Invited Paper

Ferromagnetic nanodisks for magnonic crystals and waveguides

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ABSTRACT

Spin wave excitations in dipolarly coupled nanodisks from permalloy are investigated. We address, both, one-dimensional (1D) chains and two-dimensional (2D) arrays consisting of nanodisks of different diameter. An out-of-plane magnetic field allows us to initialize the so-called vortex state in each of them. Our micromagnetic simulations show that in such 1D and 2D periodic devices the low-frequency excitations of the individual disks couple and form allowed frequency bands for spin-wave propagation. The devices operate as magnonic wave guides and magnonic crystals. The diameter is found to allow one to control both the center frequency and bandwidth of the allowed miniband in the few GHz frequency regime. We discuss a hybrid nanodisk device which might allow one to control and slow down the spin waves, i.e., the transmitted GHz signal.

Keywords: Spin waves, nanomagnets, permalloy, micromagnetic simulations

1. INTRODUCTION

Collective spin excitations in ferromagnets have regained great interest in the course of spintronics research. Recent observations such as spin-wave quantization,[1–7] localization and interference[8] have stimulated the field of magnonics where spin waves (magnons) are explored in order to carry and process information.[9,10] Spin wave propagation in individual ferromagnetic wires, i.e., magnonic wave guides,[11–23] and interconnected nanowire networks, i.e., antidot lattices,[24–29] has already been investigated extensively. The complementary structure, i.e., densely packed arrays of nanodisks,[30,31] has been studied for cellular automata applications[32,33] but rarely for spin wave propagation and possible magnonic crystal behavior with an artificial magnonic band structure.[34–36] Here we investigate the dynamic properties of periodic 2D arrays and 1D chains of nanodisks for wave guiding and magnonic crystal applications in the GHz frequency regime. In our micromagnetic simulations on densely packed chains and arrays with diameters ranging from about 200 to 800 nm we find that allowed minibands are created suggesting the coherent coupling of otherwise confined modes. The coupling allows spin waves to propagate across the chains and arrays of nanodisks. Velocities are of the order of km/s in the long wavelength limit. Relevant frequencies cover the range from about 1 to 2.5 GHz. The devices open intriguing perspectives for the manipulation of spin waves at the nanoscale and the realization of magnetic metamaterials in the GHz frequency regime.[37]

2. QUASISTATIC SPIN CONFIGURATIONS IN NANODISKS IN PERPENDICULAR FIELDS

The properties of chains and arrays of ferromagnetic nanodisks were studied using micromagnetic simulations.[38] We considered permalloy nanodisks of different diameter between about 200 and 800 nm. For the different linear chains and squared arrays of nanodisks we assume a constant edge-to-edge separation a of 13 nm between nearest neighboring nanodisks but different periods p. The diameters thus amount to $p - 13$ nm. In contrast to earlier studies,[35] we addressed magnetic states in out-of-plane fields H. This field configuration allowed us to create the vortex state which is in the focus of this paper. Note that an out-of-plane field is key to stabilize the vortex core in nanodisks where the diameter is small and the vortex core might not be formed in the remanent

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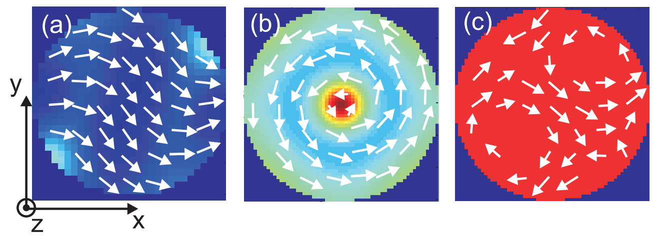

Figure 1. (Color online) Quasistatic spin configurations of a nanodisk with a diameter (thickness) of 187 nm (100 nm). Fields of (a) 66, (b) 333 and (c) 1100 mT are considered, applied in z direction. The color code is the same for each graph and presents the magnetization component M_z (red - maximum magnetization, blue - zero). The arrows indicate the orientation of the in-plane components components M_x and M_y. The arrows are normalized to the maximum in-plane component of each graph. For each state we first applied a large field to saturate the nanodisk near the out-of-plane direction. Then we relaxed the field to the value given in the caption. To consider the experimental situation where a strictly perpendicular field H might not be reached and a residual in-plane field component might break the in-plane symmetry we introduced a small in-plane component when generating the saturated state initially. The saturation field vector read (10 mT, -10 mT, 2.0 T). In the ripple-like spin configuration of (a) the magnetic moments are mainly in plane. In (b) a vortex core exists surrounded by a circular flux-closure configuration. In (c) the external magnetic field is strong and forces the magnetization perpendicularly to the plane. Still a small in-plane component exists.

