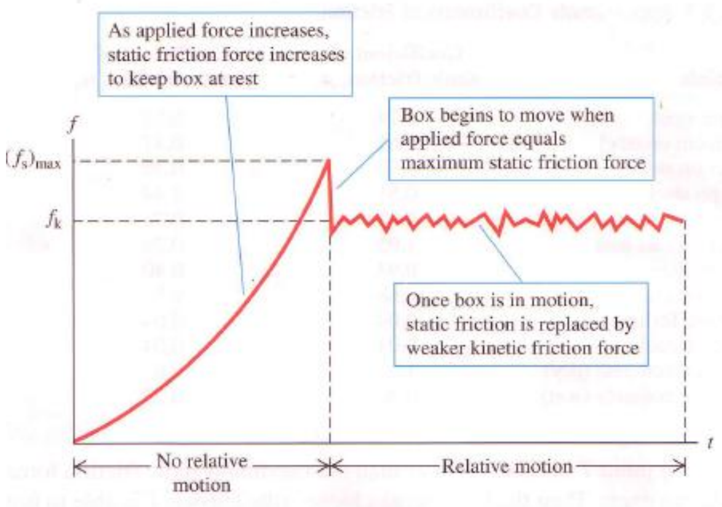




Used extensively to build complicated semiconductor sandwiches in microelectronics and optoelectronics

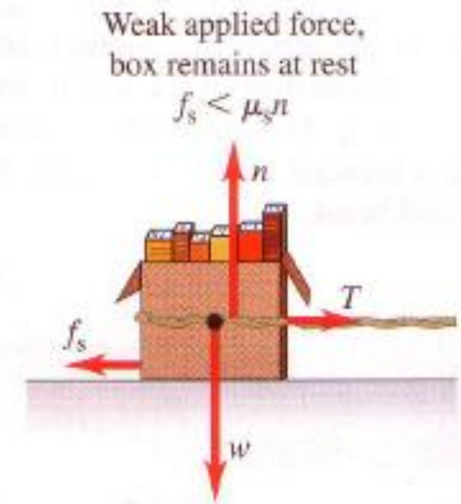
The frictional force

- Think about how the magnitude of the frictional force should change with the applied force $\boldsymbol{T}$



Start from T=0 and slowly increase the pulling force

1) Initially the frictional force f_s must increase as to exactly balance the applied force T
*→ Newton III and box is **not** in motion*

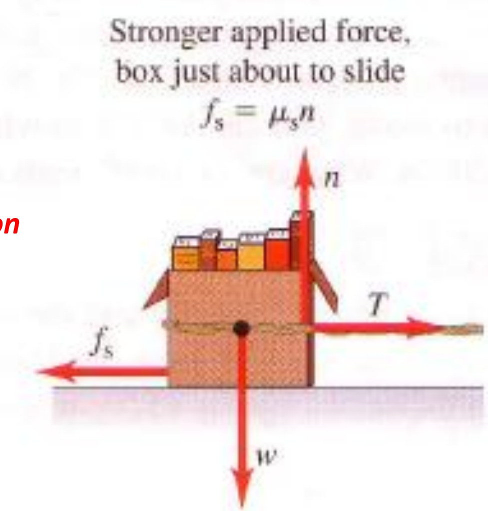


(b)

*2) At some point, the box just begins to move – here the **magnitude** of the frictional force f_s is linearly proportional to the normal force n*

$$f_s = \mu_s . n$$

Material specific parameter
→ Coefficient of static friction



*Note → **direction** of $\mathbf{f_s}$ always perpendicular to **n** and in a sense to oppose the applied force*

*3) Once object is in motion the **magnitude** of the frictional force f_s stays constant and is independent of the magnitude of the pulling force |**T**|*

$$f_s = \mu_k . n$$

→ Coefficient of kinetic friction

Some experimental observations

If an object is *not* moving under the influence of a pulling force due to friction, then $f_s \le \mu_s . n$

