

LARGE DEVIATION ESTIMATES FOR EXCEEDANCE TIMES OF PERPETUITY SEQUENCES

D. BURACZEWSKI, J. COLLAMORE, E. DAMEK AND J. ZIENKIEWICZ

ABSTRACT. We consider large exceedance probabilities of the perpetuity sequence

$$Y_n = B_1 + A_1 B_2 + \cdots + (A_1 \dots A_{n-1}) B_n,$$

where (A_n, B_n) are i.i.d. random variables with values in $\mathbb{R}^+ \times \mathbb{R}$ and the exceedance times are defined as $\tau_u = \inf\{n : Y_n > u\}$. Applying techniques based on analyzing path behavior of Y_n we provide the asymptotics of the sequence $\mathbb{P}[\tau_u = \rho \log u]$, $\rho > 0$, $u \rightarrow \infty$. Remarkable analogies and differences to random walks are discussed.

The sequence Y_n plays a central role in a variety of problems arising in both pure and applied probability. Fundamental results concerning the tail behaviour of the a.s. pointwise limit of $Y = \lim_{n \rightarrow \infty} Y_n$ proved by Kesten and Goldie found an enormous number of applications.