state at $H = 0$.[30] The dynamics of a spin vortex mode in an individual magnetic element has been described theoretically by Thiele[39] and Huber.[40] The confined low-frequency spin excitation consists of a gyrotropic motion of the vortex core around its equilibrium position. The shift of the vortex core induces magnetic surface charges on the perimeter which oscillate in phase with the gyrotropic motion. For the ideal circular disk in zero magnetic field the eigenfrequencies of the clockwise and counterclockwise motion are degenerate. For mesoscopic nanodisks typical eigenfrequencies ν are in the range of a few 100 MHz to a few GHz depending on the diameter and thickness.

First we discuss excitation with a homogeneous radiofrequency magnetic field $h_{\rm rf}$, i.e., eigenfrequencies ν of spin excitations with a wave vector $k = 0$. Second, we consider inhomogeneous $h_{\rm rf}(x, y)$ allowing us to evaluate the dispersion $\nu(k)$ across the chains and arrays. The simulated structures consisted of 64 by 64 cells in the two in-plane directions for homogeneous excitation, and 64 by 4096 cells for an inhomogeneous excitation. Here, we applied $h_{\rm rf}$ only locally to a selected number of disks and monitored the propagation across neighboring disks. We implemented two-dimensional periodic boundary conditions. The permalloy nanodisks were subdivided into up to 63 cells in transverse direction. The additional minimum 1 cell next to the disks modeled vacuum to decouple neighboring nanodisks in transverse direction. The side length of the unit cell was 4.375 nm, i.e., smaller than the exchange length in permalloy. The chain length was infinite due to the periodic boundary conditions. The parameters were saturation magnetization $M_{\rm s} = 780$ kA/m, exchange constant $A = 1.3 \times 10^{-11}$ J/m and damping constant $\alpha = 0.005$. Time-domain simulations were performed for a time period of 6 ns starting from the relevant quasistatic configuration at each H. A field pulse of 8 mT was applied with the field pointing 45° out of plane. In case of the chain (array) the in-plane component was perpendicular to the long axis of the chain (the axis of the squared unit cell). The field pulse had approximately Gaussian shape lasting from 0.1 ns to 0.11 ns in the time frame (the full width half maximum of the pulse was 0.003 ns). Eigenfrequencies have been determined by fast Fourier transformation (FFT) of the time-domain data. We consider dynamics in the linear regime.[41]

Figure 1 depicts quasistatic spin configurations of a 100 nm thick $Ni_{80}Fe_{20}$ (permalloy) nanodisk which exhibits a diameter of 187 nm. The domain configuration is found to vary as a function of H. Considering the field-dependent frequencies of eigenmodes at $k = 0$ we attribute the three states to the three different field regimes defined in Fig. 2. For small H (regime I) we find a ripple-like spin configuration [Fig. 1(a)]. For intermediate H the disk enters the state incorporating a vortex core (vortex state) [Fig. 1(b)] (regime II). The core introduces a local out-of-plane magnetization component in the central position of the disk. Only at large fields the spins overcome the demagnetization effect in z direction and align with H throughout the nanodisk [Fig. 1(c)] (regime III).

The vortex state in regime II is interesting in that the spins follow a ring-like configuration mostly in the plane of the nanodisk. Here the stray-field is minimized if compared to regimes I and III. An appreciable out-of-plane

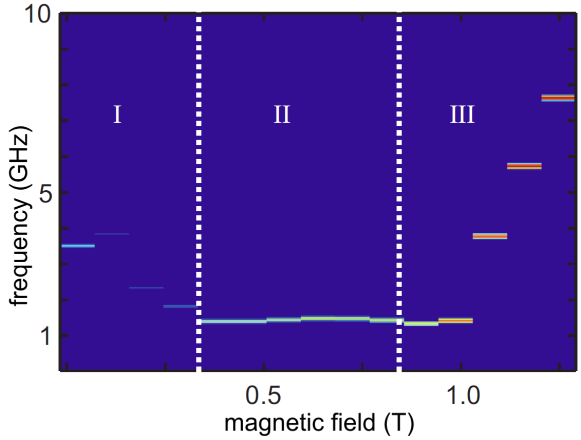

Figure 2. (Color online) Field dependent excitation spectra ($k = 0$) of the nanodisk array with $p = 200$ nm. Red (blue) color encodes resonant (non-resonant) excitation. H is applied in z direction. We highlight the three different field regimes, i.e., I with mainly in plane magnetization of the nanodisks, II with the vortex configuration, III with out of plane magnetization. The eigenfrequency ν in the vortex regime is around 1.5 GHz.