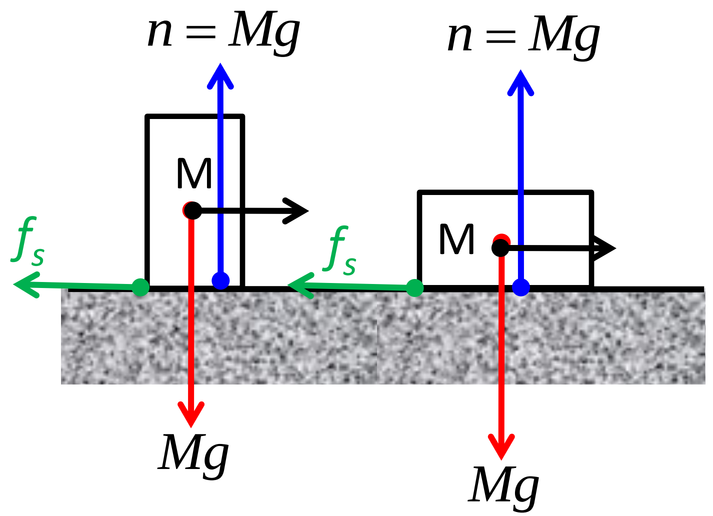
The coefficient of kinetic friction is generally smaller than the static friction coeff. $\mu_K \le \mu_s$

Experiment shows that f_s is independent of the surface area of the object in contact with the surface:

→ *Only the normal force is important – WHY ?*

Table 5.1 Approximate Coefficients of Friction

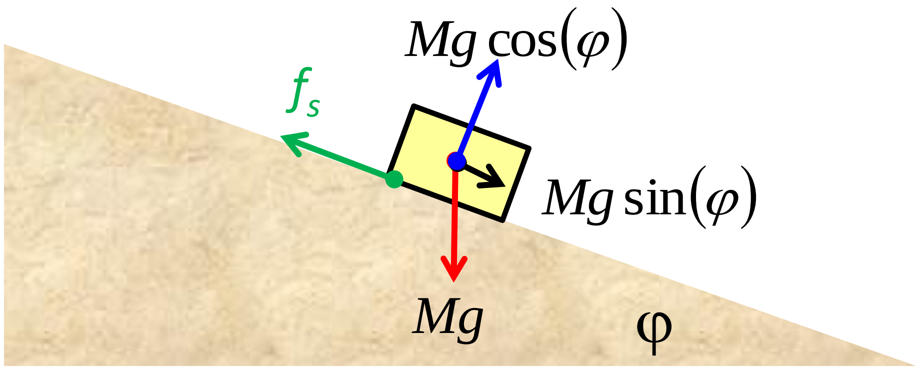
Materials	Coefficient of static friction, μ_s	Coefficient of kinetic friction, μ_k
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Copper on steel	0.53	0.36
Brass on steel	0.51	0.44
Zinc on cast iron	0.85	0.21
Copper on cast iron	1.05	0.29
Glass on glass	0.94	0.40
Copper on glass	0.68	0.53
Teflon on Teflon	0.04	0.04
Teflon on steel	0.04	0.04
Rubber on concrete (dry)	1.0	0.8
Rubber on concrete (wet)	0.30	0.25



HOW TO MEASURE ?

Measuring the static coefficient of friction

μ_s can be measured using an inclined plane



Normal force → $F_N = Mg\cos(\varphi)$

Pulling force → $F_{Pull} = Mg\sin(\varphi)$

Frictional force → $f_S = \mu_S . Mg\cos(\varphi)$

1. *As the angle of the inclined plane φ increases, the component of the weight down the slope will progressively increase until the friction becomes "critical"*

