Stochastic Compactness of Lévy Processes

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ABSTRACT

We characterize stochastic compactness of a Lévy process at "large times", i.e., as $t \to \infty$, and "small times", i.e., as $t \to 0$, by properties of its associated Lévy measure. We shall say that a Lévy process X_t, $t \geq 0$, is in the Feller class at infinity (stochastically compact at infinity) if there exist nonstochastic functions $B(t) > 0$, $A(t)$ such that every sequence $t_k \to \infty$ contains a subsequence $t_{k'} \to \infty$ with

$$\frac{X_{t_{k'}} - A(t_{k'})}{B(t_{k'})} \xrightarrow{\mathrm{D}} Y', \tag{1}$$

where Y' is a finite nondegenerate rv, a.s. We shall say that a Lévy process X_t, $t \geq 0$, is in the Feller class at zero if there exist nonstochastic functions $B(t) > 0$, $A(t)$ such that every sequence $t_k \downarrow 0$ contains a subsequence $t_{k'} \downarrow 0$ for which (1) holds.(The prime on Y' signifies that in general it depends on the choice of subsequence $t_{k'}$.) In the large times case this is accomplished by a mechanism for transferring between discrete (random walk) and continuous time results. The multidimensional case is also described. As an illustration of stochastic compactness ideas, semi-stable laws and Lévy process are considered.

This talk is based on joint work with Ross Maller.