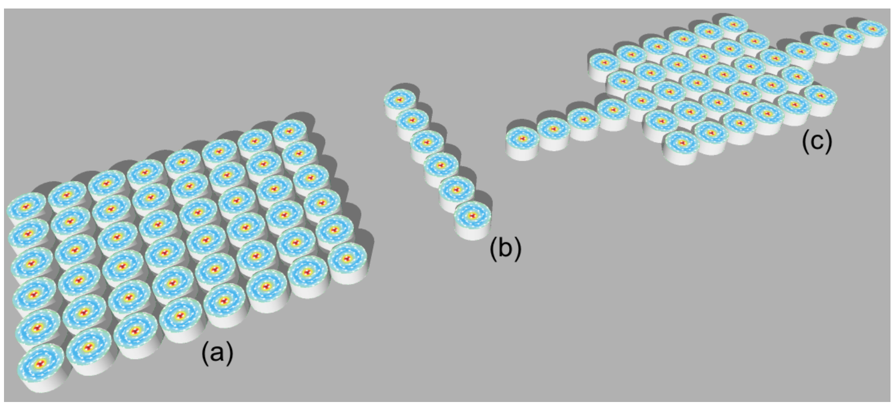

Figure 3. (Color online) Sketches of nanodisk samples: (a) 2D array, (b) 1D linear chain, and (c) hybrid device consisting of a linear chain where a segment incorporates a 2D array.

magnetization exists only in the central vortex core. In contrast to regimes I and III the vortex state in regime II obeys a magnetic configuration with a characteristic radial symmetry. This is key to, both, obtain radially symmetric coupling characteristics and form a 2D magnonic crystal with identical spin-wave dispersions $\nu(k)$ in orthogonal high-symmetry directions of the periodic lattice. This has not been achieved with nanodisks subject to in-plane magnetic fields.[36] In the following we will focus on the vortex state in a perpendicular field H and compare nanodisks of different diameter. With increasing diameter regime II is found to extend to smaller and smaller fields, i.e., for a diameter larger than 400 nm the vortex state is found to be stable down to $H = 0$.

3. SPIN WAVES AND SIGNAL TRANSMISSION IN NANODISK ARRAYS AND CHAINS

We will first discuss the dispersion and band structure $\nu(k)$ for the 2D array of densely packed nanodisks in the vortex state [Fig. 3 (a)]. We will discuss the miniband formation and, in particular, the propagation velocity for GHz signal transmission across the array. Then we will compare to the velocity found in a 1D linear chain of nanodisks where wave guiding is found. We focus on the propagation along nearest neighboring disks, i.e., the wave vector k is collinear with the unit cell sides of the periodic lattices.

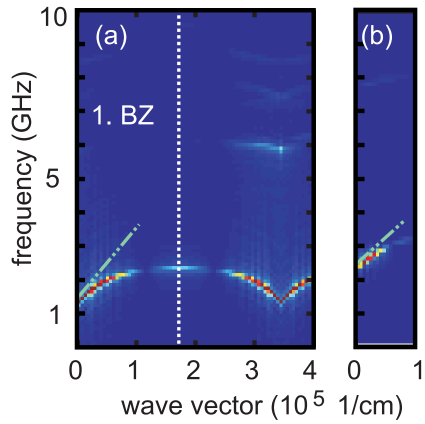

Figure 4. (Color online) (a) Simulated dispersion relation $\nu(k)$ of the permalloy nanodisk array in the vortex configuration at $\mu_0 H = 400$ mT. The period is 200 nm. The diameter amounts to 187 nm. We consider the wave vector k to be collinear with the edge of the squared unit cell of the periodic lattice of nanodisks. An allowed miniband is found with a bandwidth of $\Delta\nu = 1.0$ GHz. The boundary of the first Brillouin zone (1. BZ) is marked by the vertical dotted line. The group velocity $v_\mathrm{g} = 2\pi \frac{\partial \nu}{\partial k}$ is $v_g = 1.2\ \frac{\mathrm{km}}{\mathrm{s}}$ near $k = 0$ in the long wavelength limit (highlighted by the dash-dotted line). (b) Simulated dispersion relation $\nu(k)$ of the permalloy nanodisk chain. The overall frequencies are higher than in (a). The group velocity extracted from the dash-dotted line is $v_\mathrm{g} = 0.95$ km/s.