2. *At that point, when the body just begins to move, the frictional force is maximal*

$$f_S = Mg\sin(\varphi_{\max}) = \mu_S Mg\cos(\varphi_{\max})$$

$$\mu_S = \tan(\varphi_{\max})$$

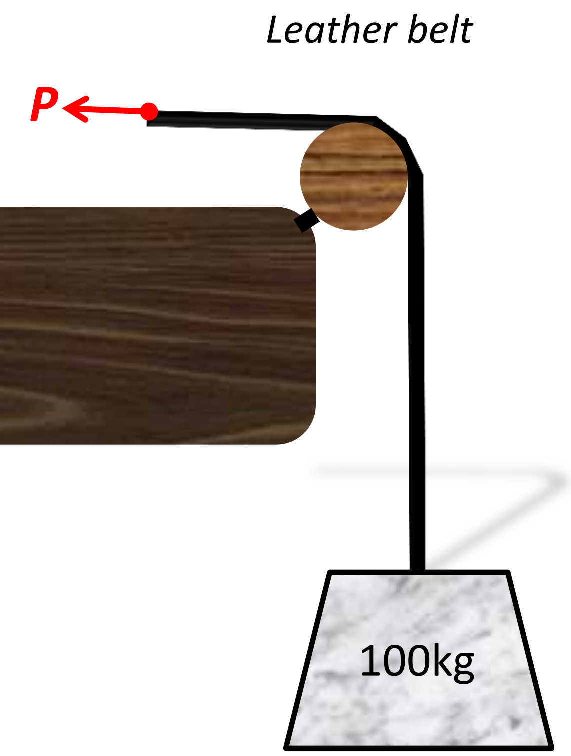
3. *Measurement of the critical angle φ_{max} will allow determination of the static coefficient of friction*

Example: Static Friction

- A weird physicist (Bush?) wraps a leather belt a quarter turn around a fixed wooden circular cylinder with radius a. The lower end holds a weight of 100kg.
  - *If the coefficient of static friction between the leather belt and the wood is μ_s=0.2, determine the force (P) needed to just begin to move the weight upward*

SOLUTION: To solve this we need to analyze the problem by considering a small portion of the belt in contact with the cylinder and applying calculus

→ Our goal is to get an equation for the "normal force" as a function of position around the wooden cylinder

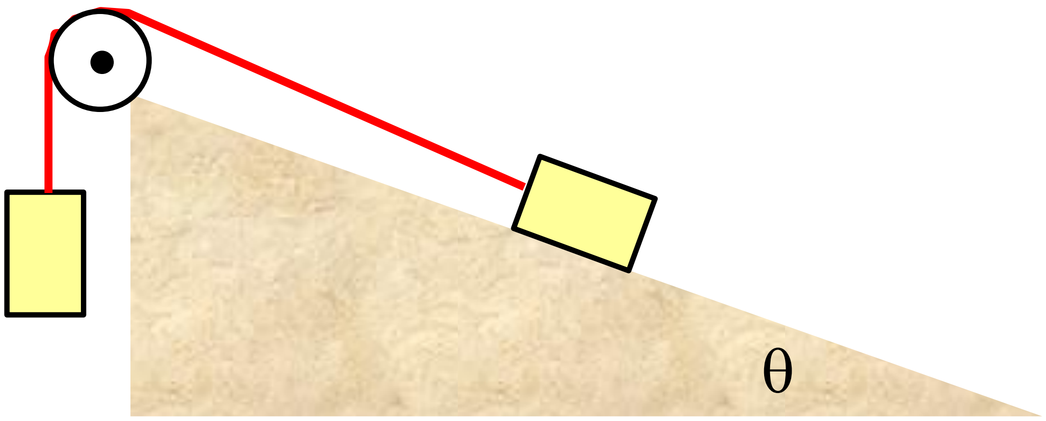


We are going to denote by $T(\theta)$ the tension in the belt segment at an angle θ and define the total normal (G_ρ) and tangental (G_θ) components of the force between the belt and the wooden cylinder

Capstan Winch at work

The worker can drag a large fallen tree across rough ground using only his hand to provide just enough tension to keep the rope from slipping . Friction between the rope and white cylinder does the heavy work of pulling the log.

Homework 2



- Two blocks of equal mass M are connected by a string which passes over a frictionless pulley. If the coefficient of dynamic friction is μ_K, what angle θ must the plane make with the horizontal so that each block will move with constant velocity once in motion ?

I do not know what I may appear to the world, but to myself I seem to have been only like a boy playing on the sea-shore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me.

Newton

Next time

Work and Potential Energy