3.1 Allowed frequency minibands in nanodisk arrays

In Fig. 2 eigenfrequencies ν at $k = 0$ for the nanodisks with a diameter of 187 nm are shown. In regime I, i.e., for the ripple-like spin configuration, ν shows a significant field dependence. At about 0.4 T the frequency has decreased to about 1.5 GHz with increasing field H. Interestingly, in regime II the eigenfrequency remains almost constant near 1.5 GHz as a function of H. This reflects the resonant low-frequency gyrotropic motion of the vortex core. The in-plane component of the magnetization forms a flux closure configuration [Fig. 1(b)] and the out-of-plane field does not vary much the resonant condition of the vortex mode. As a consequence the eigenfrequency ν stays nearly constant. This is different in regime III where the nanodisk's spins tend to align with H beyond about 1 T and the eigenfrequency increases significantly with increasing H. This is the behavior expected for the uniform spin precession of a homogeneously magnetized body in a large magnetic field.[42]

It is now interesting to study spin-wave excitations with $k \neq 0$ in regime II. We consider $\mu_0 H = 400$ mT. The band structure $\nu(k)$ taken from the simulations is shown in Fig. 4 (a). We find a dispersive character for the low-frequency vortex mode. Starting from about 1.5 GHz at $k = 0$ (c.f. Fig. 2) the eigenfrequency first increases with increasing k, levels off and then decreases again. The eigenfrequency is a periodic function of the wave number k, i.e., in reciprocal space. The low-frequency branch thus reflects an allowed miniband of a magnonic crystal. It is formed due to dipolar interaction via oscillating surface charges.[35] By this means the spin vortex core excitation propagates through the array. For $d = 200$ nm and an edge-to-edge separation of 13 nm we find a bandwidth $\Delta\nu$ of about 1.0 GHz. When we analyze the group velocity v_g as a function k we find the largest value close to $k = 0$, i.e., in the long wavelength limit. Here, v_g amounts to 1.2 km/s. This is a reasonably fast signal transmission speed if compared to spin waves in an unpatterned thin permalloy film. There, depending on the relative orientation between the magnetization and wave vector the absolute value of v_g is known to vary between zero and a few km/s.[9] Near the Brillouin zone (BZ) boundary π/p the group velocity v_g is found to be zero in Fig. 4 (a).

In Figure 5 (a) we summarize bandwidths of allowed minibands simulated at 400 mT for nanodisk arrays of different p and the same edge-to-edge separation $a = 13$ nm. Increasing p is found to provoke a decreased bandwidth suggesting a reduced coupling between the gyrotropic vortex-core motion of the individual nanodisks. In the limit of very large p (or a) eigenexcitations are expected to be confined to the individual nanodisk being independent of k. With increasing p the eigenfrequency ν at $k = 0$ [Figure 5 (a)] decreases as well. Strikingly the group velocity v_g near $k = 0$ [figure 5 (b)] increases however. v_g becomes as large as 2.3 km/s for large p. Arrays of nanodisks exhibiting the vortex state thus allow one to create artificial band structures with tailored

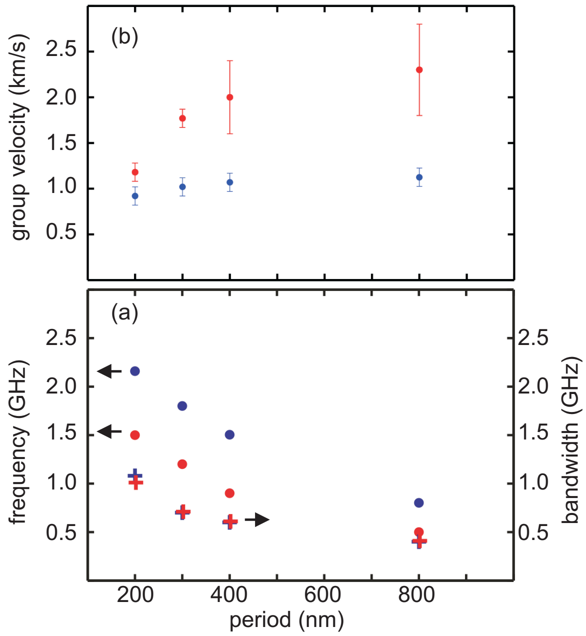

Figure 5. (Color online) (a) The upper (lower) filled circles show the eigenfrequency ν of the vortex mode at $k = 0$, i.e., Γ point, of the nanodisk 1D chain (2D array) as a function of the period p. The upper (lower) crosses represent the bandwidth of the nanodisk array (chain). Bandwidth and eigenfrequency are found to decrease with increasing p. The bandwidths are almost the same for the 1D and 2D arrangements. (b) The upper (lower) filled circles show the group velocity v_g of the nanodisk array (chain) extracted from the dispersions in the long wavelength limit, i.e., near the Γ point as highlighted by the dash-dotted lines in Fig. 4. The error bars reflect the uncertainty due to the discreteness of the wave vector values in the FFT spectra.

properties. Such magnonic crystals operating in out-of-plane magnetic fields are interesting due to the high symmetry for spin waves with wave vectors of in the in-plane directions. For periodic arrays of nanostructures exposed to in-plane fields the lattice symmetry created by the nanopatterning has been found to be reduced due to the inhomogeneous internal field.[26, 27, 43] This is not the case for the magnonic crystals considered here consisting of vortex-core states and out-of-plane magnetic fields.

3.2 Wave guiding in a nanodisk chain

We now consider a single straight chain of closely spaced nanodisks [Fig. 3 (b)] where h_{rf} is chosen such that only two of the nanodisks are excited. We follow the time evolution of this localized excitation and obtain the dispersion $\nu(k)$ from the FFT as shown in Fig. 4 (b). Due to the special form of h_{rf} the data show large contrast in the long wavelength regime in the first half of the first BZ. Here we are interested in v_g to compare with the 2D array. Again we find a relatively steep slope and relatively fast signal transmission speed as far as propagating spin waves are concerned. In the single chain of nanodisks with a diameter of 187 nm and p (a) of 200 nm (13 nm) we obtain a group velocity of v_g of 0.95 km/s. The single chain is thus found to form a magnonic waveguide in the GHz frequency regime. In the 1D chain the velocity v_g increases only slightly with p. Interestingly the wavelength $\lambda = 2\pi k$ of the relevant spin excitation is orders of magnitude smaller if compared to the wavelength of the corresponding electromagnetic wave. Coherent spin excitations in periodically patterned ferromagnets thus open the perspective of microwave guiding and circuitry on the nanoscale.

Note that the eigenfrequency at $k = 0$ is smaller for the 2D array of nanodisks in Fig. 4 (a) if compared to the 1D chain displayed in Fig. 4 (b). We attribute this to the neighboring rows of nanodisks which reduce the demagnetization effect of the individual nanodisk. By introducing additional nanodisks in transverse direction the restoring forces for a displaced vortex core are diminished if compared to a single chain and the eigenfrequency at a given field H is smaller. In Fig. 4 the discrepancy between (a) and (b) amounts to about 0.45 GHz. Note that the same frequency of about 2 GHz is obtained at $k = \pi/p$ in (a) and $k \approx 0$ in (b). The corresponding group velocities amount to $v_g = 0$ and 0.95 km/s, respectively. This observation might be used to manipulate the GHz signal transmission in nanodisks. Consider a long-wavelength spin excitation at 2 GHz propagating in a chain of nanodisks where, locally, one introduces a segment consisting of a 2D array of the same nanodisks sharing the same central line. Such a hybrid device is sketched in Fig. 3 (c). Entering the 2D array the spin wave acquires a larger wave vector (smaller wavelength) and slows down due to the modified band structure. Very recently a spin wave was transmitted from a large-area ferromagnet into a magnetic stripe.[23] There the change in wave vector was observed as assumed here. However, the propagation velocity was almost unchanged because the dispersion $\nu(k)$ of the stripe was still steep [c.f. Fig. 2 (e) in Ref.[23]]. A slowing down of the spin wave was not achieved. Hybrid devices of 1D and 2D nanodisk arrays thus offer further control of spin-wave propagation due to the tailored magnonic crystal behavior. In photonics slow light has already been addressed considering electromagnetic waves in periodically patterned dielectric materials, i.e., photonic crystals.[44]

4. CONCLUSIONS

We have investigated one- and two-dimensional arrays of permalloy nanodisks by micromagnetic simulations. The disks exhibit the vortex state. For periods and lattice constants of a few 100 nm we find spin-wave guiding and magnonic crystal behavior, respectively. In nanodisks the lattice symmetry is preserved even in an applied magnetic field. This opens further perspectives for tailoring magnonic crystals with artificial band structures with allowed minibands and forbidden frequency gaps. Our results suggest that the propagation velocity of GHz microwaves might be controlled on the nanoscale using magnetic devices which consist of an alternating sequence of 1D and 2D arrays of nanodisks. This opens further perspectives in the field of magnonics.

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